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PROCEEDINGS

OF THE

Cambridge Philosophical Society.

VOLUME XV.

Cambridge:
PRINTED BY JOHN CLAY, M.A.
AT THE UNIVERSITY PRESS.

PROCEEDINGS

OF THE

CAMBRIDGE PHILOSOPHICAL SOCIETY.

VOLUME XV.

October 26, 1908—June 6, 1910.

Cambridge:
AT THE UNIVERSITY PRESS,
AND SOLD BY
DEIGHTON, BELL & CO. AND BOWES & BOWES, CAMBRIDGE.
CAMBRIDGE UNIVERSITY PRESS,
C. F. CLAY, MANAGER, FETTER LANE, LONDON, E.C.
1910

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PROCEEDINGS

OF THE

Cambridge Philosophical Society.

The Laws of Mobility and Diffusion of the Ions formed in Gaseous Media. By E. M. Wellisch, Emmanuel College. Communicated by Sir J. J. Thomson.]

[*Read* 9 November 1908.]

The information we possess with regard to the size and structure of the ions formed in gases by the action of Röntgen rays is almost entirely derived from a knowledge of their rates of diffusion and their velocities under given electric forces*. In order to obtain this information theoretical expressions are necessary for the coefficient of diffusion (D) of an ion through a gas and for its velocity (k) under unit electric intensity. From considerations based on the kinetic theory of gases Langevin† has shown that k and D are expressible in the forms:

$$k = \frac{eL}{MV}, \qquad D = \frac{LV}{3},$$

where e denotes the charge associated with the ion, M the mass of the ion, L its mean free path through the gas, and V its mean velocity of thermal agitation. From these forms however no very exact information can be deduced inasmuch as the values of the quantities involved cannot be directly determined from experiment. In the present paper expressions are given for k and D which involve only known physical constants of the gas and from which, therefore, we may expect to derive reliable information with regard to the constitution of the ion.

A *molecule* is regarded for the present purpose as a nucleus surrounded by a "sphere of force" of radius $\frac{1}{2}s$.

* J. J. Thomson, *Conduction of Electricity through Gases*, 2nd edit. Art. 38.
† *Ann. de Chim. et de Phys.* VII. 28, p. 324, 1903.

An *ion* is regarded as involving two distinct elements:

(i) a mass (independent of the charge). Considering this element alone, let us represent the ion as a nucleus surrounded by a sphere of force of radius $\frac{1}{2}s'$.

(ii) a charge e (electrostatic units). During motion of the ion through the gas the effect of this charge is (as will be shown) to increase the radius of the force sphere of the ion without altering the mass.

A *collision* occurs between any two molecules when the distance between their centres is equal to the sum of the radii of their force spheres.

As the ion moves through the gas the charge associated with it attracts the neutral molecules; there results an increase in the mean collision frequency of the ion and consequently a diminution in its mean free path.

Expression for the ionic mean free path.

Consider the motion of an ion and a molecule regarded in the light of two interacting free particles. Let R denote the potential due to the polarisation of the molecule by the charge on the ion*, so that the force, taken as wholly radial, between the ion and the molecule is given by $\frac{dR}{dr}$ at a distance r. Let us reduce the molecule to rest and consider the relative motion as the ion approaches it so that the velocity at infinity was U. The interacting forces must now be considered as being derived from a potential $\frac{M+m}{m}R$, m denoting the mass of the molecule.

The shortest distance r to which the ion and the molecule approach is given by the equations:

$$bU = ru,$$

$$\tfrac{1}{2}M(u^2 - U^2) = \frac{M+m}{m}R,$$

where u denotes the velocity of the ion in this position and b is the length of the perpendicular from the molecule to the rectilineal path of the ion.

We deduce

$$r^2 = b^2\left\{1 + \frac{R}{\frac{1}{2}\frac{Mm}{M+m}U^2}\right\}^{-1}.$$

* The polarisation of the molecule by the electric field is regarded as negligible in comparison with that due to the ionic charge.

A collision will take place if

$$b^2 \leqslant \sigma^2 \left\{1 + \frac{R_\sigma}{\frac{1}{2}\frac{Mm}{M+m}U^2}\right\},$$

where $\sigma = \frac{1}{2}s' + \frac{1}{2}s$, the sum of the radii of the force spheres of the ion and molecule.

If $R = 0$, i.e. if the polarisation of the molecule due to the ionic charge is negligible, the condition for a collision reduces to $b \leqslant \sigma$, as is otherwise obvious.

The connection between the velocity U and the mean thermal ionic velocity V can be deduced by the application of Maxwell's law of distribution of velocities. We obtain

$$U^2 = V^2\left(1 + \frac{M}{m}\right).$$

Hence $\frac{1}{2}\frac{Mm}{M+m}U^2 = \frac{1}{2}MV^2 = \frac{1}{2}mv^2$; thus $\frac{1}{2}\frac{Mm}{M+m}U^2$ is the mean kinetic energy of the molecular motion.

The effect of the polarisation due to the ionic charge is therefore, as far as collisions are concerned, to replace σ^2 by

$$\sigma^2\left\{1 + \frac{2R_\sigma}{mv^2}\right\}.$$

Now the mean free path of an uncharged body of the same mass and dimensions as the ion is given by $\left\{\pi n\sigma^2\sqrt{1+\frac{M}{m}}\right\}^{-1}$, where n denotes the number of molecules per c.c. Hence the actual mean free path of the ion is L where

$$L^{-1} = \pi n\sqrt{1+\frac{M}{m}}\,\sigma^2\left\{1 + \frac{2R_\sigma}{mv^2}\right\}.$$

Expression for the potential R due to the polarisation of the molecule by the ionic charge.

If the molecules of a gas are polarised by an electric field of intensity X, the electric moment per c.c. is $\frac{K-1}{4\pi}X$, where K denotes the dielectric constant of the gas.

The electric moment (μ) of a molecule is therefore $\frac{K-1}{4\pi n}X$.

The force on the molecule is $\mu\frac{dX}{dr}$, which is equal to $\frac{K-1}{4\pi n}\cdot\frac{dX^2}{dr}$. The potential is therefore given by

$$R = \frac{K-1}{8\pi n} X^2$$
$$= \frac{K-1}{8\pi n} \cdot \frac{e^2}{r^4},$$

when the molecule is polarised by the field due to the ionic charge.

This expression for R assumes that the polarising field is uniform throughout the volume of the molecule. Langevin* has obtained the general expression for R in the case of a spherical molecule and finds it to be given by a series of which the above is the most important term.

Expressions for the mobility and coefficient of diffusion of the ion.

Let η denote the coefficient of viscosity of the gas, ρ its density, p the pressure in dynes per sq. cm. and l the molecular mean free path. Let n_1, ρ_1, p_1, K_1 denote the values of n, ρ, p, K respectively corresponding to a temperature of 0° C. and a pressure of 760 mm. of mercury.

The charge e carried by the ion is taken as equal to the charge (E) on the monovalent ion in the electrolysis of solutions. This equality was established from measurements of the mobility and rate of diffusion of gaseous ions†. The exact value of the ionic charge is not required in the present treatment inasmuch as e only enters in the expression $n_1 e = n_1 E$, which has been shown from experiments in electrolysis to have the value $1{\cdot}30 \times 10^{10}$, E being measured in electrostatic units. The product $n_1 E$ is denoted by A.

The gas is supposed throughout to be at a temperature of 0° C. We have the following equations:

$$k = \frac{eL}{MV}; \quad D = \frac{LV}{3}$$
$$\eta = \tfrac{1}{3} nmvl$$
$$l^{-1} = \pi \sqrt{2}\, n s^2$$
$$L^{-1} = \pi n \sqrt{1 + \frac{M}{m}}\, \sigma^2 \left\{1 + \frac{2R_\sigma}{mv^2}\right\}, \text{ where } \sigma = \tfrac{1}{2}s' + \tfrac{1}{2}s$$
$$R_\sigma = \frac{K-1}{8\pi n} \cdot \frac{e^2}{\sigma^4}$$
$$MV^2 = mv^2 \text{ (equipartition of energy)}$$
$$e = E$$

* *loc. cit.* p. 317.

† For the evidence in support of this equality the reader is referred to J. J. Thomson, *Conduction of Electricity through Gases*, 2nd edit. Art. 39.

$$n_1 E = A$$
$$\rho = nm$$
$$p = \tfrac{1}{3}\rho v^2.$$

We deduce

$$k = \frac{A\eta}{\rho_1 p} 4\sqrt{2} \left(\frac{m}{M}\right)^{\frac{1}{2}} \left(1 + \frac{M}{m}\right)^{-\frac{1}{2}} \left(1 + \frac{s'}{s}\right)^{-2} \left\{1 + \frac{4(K-1)e^2}{\pi n m v^2 (s+s')^4}\right\}^{-1}.$$

When the mass and dimensions of the ion are the same as those of a molecule we have $M = m$, $s' = s$, and the expression for the mobility becomes

$$k = \frac{A\eta}{\rho_1 p} \left\{1 + \frac{(K-1)e^2}{4\pi n m v^2 s^4}\right\}^{-1}.$$

This expression can be transformed into

$$k = \frac{A\eta}{\rho_1 p} \left\{1 + \frac{(K_1 - 1)\pi A^2 \eta^2}{2 p_1^2 \rho_1}\right\}^{-1} \quad \ldots\ldots\ldots\ldots(\alpha).$$

In a similar manner we obtain for the case when $M = m$, $s' = s$,

$$D = \frac{\eta}{\rho} \left\{1 + \frac{(K_1 - 1)\pi A^2 \eta^2}{2 p_1^2 \rho_1}\right\}^{-1} \quad \ldots\ldots\ldots\ldots(\beta).$$

Consider the expressions (α) and (β) which have been found for the mobility and coefficient of diffusion through a gas of an ion regarded as a molecule carrying a charge equal to that associated with the monovalent ion in electrolysis.

For a given medium K_1, ρ_1 and p_1 are constant; whence k varies inversely as p provided η is constant. Now by Maxwell's law* the coefficient of viscosity of a gas is independent of its density; consequently over the range of pressures for which this law holds good we should expect the ionic mobility to vary inversely as the pressure, a conclusion which has been verified by experiment. Similarly we should expect the coefficient of diffusion to vary inversely as the density over the same range.

The expressions for k and D involve only known physical constants of the gas and are therefore directly comparable with the results of experimental observation. The results obtained by substituting the observed experimental values of the quantities involved are given in Table I. The values of the viscosity coefficients and of the dielectric constants have been taken from Landolt and Börnstein's Tables (3rd edition); the constant p_1 was taken as 1,013,610 (dynes per sq. cm.).

The seventh column in the table affords an indication of the effect on the mobility of the electric polarisation of the molecules by the ionic charge; it will be seen that the effect is quite

* *Vide* Jeans, *Dynamical Theory of Gases*, p. 252.

TABLE I.

1	2	3	4	5	6	7	8	9	10	11
Gas or vapour	Formula	Molecular weight	$\rho_1 \times 10^5$	$\eta \times 10^6$	$(K_1 - 1) \times 10^5$	$\frac{(K_1-1)\pi A^2\eta^2}{2p_1^2\rho_1}$	$\frac{1}{300}\left(\frac{A\eta}{\rho_1 p}\right)$	k_{760}*		
								Calculated	Observed +	Observed −
								cm./sec.	cm./sec.	cm./sec.
Air	—	—	129	177	59	3·70	5·87	1·25	1·36	1·87
Hydrogen	H_2	2	9	85	26	5·39	40·38	6·32	6·70	7·95
Carbon monoxide	CO	28	125	163	69	3·79	5·58	1·16	1·05	1·10
Nitrogen	N_2	28	125	163	59	3·24	5·58	1·31	1·6 †	
Oxygen	O_2	32	143	191	54	3·56	5·71	1·25	1·36	1·80
Carbon dioxide	CO_2	44	196	141	96	2·52	3·07	·87	·77	·81
Nitrous oxide	N_2O	44	196	141	107	2·80	3·07	·81	·79	·86
Ammonia	NH_3	17	76	96	770	24·13	5·40	·21	·70	·76
Ethyl alcohol	C_2H_6O	46	205	83	940	8·16	1·73	·19	·32	·26
Ethyl chloride	C_2H_5Cl	64·5	288	93	1554	12·06	1·38	·11	·32	·30
Ethyl ether	$C_4H_{10}O$	74·1	330	69	742	2·77	·89	·24	·28	·30
Carbon tetrachloride	CCl_4	153·8	686	153	426	3·76	·96	·20	·29	·30

* With the exception of H_2, N_2, O_2 and air, the observed mobility values are the results of experiments, not yet published, recently made by the writer.

† Mean value.

considerable. Column 8 gives the values of the mobilities under a potential gradient of 1 volt per cm. which would be possessed by a molecule carrying a charge E if there were no retarding effect due to this polarisation. The remaining columns give the values of the mobilities as deduced from expression (α) together with the observed experimental values of the positive and negative ionic mobilities; these values correspond to a real or hypothetical pressure of 760 mm. of mercury.

Unfortunately the values of K have been determined experimentally for only a very limited number of gases and vapours; in consequence there do not appear in the table several vapours whose mobilities have been ascertained.

Considering that we are comparing the observed values of the mobility with absolute values calculated from various physical constants of the substances the agreement is in several cases quite satisfactory; however, in the case of ammonia and the vapours, there is a marked divergence between the calculated and observed values, the former values being invariably smaller. The values of the dielectric constants for the vapours appear to be extraordinately large, and, in this connection, it is interesting to note that it is also in the case of vapours that there is a marked departure from Maxwell's law, $K = n^2$, where n is the refractive index. The following table will serve as an illustration:

TABLE II.

Gas	K	$K^{\frac{1}{2}}$	n
H_2	1·000264	1·000132	1·000138
CO	1·000690	1·000345	1·000340
CO_2	1·000960	1·000480	1·000450
N_2O	1·001070	1·000535	1·000500
Air	1·000590	1·000295	1·000294
NH_3	1·0077	1·0038	1·000370
C_2H_5Cl	1·0155	1·0077	1·0010
C_2H_6O	1·0094	1·0047	1·00086

In the case of Maxwell's law the reason for the departures is probably the existence in the molecule of free periods with durations long compared with that of the incident wave period*. It is possible that, in the case under consideration, owing to free periods of long duration in the molecules of the vapours, these

* *Vide* Drude, *Theory of Optics*, Eng. trans. p. 389.

have not time to be completely polarised by the adjacent ionic charge during a collision. It is worthy of mention that the large values of K which have been experimentally obtained for ammonia and vapours have been ascribed by some authorities as being partially due to traces of conductivity which they possess.

Without dwelling further on the cause of these departures from the mobility law, it appears that, on the whole, the experimental values of the mobilities of the ions can be explained approximately on the supposition that the ion consists of a single molecule of the gas, with which is associated a charge equal to that carried by the monovalent ion in electrolysis. It has been shown from considerations based on the kinetic theory of gases that, irrespective of any theory as to the structure of the ion, the ascertained mobility values lead of necessity to the conclusion that the volume of the ion is in all cases greater than that of the corresponding gaseous molecule. The question arises: what is the nature of this volume? On the one hand, if we neglect the influence of the charge on the mean free path of the ion, we are led to the conception of the ion as a cluster of molecules held together by forces arising from the polarisation due to the electric charge. On the other hand, the effect of the charge on the collision frequency has been shown to be equivalent to an increase in the molecular sphere of force such that the resultant effective volume is sufficient to explain approximately the observed mobilities. On this view the effect of the charge is to cause the ion itself and the neighbouring molecules to deviate from their rectilineal free paths; Sutherland*, by assuming such deviations to occur in the case of gaseous molecules by reason of attractive forces between them, was able to explain accurately the observed variation of the viscosity of gases with temperature. Langevin† has obtained an expression for the ionic mobility by using the dynamical method employed by Maxwell in the kinetic theory of gases; he concluded that the experimental values of the mobilities lead to the necessity of regarding the ion as a cluster of molecules. The question as to the nature of the volume of the ion as determined from the experimental mobility values could be decided if the ratios $\frac{e}{m}$ for the different gaseous ions were known. In this connection it is worthy of mention that Professor Sir J. J. Thomson‡ has recently measured this ratio for the positive ions formed by cathode particles in oxygen at low pressures and came to the conclusion that the ion consisted of a single charged oxygen molecule.

* *Phil. Mag.* v. 36, p. 507, 1893.
† *Ann. de Chim. et de Phys.* VIII. 5, p. 270, 1905.
‡ *Phil. Mag.* v. 16, p. 680, 1908.

In the course of recent experiments made by the writer to ascertain the ionic mobilities in a series of gases and vapours it was found that the mobility (k) varied inversely as the pressure (p) over a wide range of pressures; however, there was observed a tendency of the product pk to increase at low pressures in the case of air, nitrous oxide and carbon dioxide, and a tendency for it to diminish in the case of vapours, e.g. ethyl chloride, when the pressures approached the vapour pressure at the temperature under consideration. Such deviations from the law pk = constant could be ascribed to variations in the size of the cluster constituting the ion; however, they follow readily from the expression (α) of the mobility if we take into account the deviations from the law of Maxwell which states that the coefficient of viscosity of a gas or vapour is independent of its density. In the case of gases it is known that η tends to diminish as the pressure is reduced beyond a certain value*; such a diminution would, according to the theory here given, tend to increase the value pk. In the case of vapours it has been established that η increases rapidly as the saturated state is approached; in fact, as a result of Warburg and von Babo's experiments on the viscosity of carbon dioxide at high pressures, Meyer† came to the conclusion that the experimental values above certain pressures could be explained only by supposing carbon dioxide to behave as a liquid, the density of which is practically independent of pressure; an increase in the value of η would, according to expression (α), diminish the product pk, a result in accordance with experimental observation.

In conclusion, it is of interest to note that, on the supposition given above as to the nature of the ion, a satisfactory agreement can be obtained between the calculated and observed values of the coefficients of diffusion of the gaseous ions. The values of these coefficients have been determined by Townsend‡ for four gases. Instead of using the ascertained expression (β) which is really involved implicitly in the expression (α) of the mobility, a slightly different procedure is advisable.

If D denote the coefficient of diffusion of the ion through the gas, and d that of a molecule through the gas, we then have from the preceding theory

$$\frac{D}{d} = \frac{L}{l} = \left\{1 + \frac{(K_1 - 1)\pi A^2 \eta^2}{2p_1^2 \rho_1}\right\}^{-1}.$$

The nature of the agreement between the calculated and observed values is shown in the following table:

* *Vide* Jeans, *Dynamical Theory of Gases*, p. 253.
† *Kinetic Theory of Gases*, Eng. trans. Art. 90.
‡ *Phil. Trans.* A. 193, p. 129, 1900.

Table III.

Gas	$\frac{(K_1-1)\pi A^2\eta^2}{2p_1^2\rho_1}$	d* observed	D		
			Calculated	Observed +	Observed −
Air	3·70	·150	·032	·028	·043
H_2	5·39	1·31	·205	·123	·190
O_2	3·56	·189	·041	·025	·040
CO_2	2·52	·109	·031	·023	·026

* Calculated from Loschmidt's observations. *Vide* Jeans, *loc. cit.*, p. 274.

The Radioactivity of Rubidium. By NORMAN CAMPBELL, M.A., nity College.

[*Read* 9 November 1908.]

1. In the first paper by Mr Wood and myself* on the radio- ivity of the alkali elements, it was stated that rubidium as well potassium was found to be radioactive, and that the rays from former metal were less penetrating than those from the latter. ring to the difference in the penetrating powers of the rays, the ative activity of the two elements can not be determined directly a comparison of the total ionisation caused by the rays from hick layer, but it can be determined by measurements of the ivity of layers of varying thickness. Such measurements on assium sulphate were described in a recent paper†: similar asurements have now been made upon rubidium sulphate.

2. The experiments were made in a manner exactly similar those on the potassium salt, and the same apparatus was used. e sample of rubidium sulphate used was specially prepared by hlbaum: its purity was tested by precipitation by barium oride: the weight of the sulphate precipitated agreed within limits of error (1 %) with that predicted from the accepted mic weight of Rb. Some difficulty was experienced in obtaining ers of the requisite thinness and uniformity: the method opted ultimately consisted in scattering the finely powdered t over the surface out of a vessel of the nature of a pepper-pot. The following table gives the results obtained:

ass of salt in tray (grammes)	Activity	Mass of salt in tray (grammes)	Activity
6·20	47·3	29·31	87·3
10·69	65·2	41·66	90·0
13·44	67·4	43·95	90·5
16·56	75·1	52·76	93·3
16·81	73·7	63·06	94·7
22·58	79·5		

These experimental points lie less closely on a theoretical ex- nential curve than the corresponding points for the potassium t: the rubidium rays are probably less homogeneous.

* *Proc. Camb. Phil. Soc.* XIV. 1, pp. 15—21. † *Ibid.* XIV. 6, pp. 557—567.

3. The method of calculation explained on pp. 563, 565 the last paper give directly I_0, the activity of an infinitely thi layer of the area (σ) of the tray, and the product $\frac{\lambda}{\rho\sigma}$, where λ the absorption coefficient of the material for the rays it emi and ρ the density of the material. If α is the ionising power the total radiation from 1 gramme of the material, when all th radiation is absorbed in air, then

$$\alpha = \frac{I_0}{\sigma} \cdot \frac{\lambda}{\rho}.$$

For rubidium metal we obtain

$$\alpha = 14{\cdot}47 \pm 0{\cdot}365, \quad \frac{\lambda}{\rho} = 53{\cdot}2 \pm 2{\cdot}1.$$

For potassium metal, using the figures given in the last paper, w obtain

$$\alpha = 2{\cdot}003 \pm {\cdot}0376, \quad \frac{\lambda}{\rho} = 8{\cdot}23 \pm 0{\cdot}1.$$

It appears then that, though, for layers infinitely thick an containing the same proportion of the active metal, the activit is slightly greater for potassium than for rubidium, the intrins activity of the latter metal is at least seven times as great as th of the former.

It will be noted that the statement that the penetratin power of the rubidium rays is considerably less than that of th potassium rays is confirmed. The difference is so notable that may be discovered readily by covering the active layer with a shee of stout paper. Such paper absorbs half of the rubidium rays, bu not more than 5 °/₀ of the potassium rays. It is clear, therefor that the activities of the two elements cannot be attributed to common constituent.

Prof. McLennan and Mr Kennedy* have stated that they hav been unable to detect any activity in rubidium salts. But, sinc they have revised some of their statements concerning the activit of the alkali metals, it is not impossible that agreement will b obtained in this matter also.

* *Phil. Mag.* Sept. 1908, p. 377.

On the Free Pressure in Osmosis. By L. VEGARD, Cand. al. (Research Student at the Cavendish Laboratory). [Communicated by Sir J. J. Thomson.]

[*Read* 9 November 1908.]

1. In the introduction to their paper on osmotic pressure* erkeley and Hartley mention that a method for studying osmotic henomena is given by the determination of what they call the motic force, and this method "depends on the determination of e rate at which the solvent will flow through a semipermeable embrane into a large quantity of solution when there is no ressure on the latter. A knowledge of this rate together with e frictional resistance will enable the osmotic force to be lculated in absolute units."

The authors, however, do not enter further into the question, nd we get from this no clear idea as to the strict definition of the motic force.

The following paper contains some experiments done in this rection as well as some conclusions to be drawn from the xperiments. The experiments consist in some determinations the frictional resistance and of velocities produced by osmosis hen solution and solvent are exposed to the same pressure. The ell used is the same as used in the experiments upon osmosis escribed in an earlier paper†.

Determination of the Frictional Resistance.

2. The frictional resistance depends on the velocity and is easured by the pressure that must be applied in order to press he solvent through the membrane with the velocity under conderation. If pressure and velocity are denoted by Q and V spectively we have $Q=f(V)$ provided temperature is kept conant. To find this function we have simply to determine the elocity corresponding to different pressures.

The measurements were made with the same osmometer as escribed in an earlier paper and the way of proceeding the same s used for the determination of osmotic velocities only that now he solution was replaced by the pure solvent. The relation etween Q and V has been determined in two sets of measure-

* The Earl of Berkeley and E. G. J. Hartley, *Phil. Trans. Roy. Soc.* 1906, 481.

† L. Vegard, *Phil. Mag.* Vol. XVI. No. 92, 93, 1908.

ments corresponding to a considerable difference in the maximum electric resistance of the membrane. Before each experiment the cell was electrolysed after which it was placed for about thirty hours in distilled water in order to remove possible substances soluble in water. The measurements gave the following result:

TABLE I.

Experiment 1. Date: Middle of May. Electr. Resist. 350,000 ohms at 2° C. Temp. 15° C.		Experiment 2. Date: June 8. Electr. Resist. 130,000 ohms at 7° C. Temp. 15·8° C.	
Q	*V*	*Q*	*V*
3·21 atm.	39 mm./hour	4·33 atm.	78 mm./hour
7·04 „	155 „	8·19 „	155 „
14·5 „	300 „	12·2 „	270 „
26·4 „	580 „	21·0 „	420 „

Determinations of Osmotic Velocities.

3. The velocities were measured with the osmometer earlier described for some aqueous solutions of cane sugar and no pressure was applied on the solution. The measurements including also the determination of the frictional resistance in Exp. 2 were carried out as quickly as possible in succession and in the order of increasing concentrations. The membrane was not electrolysed between each trial. Thus it was secured that the properties of the membrane were kept as nearly as possible the same under the whole series of measurements. The results are given in the following table.

The temperature was kept at 15° C. which very nearly was the temperature of the room.

The velocity is here given in an arbitrary unit as the number of mm. which the water level in the capillary from the cell* changes in one hour. For the sake of comparison with other experiments we shall find how this unit is connected to the absolute unit. To find this we must know the area of the membrane (O) and the inner diameter (d) of the capillary. For the apparatus used in Cambridge $O = 17{\cdot}0$ cm.^{2} (very nearly) and

* See description of apparatus, *loc. cit.* p. 248.

TABLE II.

Concentration, gr./litre 15°	Osmotic Pressure, Atmos.	Time for observation of velocities	Mean time corresponding to the velocity	Osmotic velocity, mm./hour
150	12·1	June 9, 3.23 p.m.—4.23 p.m.	3.53 p.m.	155
		„ 4.25 „ —6.11 „	5.18 „	152
300	28·0	June 10, 11.10 a.m.—11.28 a.m.	11.19 a.m.	366
		„ 11.35 „ —11.45 „	11.40 „	336
		„ 11.45 „ —11.51 „	11.48 „	320
		„ 11.51 „ —12.31 p.m.	12.11 p.m.	286
		„ 12.31 p.m.—12.55 „	12.43 „	268
		„ 12.55 „ — 1.18 „	1.06 „	252
		„ 1.48 „ — 3.16 „	2.32 „	244
450	52·3	June 10, 3.23 p.m.—3.46 p.m.	3.35 p.m.	402
		„ 3.46 „ —4.14 „	4.00 „	354
		„ 4.14 „ —4.58 „	4.36 „	330
		„ 5.01 „ —5.28 „	5.15 „	328
600	85·6	June 11, 10.22 a.m.—10.28 a.m.	10.25 a.m.	770
		„ 10.28 „ —10.34 „	10.31 „	700
		„ 10.34 „ —10.40 „	10.37 „	660
		„ 11.12 „ —11.35 „	11.24 „	506
		„ 11.35 „ —12.10 p.m.	11.53 „	468
		„ 12.16 p.m.—12.50 „	12.33 p.m.	458
		„ 12.55 „ — 2.01 „	1.28 „	421
		„ 2.05 „ — 3.08 „	2.37 „	398
		„ 3.12 „ — 4.12 „	3.42 „	378

$d = 0{\cdot}060$ cm., then if the velocity in absolute and relative units is denoted by V' and V respectively, we find

$$4{\cdot}61 \,.\, 10^{-9} V = V' \text{ [cm. sec.}^{-1}\text{]};$$

V' can either be considered as the number of cm.^{3} passing each cm.^{2} in one second or as the average linear velocity of the fluid close to the membrane.

Discussion of the Experiments.

4. The relation between the friction pressure and the velocity is represented in Fig. 1 B curve II, corresponding to Exp. 2, Table I. We see that as far as the observations go the points are distributed round a straight line. Consequently we have

$$Q = A V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where A is a constant characteristic of the membrane and the fluid, and dependent on the temperature. We shall call it the Specific Resistance of the system.

The law for the motion through the membrane is the same as that for the motion of a viscous fluid through narrow tubes. This fact is simplest explained by assuming that the liquid also during the passage through the membrane maintains the character of a liquid. Then in points inside the tubes of the membrane the fluid should possess a certain hydrostatic pressure and the cause of the motion at each point should be found in the pressure gradient at the point.

The values of the specific resistance corresponding to the two experiments are—using the method of least squares—found to be:

Exp. 1. $A = 0{\cdot}0463$ or $A' = 1{\cdot}02 \,.\, 10^{13}$ [dyn. cm.$^{-1}$ sec.$^{-1}$].

Exp. 2. $A = 0{\cdot}0492$ or $A' = 1{\cdot}08 \,.\, 10^{13}$ „

A corresponds to the units given in the table, A' to absolute units.

We see that we get nearly the same value for the specific resistance in the two cases and that in spite of the fact that the maximum electric resistance in the last experiment was not half of that in the first one, where the frictional resistance even is somewhat smaller. Thus the maximum electric resistance seems to give a very delicate test for the permeability of the membrane. When the membrane first is well formed the mechanical resistance against the water flow will be very little altered for very considerable changes in the maximum electric resistance.

5. In Fig. 1 A the measurements given in Table II are represented by curves giving the velocity as a function of time reckoned from the time when the solution was brought into contact with the membrane. This time was not exactly observed so there may be a fault of a few minutes in the time of beginning.

We see that the velocity begins with a rather large value, but falls very rapidly until at a certain value the curve makes a very sudden bend. In the case of the three lowest concentrations the curve assumes a horizontal direction showing that the osmosis has reached a steady state. In the case of the highest concen-

tration, however, the velocity after the sudden bend of the curve continues to fall.

The condition for a steady state and the time taken for reaching it depend on several circumstances. A nearer study of this question would require a special research. I shall here only call attention to some circumstances which seem to have an effect.

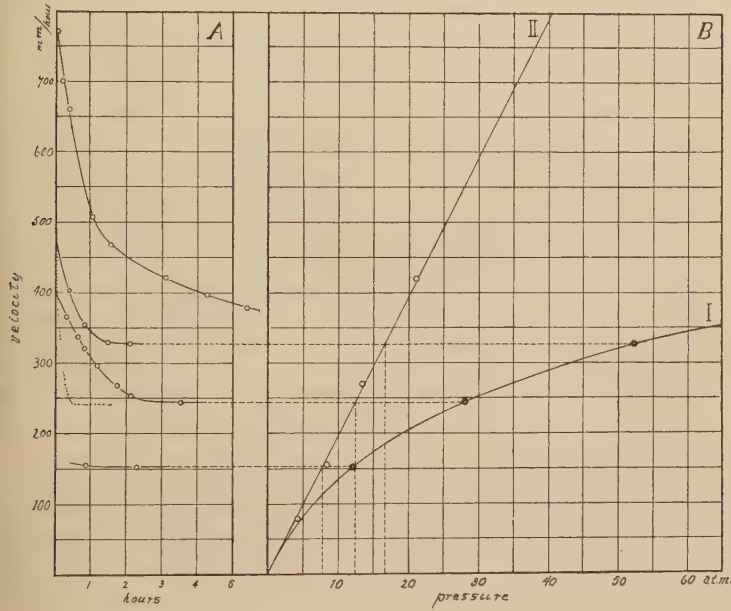


Fig. 1.

The time depends to some extent on the dryness of the membrane at the moment when the solution is brought into contact with it. Thus the membrane in the second experiment, Table II, was dried with filter paper while in the third the membrane was just so wet as to give a regular reflection of light. The result was that the steady state occurred much sooner in the third experiment and that in spite of the fact that the concentration was greater.

The steady state sets in more suddenly and is better marked for dilute solutions than for more concentrated. We shall later come back to this question. Further the steady state is better marked and more suddenly obtained when the membrane is more perfect. So *e.g.* in the measurements described in the earlier paper, where the maximum resistance was much greater, the stationary state in all cases was reached within half-an-hour. For

the sake of comparison the curve for experiment II* is put upon Fig. 1 A as a dotted line.

The power of giving a well-marked steady state after this first sudden bend of the curve is a quality that does not belong to any membrane showing osmotic activity. Even for the system here under consideration it appears to be a matter of degree just as the power of giving a reversion pressure near to the osmotic pressure is a power only possessed by the very best membranes. It will surely be of great interest to see how the different systems behave in this respect.

6. When the steady state is well defined the velocity corresponding to this state can be considered as a function of concentration, temperature, and further of some quantities dependent on the membrane and the fluid. Instead of concentration we can introduce the osmotic pressure π_0. As long as the properties of the membrane and the temperature can be considered as constants we are led to consider the variation of the velocity λ_0 with the osmotic pressure of the solution. The relation between π_0 and λ_0 is represented in Fig. 2 B, curve I. On the same figure is also drawn the curve giving the relation between the velocity and the friction pressure (curve II).

We see that curves I and II stand in a very characteristic relation to each other. *The friction line is a tangent to the curve at the zero point.*

A line parallel to the π_0 axis cuts the curves in two points $(\pi_0 \lambda_0)$ and $(Q\lambda_0)$ which we shall call corresponding points, then Q is the pressure necessary to force the pure solvent through the membrane with a velocity equal to the osmotic velocity λ_0 called forth by a solution of osmotic pressure π_0. In general we have for corresponding points $\frac{Q}{\pi} < 1$, and $\frac{Q}{\pi}$ will decrease for increasing velocities; but when the velocity decreases towards zero we get

$$\operatorname{Lim}_{\lambda_0=0} \left(\frac{Q}{\pi_0}\right) = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Let Q, π_0, λ_0 be values belonging to corresponding points.

If we calculate the quantity $\frac{\pi_0 - Q}{\pi_0 \lambda_0}$ for different values of λ_0 we find that it gives a constant value. Remembering that $Q = A\lambda_0$ we get the following simple equation for the curve

$$\pi_0 = \frac{A\lambda_0}{1 - \frac{\lambda_0}{\lambda_m}} = A\lambda_m \frac{\lambda_0}{\lambda_m - \lambda_0} \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

* L. Vegard, *loc. cit.* p. 404.

The curve is determined by two parameters A and λ_m. Of these it is only the first one for which we have a physical interpretation. As regards λ_m we see from the equation that when λ_0 approaches λ_m, π_0 approaches infinity. *Thus λ_m is the upper limit for the osmotic velocity in the stationary state.* As to its physical interpretation it will depend on A, further it must depend on the coefficient of diffusion for cane sugar in water; for it is evident as the velocity is to be produced by the action of the solution the velocity must be so small that the solution is able to maintain a certain concentration in the layer next to the membrane.

Some values of λ_m calculated for a series of velocities from the curve directly observed are given in the following table.

λ_0	λ_m
100 mm./hour	470 mm./hour
200 „	430 „
300 „	463 „
350 „	480 „

Mean value 461 mm./hour = $2{\cdot}13 . 10^{-6}$ [cm. sec.$^{-1}$].

7. From the very intimate relation between the friction line and the velocity curve we can draw some important conclusions as regards the mechanism of osmotic flow.

The work (a) required in unit time for pressing the water through the membrane with a velocity λ_0 is

$$a = \kappa Q \lambda_0,$$

where κ is a constant dependent on the units. In the case of osmosis we must be able to assume that in order to bring the water through with a velocity λ_0 an amount of energy (a') is required which cannot be less than the energy required for pressing pure water through with the same velocity. Then we must have

$$a' \geqq a.$$

But on the other hand the maximum of energy E which the system can deliver in unit time is equal to the osmotic pressure multiplied with the volume of solvent, which enters into the solution in unit time, or

$$E = \kappa \pi_0 \lambda_0.$$

Now the work (a') necessary to call forth the motion of solvent through the membrane cannot be greater than E, consequently

$$\kappa \pi_0 \lambda_0 \geqq a' \geqq \kappa Q \lambda_0.$$

This relation holds for all velocities. Letting λ_0 converge towards zero we get

$$1 \gtreqless \operatorname*{Lim}_{\lambda_0=0}\left(\frac{a'}{\kappa\pi_0\lambda_0}\right) \gtreqless \operatorname*{Lim}_{\lambda_0=0}\left(\frac{Q}{\pi}\right).$$

Regarding equation (2)

$$\operatorname*{Lim}_{\lambda_0=0}\left(\frac{a'}{\kappa\pi_0\lambda_0}\right) = 1$$

or

$$\operatorname*{Lim}_{\lambda_0=0} a' = \kappa\pi_0\lambda_0 = \kappa Q\lambda_0 = a = E \quad \ldots\ldots\ldots\ldots\ldots\ldots (4).$$

Thus we see that for small velocities the whole energy of the system is made available for the motion through the membrane and the work (a') required for the motion under osmosis is just the same as the work (a) required for pressing the pure solvent through with a velocity equal to the osmotic velocity.

As the work is the same we must assume that also in the case of osmosis *we have a motion of pure water through by far the greatest part of the membrane.* But the motion of pure water must have its cause in the fall of hydrostatic pressure in the direction of the motion just sufficient to counteract the friction corresponding to the osmotic velocity.

8. From this we are led to consider the variation of hydrostatic pressure through the membrane. In those cases where there is a layer next to the solvent where there is only pure solvent we shall in this layer have a fall of pressure in the direction of the motion, and when the pressure on the solution is less than the corrected reversion pressure (π_0')* we find that if we pass through the membrane from the side of the solvent the hydrostatic pressure will first fall to a minimum value and then increase to the pressure of the solution. In the latter part the motion takes place against the pressure fall and cannot be a motion of the fluid in bulk, but is an intra-molecular motion maintained by the energy made available when the two liquids are brought into contact.

The difference between the pressure on the solvent and the minimum pressure we shall call the *free pressure* of osmosis. The general features for the variation of hydrostatic pressure through the membrane is indicated in Fig. 2. AF_1, AF_2 etc. represent the free pressure.

If the distance from the minimum point to the surface next to the solution is nl, where l is the average thickness of the membrane, then an approximate value for the free pressure q is given by the equation

$$q = (n-1)\,A\lambda.$$

* See L. Vegard, *loc. cit.* p. 264.

The pressure gradient is $\frac{A}{l}\lambda$ and if we knew l this could be found; but at all events we see that as far as $\frac{A}{l}$ can be considered constant λ would give a relative measure for the pressure gradient near the solvent surface. In the case where n is a very small quantity we get

$$q = A\lambda = \text{Friction Pressure} \ldots\ldots\ldots\ldots\ldots\ldots (5).$$

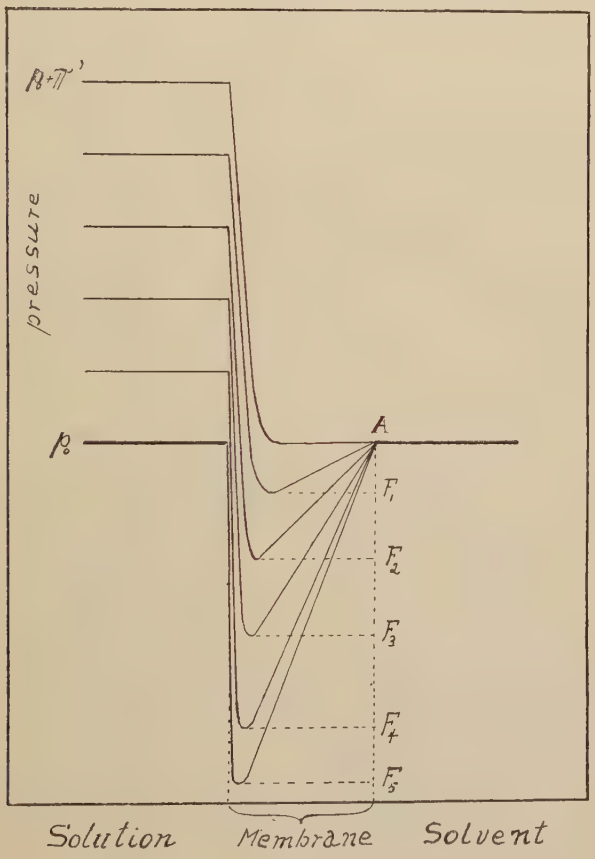


Fig. 2.

In the case of our system we have seen that for small velocities n is very small and the free pressure is determined by equation (5). Regarding equation (2) we further see that *for small velocities the free pressure is equal to the osmotic pressure.*

9. In order to explain the properties shown by the osmotic velocity we are led to assume that also in the case of higher concentrations the quantity n will be very small in all those cases

where the stationary state is well marked. We shall here mention some reasons that strongly support this view.

(1) The velocities for higher concentrations are connected to those for lower with a very simple function, which naturally must support the assumption of a similar mechanism.

(2) The form of the curve in the case when the stationary state is not obtained shows clearly that the first sudden diminution of the velocity has a cause independent of that which causes the diminution after the sudden bend. The first diminution is naturally explained by the fact that the concentration next to the membrane is diminished on account of the flow of solvent, the latter is naturally explained by assuming that the solution on account of the high free pressure and of want of semipermeability is gradually forced into the membrane; for then the path along which the motion takes place as a diffusion will be augmented and the velocity diminished. From this point of view it seems to be a necessary condition for the forming of a well marked steady state that n maintain a small value.

(3) Another support for our assumption we get by comparing the velocities here found with the velocities corresponding to $\pi = 0$ given in the earlier paper*. In spite of the great difference in the maximum electric resistance of the membrane all the velocities very nearly give the same curve. On the other hand we saw that the frictional resistance kept very nearly constant from one experiment to another. Now the value of n, however great or small it is, must depend on the degree of semipermeability or upon the maximum electric resistance. If then n had a considerable value, we should expect that the velocity in the stationary state should vary greatly with the electric resistance, this being not the case we must assume n to be a small quantity.

10. As long as n is a small quantity the free pressure is determined by equation (5) and we get the following rule:

Let $(q\lambda_0)$ *and* $(\pi_0\lambda_0)$ *be corresponding points, then* q *is the Free Pressure developed in the stationary state of osmosis with a solution of Osmotic Pressure* π_0.

The highest free pressure in the steady state will be $A\lambda_m$, which in the case considered is only 23·3 atmospheres.

When n is small in the steady state it must be the more so before this state is reached and even when the stationary state is not well defined we must be able to assume that n is very small at least at the moment the velocity sets in. In the case of the highest concentration the free pressure at the beginning of osmosis has a value of about 40 atmospheres.

As the free pressure cannot be greater than the osmotic pressure the possible velocities must lie between $\frac{\pi_0}{A}$ and $\frac{A}{q}$ where

* *loc. cit.*, Exp. I, II, III.

π_0 is the osmotic pressure and q the corresponding free pressure in the steady state. From equation (3) follows $\lim_{\lambda_0=0}\left(\frac{\pi_0 - q}{A\lambda_0}\right) = 0$. This gives an explanation to the fact that the osmosis for small concentrations immediately assumes a velocity very near to that of the steady state.

11. When the pressure on the solution is augmented, the properties are no longer so simple as they are when $\pi = 0$. The degree of semipermeability begins to play a more important part as regards the velocity. As stated in the earlier paper the absolute value of the velocity for pressures less than the reversion pressure and when other circumstances are the same is greater for a more perfect membrane. The increase of pressure was accompanied by a sudden decrease in the velocity, and this decrease is the greater the less perfect the membrane is. If the membrane was quite stable for pressures and quite semipermeable we should expect that an increase of the pressure on the solution with an amount π would have the same effect on the velocity as if the concentration was diminished to a value C corresponding to an osmotic pressure $\pi_0 - \pi$. As a consequence of this the velocity curve in the interval $0 < \pi < \pi_0$ would be

$$\pi_0 - \pi = -\frac{A\lambda}{1 - \frac{\lambda}{\lambda_m}}$$

where A and λ_m should have the same values as before if we assume the qualities of the membrane and the temperature to be the same. The direction of the tangent at the reversion point should be that of the friction line. In general we find at this point $\frac{d\pi}{d\lambda} > A$, only if the characteristic point lies near to the reversion point we find $\frac{d\pi}{d\lambda}$ nearly equal to A. This seems to be a special case of the more general rule that the direction of the velocity curve* just before reaching the characteristic point at least for more perfect membranes is very near to that of the friction line corresponding to the same membrane and temperature.

12. From the preceding considerations we see that the experiments are very well explained by the assumption of a hydrostatic pressure inside the membrane which leads to the theory of the Free Pressure. This theory, however, gives no explanation of the manner in which the semipermeability is brought about in the layer next to the solution. The effect of the membrane is equivalent to a very great resistance against the flow of solution in bulk whatever may be the manner in which this resistance is brought about.

* See L. Vegard, *loc. cit.*, Exp. I, II, III.

Therapeutic Inoculation for Generalised Bacterial Infections. By L. Noon, B.C., F.R.C.S., John Lucas Walker Student of Pathology in the University of Cambridge. [Communicated by Professor Woodhead.]

[*Read* 23 November 1908.]

It will be conceded that recovery from a generalised bacterial infection depends on the defence-mechanism of the patient being called into action. To an invasion of bacteria Nature replies with a reaction which has for its object the increase of the natural defences. Since, however, this reaction often fails, it is the business of the medical man to ask whether Art may not improve on Nature in this respect. If the natural reaction is delayed or incomplete, that is, if it is not the best, the most efficient reaction of which the patient is capable, we had better seek to improve matters by the use of an appropriate stimulus.

The following experiments are directed to shew that the presence of large numbers of infecting organisms throughout the system is not generally a sufficient stimulus to set the protective mechanism of the tissues into action at once; while on the other hand a subcutaneous injection of a suitable dose of the killed* bacteria provides a generally efficient stimulus.

The organism used was the *B. pseudotuberculosis rodentium.* When this was injected into the peritoneal cavity of a rabbit or guinea-pig there followed a general peritonitis, associated with the formation of miliary abscesses in the liver and spleen, and in guinea-pigs also in the lungs. The abdominal lymphatic glands, as also frequently the mediastinal glands, become caseous. At death the blood was found to give a pure culture of the organism in every case. The bacillus is nearly related to that of plague (*B. pestis*). The disease is nearly always fatal in guinea-pigs; of rabbits, which received an infecting dose of $\frac{1}{20}$ of a 24 hours' agar slope culture, nearly one half died.

The disease is therefore a severe one, and widely spread in the body of the experimental animal. The natural defensive reaction against this disease consists first of an increase of opsonine. Later on an agglutinin appears, but no bactericidal or antitoxic substances can be demonstrated satisfactorily.

I traced the evolution of opsonic resistance in rabbits which had been infected as described above, and I found at once that a period of inertia follows on the infection. During this period the

* The killed organisms referred to in this paper were subjected to a temperature of 60° C. for half an hour.

disease progresses while the defensive mechanism makes no answering effort. There is no considerable rise in the opsonic index of the blood for the first five days of the disease (Fig. 2). This phenomenon was observed in nine rabbits, none gave a contrary result. Three of the rabbits died at the end of this critical period. In those which survived, the opsonic index usually rose rapidly, and recovery set in. In guinea-pigs a similar period of inertia was observed, most of the animals died about the fifth day, a few lived longer and shewed a rise in the opsonic index, but only one recovered.

Fig. 1 gives the opsonic histories of a batch of ten guinea-pigs, which were all infected with similar doses on the same day, and which have been arranged in three groups according to the length of time during which they survived an infection. It is seen that the three guinea-pigs which lived longest had shewn high indices previous to their infection, and also shewed a capability to produce a relatively large increase in opsonine in response to the disease. The animals which died earlier were those with a past history of medium or low indices. Their indices remained low during the disease.

From the above it is clear that the natural reaction to the disease is delayed so late, that many rabbits and most guinea-pigs may be said to die without a struggle. An injection of killed bacilli beneath the skin of a normal rabbit is followed by a very different sequence of events (Fig. 2). In this case the animal responds to the stimulus with a prompt increase of opsonine, which reaches about double the initial value within 48 hours. The same result was obtained with five rabbits which were given various doses between 5000 million and 80,000 million killed bacilli per kilogramme of body weight. A smaller dose evoked a doubtful response or none at all. A similar contrast between the diseased and inoculated rabbits, also appeared with regard to the formation of agglutinin (Fig. 3). Five rabbits which were inoculated with killed bacilli, all had strongly agglutinating sera on the fifth day. Three diseased rabbits, which were tested, produced agglutinating sera first on the ninth day, and even then the observed agglutinating power was relatively small.

It cannot be held that, in disease, the protective mechanism is held in check by an inhibitory influence. If a normal rabbit responds readily to a subcutaneous inoculation, a diseased rabbit responds still more readily to such a stimulus, as is shewn on Fig. 4. After a dose of 250 million dead bacilli per kilogramme of body weight on the fourteenth day of the disease, the opsonic index of rabbit 10 rose from 1·5 to 2·5 within 24 hours. In dealing with diseased animals, however, the dose of vaccine must be carefully regulated. An excessive dose produces an irregular reaction (Figs. 5 and 6).

Conclusion.

The presence of a generalised bacterial infection is not an efficient stimulus to call forth the best protective reaction of which an experimental animal is capable.

Description of Figures.

Fig. 1. Curve I, mean of opsonic indices of three guinea-pigs which survived infection more than 12 days (one recovered).

Curve II, mean of indices of six guinea-pigs which survived from 6 to 10 days.

Curve III, index of guinea-pig which survived 5 days.

(*a*) Date on which all the guinea-pigs were infected with equal small doses of B. pseudotuberculosis, intraperitoneally.

(*b*) One guinea-pig died.

(*c*) Four died.

(*d*) Two died.

The first dot of each curve represents two separate estimations of the index of each guinea-pig.

Fig. 2. Continuous line, composite curve of opsonic indices of nine rabbits which were infected with living bacilli on day 1.

(*a*) Three of these animals died.

Interrupted line, composite curve of indices of five rabbits, inoculated with killed bacilli on day 1.

Fig. 3. Agglutinating power of rabbits of Fig. 2.

Continuous line, diseased rabbits (three animals).

Interrupted line, inoculated animals.

The numbers at the bottom refer to the same days as those in Fig. 2.

Fig. 4. Opsonic index of rabbit which received a subcutaneous dose of 250 million killed bacilli per kilogramme of body weight on the 14th day of the disease (*a*).

The dots represent independent estimations, the curve is drawn through the mean of each pair of estimations.

Fig. 5. Opsonic index of a diseased rabbit which received a subcutaneous dose of 5000 million killed bacilli per kilogramme of body weight. Pairs of independent estimations.

Fig. 6. Opsonic index of the rabbit of Fig. 5, after receiving a subcutaneous dose of 500 million killed bacilli per kilogramme of body weight (*a*). Pairs of independent estimations.

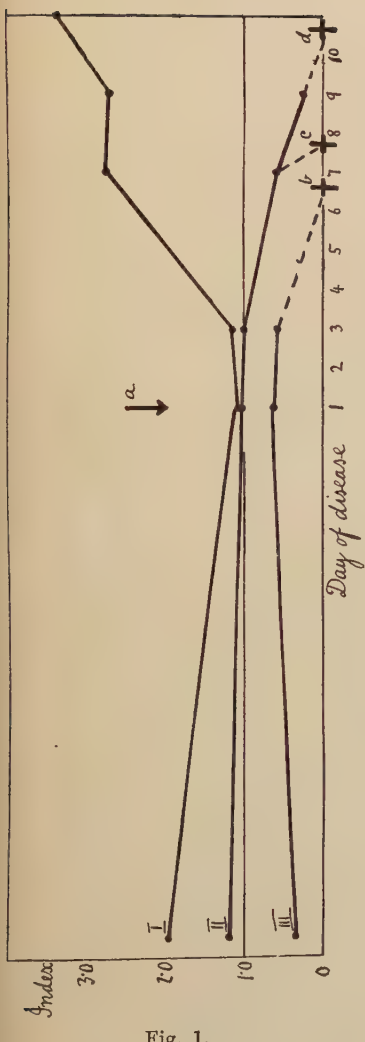

Index
3·0
2·0
1·0
0
I
II
III
a
b
c
d
Day of disease 1 2 3 4 5 6 7 8 9 10

Fig. 1.

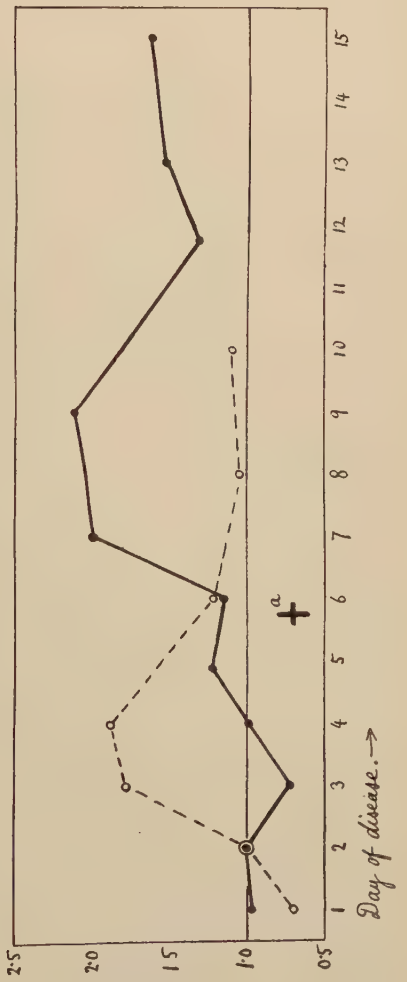
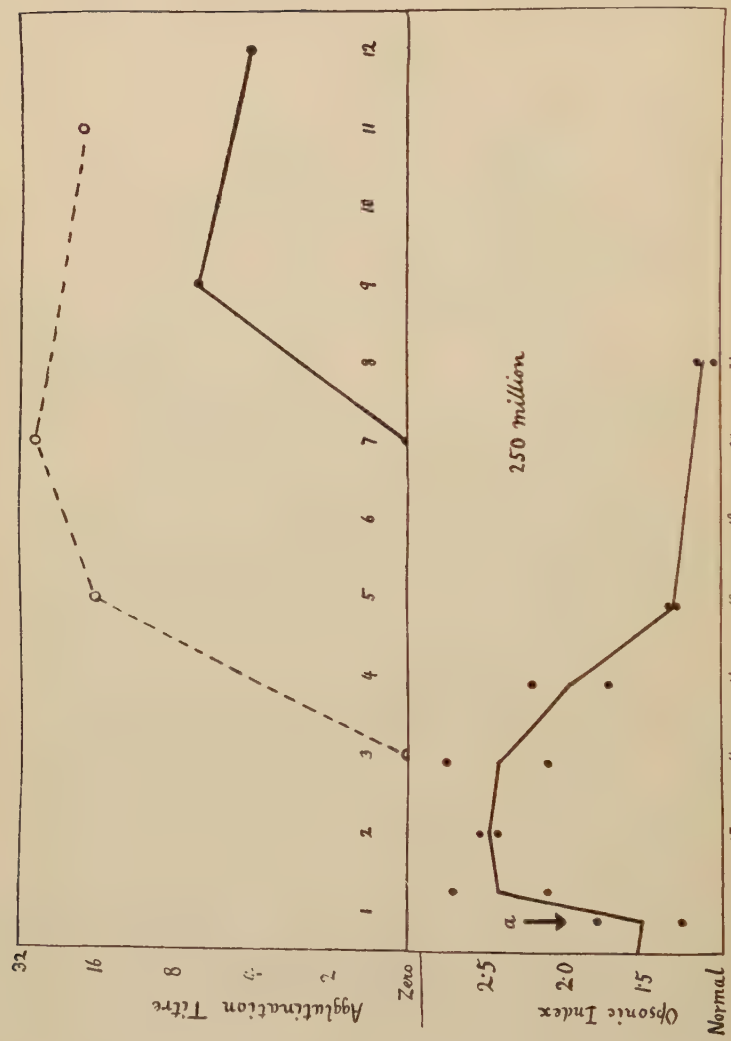

2·5
2·0
1·5
1·0
0·5
a
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Day of disease.→

Fig. 2.


32
16
8
4
2
Zero
Agglutination Titre
1
2
3
4
5
6
7
8
9
10
11
12
Fig. 3.
2·5
2·0
1·5
Normal
Opsonic Index
a
250 million
14
15
16
17
18
19
20
21
Day of disease
Fig. 4.

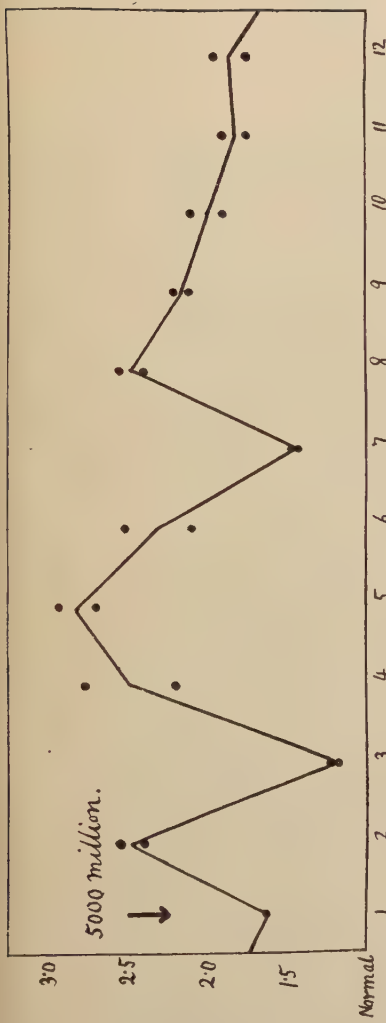

5000 million.
3·0
2·5
2·0
1·5
Normal
1
2
3
4
5
6
7
8
9
10
11
12

Fig. 5.

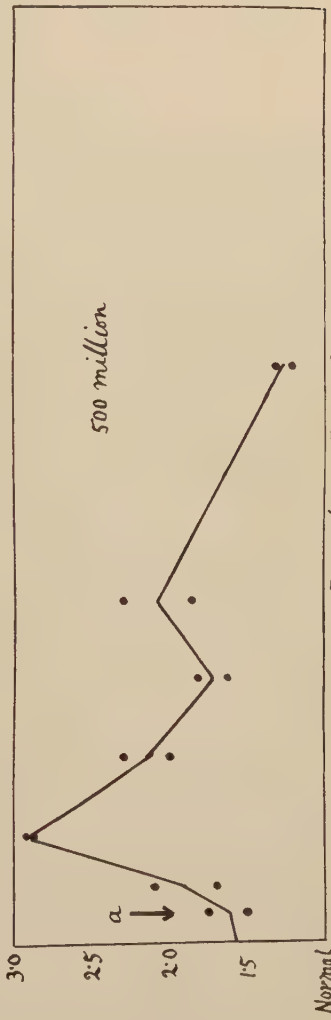

500 million
a
3·0
2·5
2·0
1·5
Normal
1
2
3
4
5
6
7
8

Fig. 6.

On the examination of living leucocytes in vitro. By CONSTANT PONDER, M.A., Emmanuel College. [Communicated by Mr W. E. Dixon.]

[*Read* 23 November 1908.]

The satisfactory examination of living leucocytes has presented many technical difficulties. Methods have been mainly employed whereby the movements, and escape of the leucocytes from blood vessels, have been observed in the tissues of a living animal, or the leucocytes have been obtained, in the fluid from serous cavities, abscesses, or blisters, and studied in suspension, in a "hanging drop." It is possible to obtain a fairly clean preparation by centrifugalizing citrated blood, as in Wright's opsonic method, and by the usual laboratory process, in which a thin film of blood, prevented from drying, is examined on a warm stage; a few white cells can be found, but their appearance is much masked by the presence of the red cells.

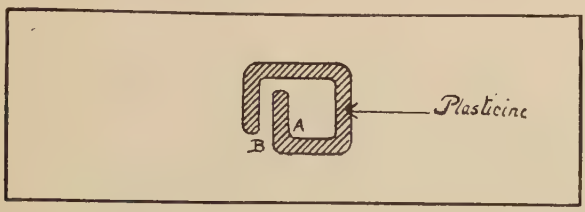
The method which I am going to describe gives a perfectly clean preparation of a great quantity of leucocytes obtained direct from any blood; the leucocytes can be kept alive for some time, in order that their movements and other physical properties can be studied, while the manipulations are so simple that they may be carried out by a class of students. I believe, moreover, that the method may be useful in research work on these most important cells, for it is also possible in this process to allow leucocytes free movement for some hours, in relation to extraneous substances introduced in their midst, in other words to study their chemiotaxis "in vitro," or again, one can actually watch their behaviour, when moving amongst a suspension of bacteria. One can also suggest the interest of observing the behaviour of pathological cells, such as those obtained from the blood of a patient suffering from one of the severe anaemias.

The method. The necessary apparatus is to be found in all pathological and physiological laboratories, the only unusual material needed is modelling clay or "Plasticine," which can be obtained anywhere.

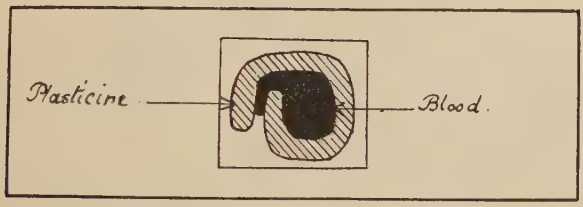
The essential point is the preparation of a blood-chamber, whereby the white cells are allowed to escape from the clot and adhere to the surface of a slide, or coverslip, the clot being afterwards removed.

To make this chamber a morsel of plasticine, half the size of a pea, is rolled out until it is as thin as the lead of a pencil and about an inch and a half long; this is then taken and gently

fixed on a clean slide, so as to wall in a small chamber, with an entrance passage leading into it, thus:



A drop of blood is allowed to fall into the chamber at *A*, a coverslip is superimposed and gently pressed down with a glass slide, so that as the plasticine is flattened, blood and air are driven out of the passage *B*, which must be kept patent, so that the chamber is completely filled with an even layer of blood, thus:



This needs a little practice to perform successfully, but under the usual circumstances, a small air bubble in the chamber does no harm.

The chamber is now incubated, at about blood temperature, for any length of time, from ten minutes to three or four hours, according to requirements.

For a class of students a very simple arrangement, doing away with the need for an incubator, is as follows. A warm stage is prepared under a microscope, by means of a copper strip, one end of which, resting on the microscope stage, is kept at blood temperature (the melting point of a fragment of cocoa butter can be used as a rough indicator), when the other end is heated by a very small flame; on the end above the flame, a flat white dish, filled with normal (·75 °/₀) saline solution, is placed, at a point on the copper (easily found by experiment) where the saline solution also keeps warm at a temperature of 38° C., or a little below. The whole slide, on which the chamber has been prepared, is immersed in the dish of saline, at blood temperature, and so incubated for at least ten minutes.

During the period of incubation the blood clots and the leucocytes escape and adhere firmly in hundreds to the surface

of the slide and coverslip, which form the glass floor and roof of the blood-chamber; it is now only necessary to clean away the clot. To do this, the coverslip is removed while under the surface of the warm saline solution by passing beneath it the point of a knife or needle, and what remains of the clot and plasticine is scraped away from the slide with a small knife. The slide should now be well washed in the warm saline until all free red cells have been rinsed off; this is shown to have taken place completely by the disappearance of any reddish colour, and on holding up the slide to the light a grey film can be seen, which consists entirely of leucocytes, adhering to the surface of the slide.

If now only a temporary preparation is needed, it is merely necessary to take the slide out of the warm saline bath, superimpose a coverslip, taking care that plenty of saline lies beneath it, and examine it on the warm stage under the microscope. If, however, the preparation is to be kept some time, or it is desired to bring in contact with the leucocytes some other fluid such as serum, containing a suspension of bacteria, it is necessary to construct on a coverslip, a plasticine chamber, similar to the one originally described for obtaining the leucocytes—though in this case with walls made as thin as possible—and having carefully filled this with a large drop of the saline, or fluid, with which it is desired to bring the leucocytes in contact, to press the slide down on it, so that the chamber is completely filled with the fluid, and the leucocytes on the surface of the glass become situated on its floor.

As an alternative preparation, the coverslip of the original blood-chamber which is also covered with leucocytes, may be taken, cleaned as described above, and pressed down similarly on a chamber (made this time on a slide) which contains a drop of saline or the fluid with which it is desired that the leucocytes shall come in contact; the leucocytes in this case are on the under surface of the coverslip, so that the preparation resembles that of a "hanging drop."

The actions of the leucocytes can by this method be studied in two ways. Firstly, they may be allowed to live and move in the primary blood-chamber for some hours, the preparation being kept in an incubator (evaporation does not take place, as the blood dries across the narrow entrance and seals it), and then, when it has been cleaned, fixed, and stained, the positions the leucocytes have taken up, with regard to any foreign substance, can be observed. Secondly, if the primary preparation is only incubated for ten minutes, and the leucocytes are then transferred to a second chamber, we have a means whereby their movements and interactions with foreign substances can take place while under observation.

Throughout this process the question of temperature is of some importance; it is advisable that the slide to which the leucocytes are adherent should be kept at a temperature not much below that of the blood, otherwise the cells become circular and are liable to be washed off; on the other hand, if the preparation is heated much over 40° C., the leucocytes become disintegrated and disappear.

I have found that the leucocytes will continue their movements for about three or four hours in the primary chamber, or for about an hour in the secondary; this is probably to be explained by the fact that the CO_2 tension becomes so high as to terminate their existence sooner when the red blood cells have been removed.

On the Relation between Ionization and Pressure for Röntgen Rays in different Gases. By J. A. CROWTHER, B.A., St John's College. [Communicated by Sir J. J. Thomson.]

[*Read* 23 November 1908.]

1. In the absence of any secondary radiation, the amount of ionization per cubic centimetre produced in a given gas by the passage of Röntgen rays of given type and intensity should be proportional to the mass of the gas present per cubic cm.; that is, if the temperature is constant, to the pressure of the gas. Thus in the absence of any secondary radiation, the ionization-pressure curve should be a straight line passing through the origin.

If, however, the action of the rays on the gas generates secondary rays, this will no longer be the case.

It was shewn in a previous paper* that in the cases of air and carbon dioxide, the energy of the penetrating secondary radiation is simply proportional to the pressure of the gas, and this result has subsequently been confirmed for the more powerful radiators.

Since this type of radiation is sufficiently penetrating to pass through the whole of the gas between the electrodes in the apparatus employed, the ionization produced by it will be proportional to the intensity of the secondary radiation and to the pressure of the gas; that is, to the square of the pressure of the gas. The ionization-pressure curve should thus have the form

$$I = Ap + Bp^2,$$

where I is the ionization and p the pressure. For most gases at the pressures employed in the present experiments the energy of this secondary radiation is too small to make any appreciable alteration in the shape of the curve. Taking Barkla's value (·00024 times the primary) for the energy of secondary radiation from a cubic cm. of air as being approximately correct, it can be shewn that in the case of the most efficient radiator used, namely ethyl bromide, the energy of secondary radiation at a pressure of 160 mm. of mercury is only about 3 °/₀ of that of the primary beam; or since the secondary radiation is in this case about three times as absorbable as the primary, the secondary ionization should be about 9 °/₀ of that produced by the primary rays at the pressure named. Since ethyl bromide gives off more than five times as much secondary radiation as methyl iodide, and two hundred times as much as air, it is evident that the ionization produced by the penetrating secondary rays in these gases is negligible.

* Crowther, *Phil. Mag.* XIV. Nov. 1907, p. 653.

It should, however, be quite perceptible in the ionization pressure curve for ethyl bromide, and on turning to Fig. 1 it will be seen that this is the case.

The curve for ethyl bromide, unlike those obtained for other gases, has a distinct upward tendency, and from the magnitude of its departure from the tangent through the origin, it is easy to deduce that, at a pressure of 160 mm., the ionization due to the secondary rays is about 16 °/₀ of that due to the primary.

For the other gases tried the curves shew that the effect of the penetrating secondary rays may be neglected.

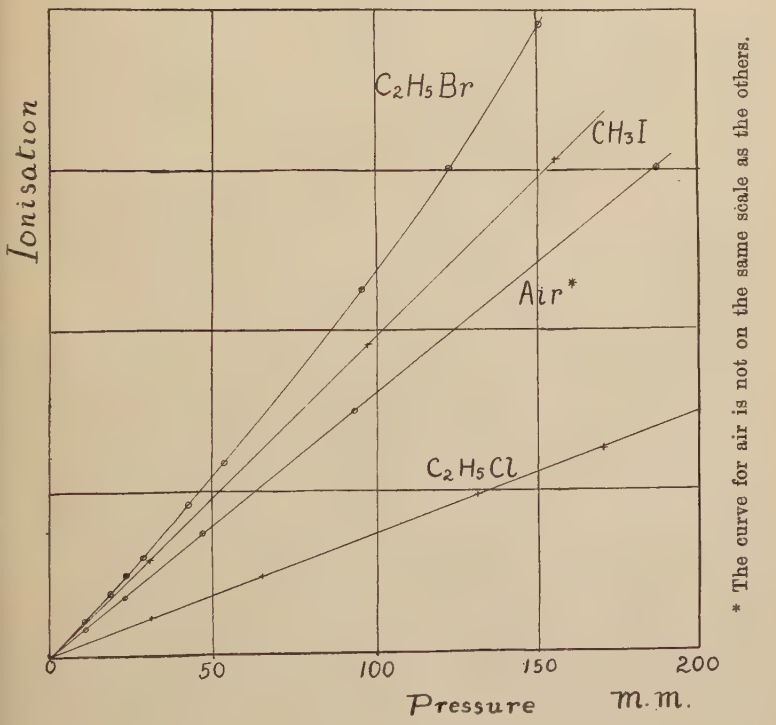


Fig. 1.

2. It was thought possible that in addition to the penetrating secondary rays there might be some soft secondary radiation (possibly soft β-rays) emitted by the gas under the action of the Röntgen rays. It was with the special object of investigating this point that the present experiments were undertaken. It can easily be seen that if the pressure of the gas is so low that the soft secondary rays from the gas are not completely absorbed in

the gas before reaching the walls of the ionization chamber, the ionization-pressure curve as before should have the form

$$I = Ap + Bp^2.$$

When, however, the pressure is high enough to cause the complete absorption in the gas of the soft secondary rays from the gas, the ionization produced by these secondary rays will be simply proportional to their energy, that is, simply proportional to the pressure of the gas; and the whole ionization in the gas will again follow a simple pressure law.

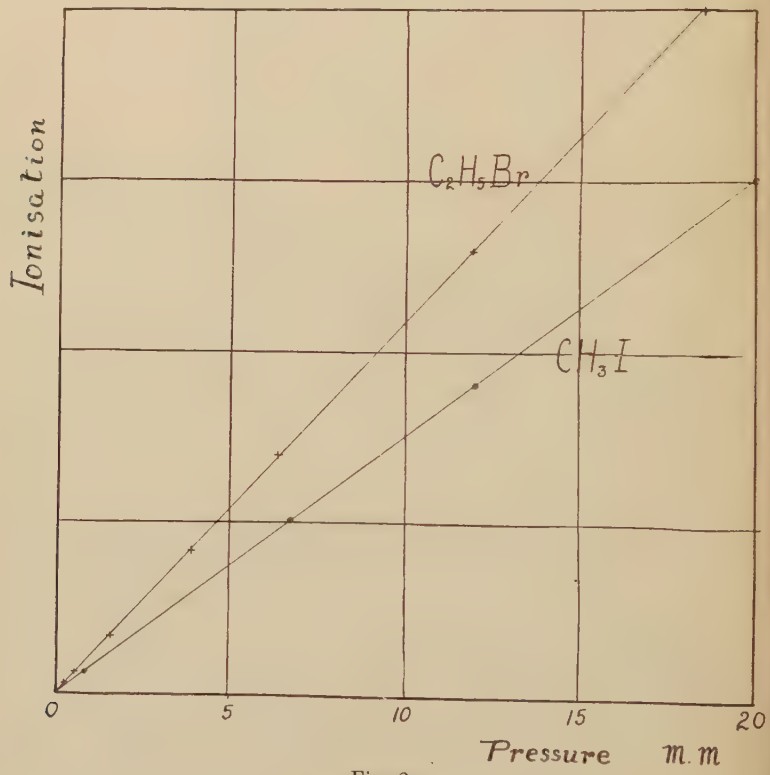


Fig. 2.

The apparatus employed consisted of a gas-tight brass box with an aluminium window to admit the rays; the ionization was measured between a pair of parallel plates placed inside the box but outside the direct line of action of the primary beam, which was limited and defined by perforated lead screens.

A second aluminium window at the far side of the box allowed the primary rays to pass out of the box, so that the amount of

absorption undergone by them could be measured. This was necessary in order to correct the results obtained, for the absorption of the primary rays before reaching the electrodes, which in order to avoid secondary rays from the aluminium window were placed some considerable distance within the box. This correction (which varies with the pressure) has been applied to all the results given in this paper. A second similar chamber was filled with air, and used as a standard.

3. The first experiments were made with the plates 6 cms. apart. These failed to give any indication of any departure from the simple pressure law (except, as stated above, in the case of ethyl bromide at the higher pressures). It was thought that this might be due to the distance between the plates being sufficient to absorb completely any soft secondary rays emitted by the gas, even at the lowest pressures employed. The apparatus was then altered so that the distance between the plates was only 5 mm.; the aperture of the primary beam being suitably reduced by means of lead slits to a width of about 2 mm., and the experiment repeated.

The results obtained connecting pressure and ionization are given in Figures 1 and 2, Fig. 1 giving the results for comparatively high pressures (up to 200 mm.) and Fig. 2 the results for ethyl bromide and methyl iodide at pressures below 20 mm. of mercury. The upward trend of the ethyl bromide curve is evident from Fig. 1. Apart from this all the curves are sensibly straight lines passing through the origin. There is thus no evidence of any appreciable amount of soft secondary radiation from the gas itself. If gases do emit these soft secondary rays they are either too absorbable to pass through $2\frac{1}{2}$ mm. of gas (the half distance between the electrodes) even at the lowest pressures employed, or else they are too small in amount, compared with the ionization produced by the direct action of the primary rays, to be appreciable.

The former hypothesis is not probable. By allowing the primary rays to graze one of the aluminium electrodes it was shewn that the soft secondary rays emitted by the aluminium under the action of the Röntgen rays passed through the whole distance between the plates without being completely absorbed by ethyl bromide at a pressure of 20 mm. of mercury. It seems probable therefore that practically the whole of the ionization is produced by the direct action of the primary rays.

On the Relative Ionization produced by Röntgen Rays in different Gases. By J. A. CROWTHER, B.A., St John's College. [Communicated by Sir J. J. Thomson.]

[*Read* 23 November 1908.]

It may be of interest to place on record the following results which have been obtained during the course of a series of experiments, still in progress, on the passage of Röntgen rays through gases. The apparatus was that described in a previous paper*. A beam of Röntgen rays was fired through the gas, between two parallel plate electrodes, care being taken to avoid any portion of the beam falling upon either of the electrodes. If this precaution is neglected very different values are obtained owing to the large amount of soft secondary β radiation given off by the electrodes under the action of the primary rays.

The measurements given under the column headed "soft rays" were made with the softest rays which would produce any appreciable amount of ionization, and the aluminium window was made very thin (about $\frac{1}{10}$ mm.) in order to cut off as little as possible of the soft radiation. The measurements labelled "hard rays" were made with the bulb as hard as it was possible to work it with a Rudge induction coil worked by a turbine mercury interrupter.

It has been suggested that the relative ionization would tend to follow a density law as the rays became harder. The results obtained in the present experiments do not give any indication of such a result. Compared with air as a standard, methyl iodide, methyl acetate and carbon dioxide shew a decrease as the rays get harder in the relative amount of ionization produced; hydrogen and ethyl bromide, on the other hand, give a distinct increase; while ethyl chloride and carbon tetrachloride remain nearly constant. The exceedingly small value obtained for hydrogen with very soft rays (only 1 °/₀ of that of air at the same pressure) is very remarkable. The increase in the case of ethyl bromide, the value for which for soft rays is already considerably above that required on a density law, may also be noticed.

* *Proc. Camb. Phil. Soc.* Vol. xv. p. 34, 1908.

TABLE.

	Soft rays	Hard rays
Air	1·00	1·00
H_2	·01	·18
CO_2.....	1·57	1·49
$CH_3CO_2CH_3$.	4·95	3·90
C_2H_5Cl	18·0	17·3
CCl_4	67	71
C_2H_5Br	72	118
CH_3I	145	125

Relative Ionization.

The Relationship between Human and Bovine Tuberculosis. By G. Sims Woodhead, Professor of Pathology.

[*Read* 23 November 1908.]

Before the infective nature of tuberculosis had been demonstrated by modern methods there was considerable difference of opinion as to the specificity of the tuberculosis occurring in animals and in human subjects. Koch in his earlier observations seems to have had little doubt as to the identity of the tubercle bacillus in all forms of tuberculosis, though Klein at a very early date held that the bovine tubercle bacillus differed somewhat from that found in the human subject not only in its manner of growth and in its relation to the tissues but also in virulence. Theobald Smith, Dinwiddie and others as the result of a series of very careful observations called attention to what they believed were almost specific differences as regards size, mode of growth, virulence and chemical products, between the human and the bovine types of tubercle bacilli. During the time that I held the Grocers' Company Scholarship I was led to make a careful examination of the tuberculous material that fell into my hands with the view of obtaining some light on the subject of caseation in tubercle and of the relations of the various elements found in tubercle to the spread of the disease not only amongst human beings but amongst cattle. I was very early struck by the large proportion of cases of abdominal tuberculosis (*tabes mesenterica*) met with in extremely young children and after comparing my own observations with those already made by Bang, Rilliet, and Barthez I was convinced that, in children at any rate, many of the cases of tuberculosis were the result of a kind of natural infection through the intestine. I found that in 127 cases of tuberculosis in children on whom I had the opportunity of making a post-mortem examination tubercular ulceration of the intestine was found in forty-three. Only one of these cases had succumbed during the first year after birth, but 14 died within two years and a half of birth, 10 between three and five years, 7 from six to seven and a half years, 5 from eight to ten years, and 6 between eleven and fifteen years. Although there was tuberculous ulceration in forty-three cases only, there was distinct tuberculous degeneration of the mesenteric glands in no fewer than 100 cases or nearly 79% of

the whole. The age at which these tubercular glands in the mesentery were found is significant. During the first year of life there were 4 cases; from one to two and a half years, 33; from three to five and a half years, 29; from six to seven and a half years, 12; from eight to ten years, 13; and from eleven to fifteen years, 9. In fourteen of these cases the mesenteric glands only were affected, that is no tubercle was found in any other part of the body. Here again more than half the cases appeared between two and five and a half years. Since these figures were published, Still, Shennan and the Royal Commissions on Tuberculosis in this country and in Germany have published corresponding statistics which in the main agree with those I have given and I only use my own figures because they were the factor that influenced, perhaps even biassed, my own conclusions. I should now like to point out that whilst the child is suckled by its mother, i.e. during the first year after birth, it is not nearly so liable to contract mesenteric trouble as at a later period; whilst during the next two periods during which children are living on mixed diet and, usually, are taking some milk from the cow, the number of cases of tuberculosis rises very rapidly. Dr B. Hubermaas studying tuberculosis of the breast pointed out that although tuberculosis is so common in young married women, tuberculosis of the breast is exceedingly rare. This observation has been fully confirmed by later observers. It may, of course, be suggested that tuberculosis is a disease of such slow development that even if it were conveyed from other sources than by food and by other channels than by the intestinal canal it would take some time for the disease to manifest itself, and therefore would not kill the child until it had reached an age beyond the first year. It must be remembered, however, that in my investigation a very careful search was made with the special object of finding tubercle and that therefore tuberculous disease even though it had not given rise to any symptoms during life would scarcely be overlooked. I do not wish to minimise the importance of infection by the tuberculous mother kissing and fondling her child, but this factor (except that the process is not extended over such a long period) is just as likely to be brought into play in the early years of life, as later. We should not expect to have the very sudden rise that we meet with if this were the only factor at work. It is evident, however, that after the first year a second factor comes in—food, in this instance milk, and so far as the reports of the Royal Commission on Tuberculosis have been published and from the reports from various laboratories there seems to be little doubt that tubercle bacilli in enormous numbers are often present in the milk that is supplied for human consumption. Quite apart from anything else then, I am satisfied that these statistics as to

mesenteric tubercle plus the presence of the tubercle bacilli in such a large proportion of milk taken by these children are closely associated.

Let us now come to another aspect of the question. All those who have studied tuberculosis are satisfied that very numerous and very distinct species are affected by tuberculosis, and as Sir John McFadyean points out, we know of no single organism which will produce disease in more than half a dozen species and which affects either the human subject or cattle which does not also affect them both. It has of course been pointed out that the tubercle bacilli met with in various animals exhibit very varying degrees of virulence. They also differ considerably as to their rate and luxuriance of their growth, and it is quite possible that they may differ in other respects and perhaps differ very materially. In this, however, they resemble many of the other infective organisms. Washbourn and Eyre, for example, pointed out that not only could they modify the virulence of the pneumococcus by passing it through rabbits and mice, but that by cultivating it as a saprophyte on sterile media in test tubes, they were able to render its growth much more luxuriant and at the same time to diminish its virulence, these alterations taking place much more rapidly in certain of the individual organisms than in others, as after cultivating them for some time they could separate from the same culture highly virulent organisms, growing slowly, and slightly virulent organisms, growing rapidly and luxuriantly. Pasteur's experiments on hydrophobia afford another example of increase and diminution of virulence by passage through different species of animals. Numerous observers have obtained similar results by the passage of a streptococcus through different species of animals; the streptococcus passed through the rabbit becoming more virulent for that animal and less so for the mouse, whilst a similar organism passed continuously through a series of mice acquires increased virulence for that animal, this being accompanied by a corresponding diminution in virulence of that organism for the rabbit. It is unnecessary to multiply examples. Anyone who will take the trouble to go through the appendix to the Second Interim Report of the Royal Commission on Tuberculosis will be very much struck by the marked differences in virulence of the tubercle bacilli separated from different cases and inoculated into animals. They will find that, speaking generally, the tubercle bacillus, that is what the Commission terms "dysgonic," is distinctly more virulent than the rapidly and luxuriantly growing (eugonic) form. All varieties both as to growth and virulence have been found in the human subject, some appearing to produce extensive and rapidly generated lesions, others producing minimal lesions or, in some cases, none at all. It is known that alterations

in temperature of nutrient media, etc. may affect the bacillus of anthrax and render it almost inert, and that it may take some time and special conditions for the restoration of the original virulence. It is recognised too that the resistance of individuals of the same species may vary enormously even to the same organism, and that this comes out most markedly in the case of an organism of modified virulence. If this holds good in the case of the anthrax bacillus, may it not be equally true in the case of the bacillus tuberculosis?

Here we have to remember however that we have an organism that grows comparatively slowly. In the first instance it sets up localised lesions which in most cases at any rate are of slow development.

A slowly growing organism and one that causes such comparatively slowly developing lesions will in all probability undergo comparatively slow and slight modifications when it is placed in new environments and is subjected to new conditions. The direct evidence on these points is at present very small in amount and unsatisfactory in character. We have little evidence of a morphological character. Pathologists have had so much to undertake in connection with the study of the bacillus in the tissues and in ordinary "culture" conditions that they have had little time or opportunity to devote to the changes in morphological and biological characters of the tubercle bacillus outside these two limited areas.

The tuberculin reaction however has given us some evidence that the differences insisted upon by Koch and his followers are not specific, indeed are scarcely to be dignified by the term varietal, but the large field of biological and morphological investigation that is just being broken, will I believe prove a most fruitful field to men trained in these branches of study, and how warmly their cooperation will be received it is scarcely necessary for me to emphasise.

As far as can be gathered from the information now at our disposal, the character of the lesions set up by different varieties of the tubercle bacillus, the specificity of the reactions obtained with tuberculin derived from bacilli derived from human and bovine sources, the evidence slight though it be of modification of morphological and biological characters, and especially the more marked evidence of gradation of virulence in the two types as usually described, we are, I think, justified in assuming that some time or other we should be able to find links connecting even the distant extremes, in spite of the great difficulties encountered in tracing these links, difficulties that arise out of the comparatively slow growth of the tubercle bacillus both in culture and in the tissues, and the correspondingly slow development of the tuberculosis induced by them.

So far as mode of growth and lesions produced in the tissues, size of organism, tuberculin reaction and all known characters give us any lead we are I think bound to maintain that the differences between bacilli obtained from tuberculous lesions in the human being and those obtained from similar lesions in cattle, are not specific, and that for this reason, the tubercle bacillus, from whatever source it may be derived, constitutes a distinct, and even grave, danger to any human or brute subject to which it may gain access and I for one should be unwilling to accept any responsibility for suggesting the relaxation of the laws dealing with any thing in which this highly infective agent is concerned. Indeed I would go further than this as I consider it essential that it is our duty to do all that we possibly can to strengthen the hands of Medical Officers of Health and their Inspectors in the difficult task with which they are now confronted.

The radiation of various spectral lines of neon, helium and sodium in a magnetic field. By J. E. Purvis, M.A., St John's College.

[*Read* 23 November 1908.]

Neon. The author had already photographed the principal lines of neon vibrating in a magnetic field, when a paper by Lohmann appeared giving a short account of the effect on the lines when observed by means of an echelon spectroscope*: and since then, Lohmann has completed the measurements†. He used an echelon of 32 plates, the thickness of each plate being 1 cm., and the strength of the magnetic field was usually 11,000 units. He states he was able to see and to photograph the lines divided into 9, 12 and 15 constituents.

Field strengths were used in the author's experiments of 26,100 and 24,000 units respectively, as the distances between the poles of the magnet had to be adjusted to the diameters of the neon tubes. In the first series of experiments, the first order of Professor Liveing's 21-foot radius grating spectroscope was used, and, usually, there was an exposure of 20 minutes of the photographic plates. On examining the photographs with a low power microscope, the author was unable to see the more complicated divisions of some of the lines described by Lohmann.

Lately, the author has repeated the experiments with an echelon of 18 plates, the thickness of each plate being $7\frac{1}{2}$ mm.; and using the same field strengths as were used in the Rowland grating experiments. But, when the lines were divided into more than three or six constituents, there was a great difficulty in distinguishing clearly between the images of other orders and the images of some of the constituents of the divided lines. For instance as regards the line 6402, described by Lohmann as giving 15 constituents, one could only be sure of five constituents; and of the line 5944 divided into 12 constituents by Lohmann, four constituents could be distinguished, of which there were two fairly sharp ones

* *Physik. Zeits.* 7 Jahrgang, No. 22, Seite 809—811.

† "Beitr. zur Kennt. des Zeeman-phänomens," *Inaug. diss.*, Halle, 1907.

vibrating parallel to the lines of force, and two very diffuse ones showing doubtful signs of division and vibrating perpendicularly to the lines of force. The importance of this difficulty cannot be exaggerated; for when the gas is vibrating in the magnetic field, its luminosity is greatly increased; and the very faint images of other orders, not clearly visible when the gas is not vibrating in the field, would become better marked and clearer. The constituents of these images would, therefore, interfere with a clear differentiation of the constituents of any particular line under observation.

The advantage of Lohmann's apparatus was the greater dispersive power of his echelon spectroscope of 32 plates as compared with the one of 18 plates used in these experiments, and the stronger field used in the present experiments did not appear to compensate for the less dispersion of the echelon. So that until the conditions of the experiments are more equal, no exact comparison of the observations can be made: and there is still the more important difficulty of distinguishing the real constituents of lines from the images of other orders. However, the distances have been measured of the constituents of the lines unmistakably divided into triplets, and, also, those of the line 6383 undoubtedly divided into six constituents.

The diameters of the capillary parts of the neon tubes were different, so that the distances between the magnet poles had to be altered to accommodate them: and the strengths of the field were 24,000 and 26,100 units respectively. The following table gives the distances of the two constituents of the triplets vibrating perpendicular to the lines of force from the one vibrating parallel thereto, measured on the scale of vibration numbers, i.e. the number of vibrations in a path of one centimetre*. The positive signs denote the distances in the direction of greater wave length, and *s* and *p* that they vibrate perpendicular or parallel to the lines of force. It may be of use to workers in this kind of research to state that the analysing Nicol prism should be placed between the magnet and the quartz condensing lens when analysing the various constituents of a divided line; for, if it is placed between the lens and the slit of the spectroscope, the rotation of the polarised constituents by the quartz lens produces a complete inversion of the images when photographs are being taken in the ultra violet part of the spectrum, so that the two outside constituents of a triplet would appear to vibrate parallel to the lines of force, and the middle one perpendicular thereto. It is possible that the abnormal polarisations of certain triplets of various elements which have been recorded have been produced in this way.

* Runge and Paschen, *Sitz. d. Akad.*, Berlin, 1902 (1), p. 721, "Zeeman-effect entsprechender Serienlinien."

λ	Strength of field 26,100 units.	Strength of field 24,000 units.
6717·2	+ 1,237 *s*	+ 1,126
	0 *p*	0
	− 1,226 *s*	− 1,125
6533·1	+ 0,817 *s*	+ 0,766
	0 *p*	0
	− 0,811 *s*	− 0,788
6266·66	+ 1,176 *s*	+ 1,129
	0 *p*	0
	− 1,184 *s*	− 1,134
6163·79	+ 1,666 *s*	+ 1,544
	0 *p*	0
	− 1,587 *s*	− 1,518
6074·52	+ 1,788 *s*	+ 1,713
	0 *p*	0
	− 1,722 *s*	− 1,692
5852·65	+ 1,249 *s*	+ 1,161
	0 *p*	0
	− 1,248 *s*	− 1,195

The line 6383·15 is divided into six constituents, and the distances between corresponding constituents were measured.

λ	Strength of field 26,100 units.	Strength of field 24,000 units.
6383·15	+ 1,804 *s*	+ 1,656
	+ 0,878 *s*	+ 0,812
	+ 0,878 *p*	+ 0,812
	0	0
	− 0,878 *p*	− 0,812
	− 0,878 *s*	− 0,812
	− 1,804 *s*	− 1,656

In order to compare these numbers with Lohmann's, the following table contains the values obtained when they are calculated in terms of the strengths of the field.

λ	Purvis $-\frac{d\lambda}{\lambda^2 \times 26{,}100} \times 10^{13}$.	Purvis $-\frac{d\lambda}{\lambda^2 \times 24{,}000} \times 10^{13}$.	Lohmann $-\frac{d\lambda}{\lambda^2 \times H} \times 10^{13}$.
6717·2	− 4,73 *s*	− 4,69	− 4,70
	0 *p*	0	0
	+ 4,69 *s*	+ 4,68	+ 4,70
6533·1	− 3,12 *s*	− 3,11	− 3,13
	0 *p*	0	0
	+ 3,10 *s*	+ 3,28	+ 3,13
6266·66	− 4,50 *s*	− 4,70	− 4,40
	0 *p*	0	0
	+ 4,53 *s*	+ 4,72	+ 4,40
6163·79	− 6,38 *s*	− 6,43	− 6,07
	0 *p*	0	0
	+ 6,07 *s*	+ 6,32	+ 6,07
6074·52	− 6,84 *s*	− 7,13	− 6,80
	0 *p*	0	0
	+ 6,59 *s*	+ 7,05	+ 6,80
5852·65	− 4,78 *s*	− 4,83	− 4,74
	0 *p*	0	0
	+ 4,78 *s*	+ 4,97	+ 4,74
6383·15	− 6,91 *s*	− 6,90	− 6,63
	− 3,36 *s*	− 3,38	− 3,38
	− 3,36 *p*	− 3,38	− 3,38
	0	0	0
	+ 3,36 *p*	+ 3,38	+ 3,38
	+ 3,36 *s*	+ 3,38	+ 3,38
	+ 6,91 *s*	+ 6,90	+ 6,63

The agreement between these numbers is moderately close; the chief differences being the two outermost constituents of the sextet from 6383; although the numbers obtained from both my measurements are fairly comparable. Also, there is no very close agreement between the numbers obtained from the triplet of 6163, except that the numbers obtained from my own measurements are comparable.

Lohmann did not appear to observe if there were any shift of the lines towards the red end of the spectrum when the neon was vibrating in the magnetic field. It would be difficult to do this with an echelon, for it would be necessary to move the photographic plate and there would be a consequent source of error. But, with this object in view, some photographs were taken by

the author with the Rowland grating. To accomplish this, a copper slide was attached to the slit of the spectroscope, and slotted in such a way that, in one position, the upper half of the image was cut off during the normal vibrations of the gas, and, in another position, the lower half was cut off when the vibrations were under the influence of the magnetic field. No other alteration was made; the position of the photographic plate was not altered; the neon tube was not touched; and the discharge through the gas was exactly the same in both series of experiments. The two images were consequently photographed directly over each other; and the following table contains the numbers, in thousandths of an Ångström unit, of the shift towards the red end of the spectrum of the middle constituent of the triplets, as well as of the shift observed in the sextet from 6383·15, produced by a field of 24,000 units.

λ	Shift in thousandths of an Å.U.	λ	Shift in thousandths of an Å.U.
6717	+ 90	6074	+ 42
6533	+ 100	5852	+ 32
6266	+ 32	6383	+ 60
6163	+ 101		

The + sign meaning, as before, that the shift is in the direction of greater wave lengths.

Efforts were also made to measure the shifts of some of the lines which give more than three constituents. For example, Lohmann states that the line 6096 is divided into nine constituents. On my plates, there are three diffuse constituents, and there are doubtful signs of further subdivisions; but the latter are not sufficiently well marked, so that the measurements were made from the centre of the middle diffuse constituent. The same remark applies to 6678 and 6506 which are stated by Lohmann to divide into nine constituents, and to 6402 which is stated to divide into 15 constituents. There is no doubt that there is a shift of these lines towards the red end of the spectrum when they are vibrating in the magnetic field, but for the reason just stated, the measurements are not of the same degree of certainty as those obtained from the above triplets.

λ	Shift in thousandths of an Å.U.
6678	+ 90
6506	+ 80
6402	+ 45
6096	+ 74

The numbers indicate that the shift is different for different

lines; and these differences eliminate any sources of error caused by a movement of the instrument. If there had been any such movement, it is obvious that the shifts would have been equal.

There was a considerable increase in the luminosity of the neon when it was vibrating in the magnetic field; and although the times of exposure and the other conditions were the same, the intensities of the unaltered lines when the gas was vibrating normally were weaker than the strongest constituents when they were divided. And in no case was the sharpness of the constituents equal to that of the unaffected lines, so that there was always some difficulty in measuring any shifts or separations.

With regard to the intensities of the normal undivided neon lines it may be mentioned that my observations do not coincide exactly with those of Baly* or Lohmann, as the following table shows. But not much importance can be attached to observations of this kind, unless the differences are clearly marked; and possible variations in the efficiency of the dyed emulsion in various parts of the photographic plates should not be left out of consideration, although there are some differences which may indicate the influence of varying conditions of pressure and discharge in the tubes, as for example, in 5852 and 6717.

	Intensities		
λ	Baly	Lohmann	Purvis
5852	20	20	9
5882	8	10	8
5944	10	11	9
5975	8	9	5
6030	10	9	6
6074	10	11	8
6096	10	12	9
6143	10	15	10
6163	10	11	8
6217	8	9	6
6266	10	10	9
6304	8	9	8
6328	6	—	—
6335	—	10	10
6383	8	10	10
6402	10	20	15
6506	6	10	10
6533	4	9	8
6599	4	9	9
6678	—	9	9
6717	1	9	8

* "The spectra of Neon, Krypton and Xenon," *Phil. Trans.* A. 202, 1904, p. 183.

Also, in agreement with Lohmann's observations, and opposed to Baly's, no neon line at 6328·38 was observed on any of the plates; but, as in Lohmann's neon, there was a well marked one at 6335.

Sodium. When the discharge through the neon was going on, the influence of the magnetic field was to force it against the side of the glass capillary, so that small particles of the glass were illuminated, and the D_1 and D_2 lines of the sodium in the glass were seen quite clearly. The divided constituents of these two lines were sharp and well defined, and they could be seen more easily than when a sodium flame was placed between the magnet poles. Lohmann used the same method in examining these lines, and the following table contains the results of the values obtained from my measurements compared with his, and also with those of Runge and Paschen, who used a Rowland grating and a field strength of 31,000 units*.

λ	Lohmann $-\frac{d\lambda}{\lambda^2 \times H} \times 10^{13}$.	Runge & Paschen $-\frac{d\lambda}{\lambda^2 \times H} \times 10^{13}$.	Purvis $-\frac{d\lambda}{\lambda^2 \times 24{,}000} \times 10^{13}$.
	$-5{,}89\ s$	$-6{,}00$	$-6{,}40$
	$-2{,}92\ p$	$-2{,}97$	$-3{,}16$
D_1. 5896·2	0	0	0
	$+2{,}92\ p$	$+3{,}02$	$+3{,}16$
	$+5{,}89\ s$	$+5{,}95$	$+6{,}30$
	$-7{,}35\ s$	$-7{,}18$	$-7{,}42$
	$-4{,}43\ s$	$-4{,}53$	$-4{,}75$
	$-1{,}46\ p$	$-1{,}48$	$-1{,}44$
D_2. 5890·2	0	0	0
	$+1{,}46\ p$	$+1{,}48$	$+1{,}44$
	$+4{,}43\ s$	$+4{,}32$	$+4{,}75$
	$+7{,}35\ s$	$+7{,}39$	$+7{,}42$

The agreement between these numbers is only moderately good. It is difficult to explain the variations, for the ratio of 2 : 1 of the constituents of D_1 is fairly well maintained. The values are the close mean of 12 separate measurements on three separate plates; and no changes in the strength of the field were observed during any of the experiments.

Helium. One of the neon tubes contained some helium, for the triplet from the line 5875·618 was well marked in the photo-

* *Sitz. der Berl. Akad.* 1902, p. 722.

graphic plates, and the following table contains the numbers obtained from the measurements of the separated constituents, compared with those obtained by Rayleigh quoted by Lohmann and by Lohmann himself.

λ	Rayleigh $-\frac{d\lambda}{\lambda^2 \times H} \times 10^{13}$.	Lohmann $-\frac{d\lambda}{\lambda^2 \times H} \times 10^{13}$.	Purvis $-\frac{d\lambda}{\lambda^2 \times 24{,}000} \times 10^{13}$.
Helium 5875·618	$-4{,}09$ *s*	$-4{,}33$	$-4{,}58$
	0 *p*	0	0
	$+4{,}09$ *s*	$+4{,}33$	$+4{,}58$

Lohmann's were eye observations, and they might not be so accurate as photographic ones.

Furthermore, it may be that small variations in the amount and nature of the constituents of the glass of the tubes produce changes in the field between the magnetic poles; and it is conceivable that some of the energy would be absorbed by these constituents, so that the full strength of the magnetic field would not act upon the gaseous particles of the vibrating gas. This view may explain the differences which have been noted in the numbers obtained when the Zeeman phenomena have been observed in gases.

General Results.

The results of these observations are :—

(1) A comparison of the measurements of the constituents of various divided lines of neon, helium and sodium observed with an echelon grating by Lohmann and with a Rowland grating by the author.

(2) The important difficulty in distinguishing the real constituents of a divided line from those of the adjacent images of other orders when an echelon grating is used for the observations, particularly when the line is divided into more than three constituents.

(3) The measurements of the shifts of the divided constituents of various neon lines towards the red end of the spectrum, and that the shifts are different for different lines.

(4) A comparison of the intensities of the normal undivided lines of neon with those of Baly and Lohmann.

I have to thank Professors Liveing and Sir James Dewar who were good enough to lend me two tubes of neon used in these experiments.

The transmission of Trypanosoma lewisi by fleas and lice. By G. H. F. Nuttall, Sc.D., Quick Professor of Biology.

[*Read* 23 November 1908.]

The author described experiments, conducted in the Quick Laboratory, which demonstrated that *Ceratophyllus fasciatus* and *Haematopinus spinulosus* are capable of transmitting *Trypanosoma lewisi*. In one experiment, 3 fleas, transferred from a diseased to a healthy rat, gave a positive result. On the other hand, 30—60 lice were required for the transmission of the trypanosome. No signs of any development of the trypanosomes were observed in the bodies of the lice.

The presence of anticoagulin in the salivary glands of Argas persicus. By G. H. F. Nuttall, Sc.D., Quick Professor of Biology.

[*Read* 23 November 1908.]

Experiments conducted with Mr C. Strickland have shown that the salivary glands and intestine of *Argas persicus* contain an anticoagulin which is inactivated by exposure to a temperature of 80° C. for 10 minutes. The organs of the tick do not contain haemolysins.

The mode of action of specific substances. By W. E. Dixon, M.A., and P. Hamill, B.A.

[*Read* 23 November 1908.]

The action of secretin was first analysed, and it was shown that this substance produces its effect by chemical combination with the proferments in the pancreas. It was suggested that such substances of the hormone type have no direct action on living protoplasm. Other evidence was brought to show that drugs having a specific action on a definite tissue do not bring about that effect by chemical combination with protoplasm or with a constituent of the living cell. It was concluded that the mode of action of Galenical drugs was different from that of the hormones.

The action of specific substances in toxaemia. By W. E. Dixon, M.A., and W. H. Harvey, B.A.

[*Read* 23 November 1908.]

It was shown that certain toxins such as that of diphtheria cause death by vaso-motor failure. It was found that in animals affected with such a toxaemia death can be greatly delayed by the injection of normal saline solution. The action of drugs becomes progressively less according to the degree of toxaemia; those drugs which act on the central nervous system are the first to lose their effect and those which act on muscle-fibre retain their characteristic effect longest.

The Migration Constants of Dilute Solutions of Hydrochloric Acid. By C. Chittock, M.A., Trinity College.

[*Read* 23 November 1908.]

Introduction.

The experiments described in the present paper were undertaken with the object of throwing light on the cause of the abnormally low electrical conductivity of dilute aqueous solutions of strong acids and alkalies. It has long been known that whereas the equivalent conductivity of a neutral salt becomes approximately constant at great dilution, that of a strong acid or alkali attains a maximum value at a dilution of about one-thousandth normal, and then decreases rapidly as the concentration is still further diminished. This decrease might be due to one of two causes; first, to some interaction between the ions of the acid or alkali and the solvent, which might lead to a decrease in the amount of ionization or in the velocity of one or both of the ions; or secondly, to some interaction with the small quantities of impurity which are always present in the most carefully prepared water.

It was thought that measurements of the migration constant of an acid at varying degrees of dilution might be of assistance in deciding between these alternatives. If a small current is passed for a given time through a solution of some acid such as hydrochloric acid, hydrogen will be evolved at the cathode and oxygen at the anode. The total quantity of acid present will remain unaltered, but near the cathode there will be a loss of acid, and near the anode an equal gain. If it is assumed that the whole of the conductivity of the solution is due to the ions of the acid, measurements of these changes of concentration give us the means of calculating the ratio of the ionic velocities of hydrogen and chlorine. Such experiments would therefore enable us to determine whether the abnormally small conductivity of a dilute solution is associated with an abnormally small mobility of one of its constituent ions.

Let u, v represent the velocities of the chlorine and hydrogen ions respectively under unit potential gradient. Then the migration constant for the anion is given by $p = u/(u+v)$. If a quantity of electricity Q coulombs is passed through the solution, the number of gram-equivalents of acid gained at the anode and lost at the cathode will be equal to $\frac{pQ}{q}$, where q (= 96440 coulombs) is the charge carried by one gram-equivalent of either ion.

Now let a definite volume V c.c. of the solution round the cathode be separated from the rest and stirred, this volume being large enough to contain the whole region in which any change of concentration has taken place. The concentration of this portion will then be less than its original value by an amount

$$\delta n = 1000 \frac{pQ}{qV},$$

δn being measured in gram-equivalents per litre. By measuring the change of concentration we can therefore find the value of p, and hence that of u/v. A second determination of the same quantity can be obtained in a similar way from the change of concentration at the anode.

In determining the migration constants of very dilute solutions, it would be quite impossible to measure the changes of concentration by the ordinary methods of chemical analysis. The measurement of the electrical conductivity of such a solution affords however a sufficiently delicate method of obtaining its concentration, and this method has been employed in the present work.

The experiments here described show that the apparent value of the migration constant of a solution of hydrochloric acid increases considerably as the concentration is diminished. It was thought that this result indicated a decrease in the velocity of the hydrogen ion as the cause of the low conductivity of the solution. A recently published paper by Whetham and Paine* shows however that another explanation is more probable. These authors have carried out a series of measurements on solutions of sulphuric acid, by a method similar in principle to that which has been employed by the present writer, and have found a change in the migration constant, which is similar, though smaller in amount, to that which has been obtained in the case of hydrochloric acid. They find that the change can be explained on the supposition that the conductivity of the solvent water is partly due to the presence of a salt formed from a weak acid and a weak base, such as ammonium carbonate; this substance may be present in sufficient quantity, owing to the absorption of atmospheric ammonia and carbon dioxide. A small quantity of ammonium carbonate would account for the observed change in the migration constant, and also for the decrease in the apparent equivalent conductivity of a dilute solution of acid or alkali. They therefore conclude that their experiments give no evidence in favour of the hypothesis that the velocity of the hydrogen ion becomes smaller at great dilution.

The writer's experiments on hydrochloric acid had not been

* *Proc. Roy. Soc.* LXXXI. A, p. 58.

completed when the work of the above authors became known to him. The results already obtained had shown a large increase in the apparent value of the migration constant; the degree of accuracy attained with the most dilute solutions was however not very satisfactory. The experiments have not been continued, since it appeared that no information as to the existence of any variation of the true mobility of the hydrogen ion would be obtained; a short account of the method and of the preliminary results is however given.

Experimental.

In conducting migration experiments with hydrochloric acid, it was considered advisable to keep the electrolysing current quite small, in order to prevent any evolution of chlorine at the anode; the values deduced from the anodic and cathodic changes of concentration should then be equally trustworthy. For this reason the vessel used to contain the solution was arranged in such a way that a comparatively small volume of liquid near each electrode might be separated from the rest; the resulting change in conductivity would thus be greater than if the liquid after the passage of the current were merely divided into two equal portions.

The cell is shown in Fig. 1. The main portion consists of a glass tube 1·3 cm. in diameter, bent into the form shown, the total length of the column of liquid from A_1 to A_2 being about 80 cm. The current enters and leaves the solution by the electrodes A_1, A_2, which are of stout platinum foil, platinized and subsequently heated to redness. B_1C_1, B_2C_2 are two pairs of electrodes which are used for measuring the resistance of the solution, connexion being made through the narrow tubes D, which are slipped over the platinum wires which support the electrodes, and fixed by sealing-wax.

The upper end of each main tube is closed by the indiarubber bellows F, which forms an air-tight joint, and at the same time allows the electrode A to be raised and lowered for the purpose of stirring the liquid. The separation of the anodic and cathodic portions of the solution is carried out by increasing the pressure of the air at K; the effect of this is to depress the liquid below the level of the bends at S, S, and at the same time to cause it to rise at each end of the tube to some distance above the electrodes B, C.

The solution is made up in the stoppered pipette M. A quantity of distilled water is placed in it, and the required amount of a standard solution of acid is then run in from a small filling vessel, which is weighed before and after the operation. The pipette with

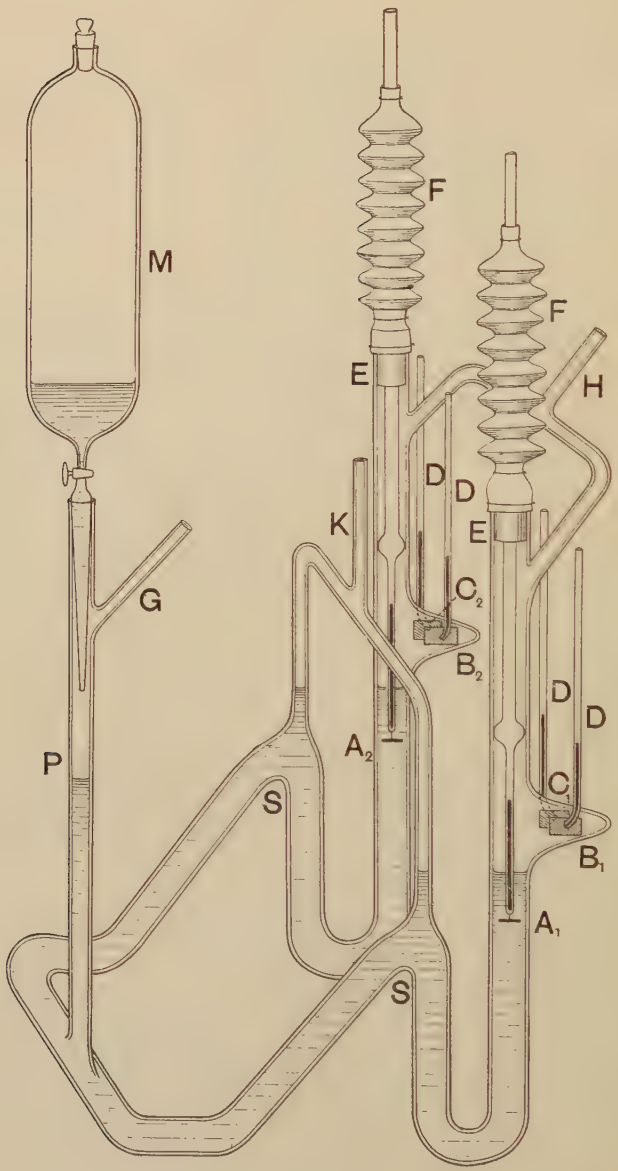


Fig. 1.

its contents is now weighed, and the concentration of the solution can thus be calculated when that of the stock solution is known. Three such stock solutions were prepared; the strongest (·2154 normal) was estimated by precipitation as silver chloride, and the other two were obtained by diluting this with weighed quantities of water. The water used had an average conductivity of about 10^{-6} reciprocal ohms at 18° C.

The cell was immersed in a tank of water, which was well stirred and maintained at a temperature of 18° C. by means of a toluene regulator.

The method of carrying out an experiment was as follows. The solution was made up in the pipette M, of which the delivery-tube was ground to fit the upper end of the vertical tube P. A current of air, purified from ammonia and carbon dioxide, was then slowly drawn through the cell, being introduced through the tube K and led out at G and H. The liquid was then run in from the pipette up to a mark on the tube at P, the volume of solution used being thus the same in all the experiments. The pressure over K was now increased, so that the liquid covered the electrodes B, C, and the resistance at each end measured several times by means of a commutator and galvanometer.

The liquid was now brought back to its original position, and a current passed through the solution from A_1 to A_2, usually for about an hour. The current was obtained from a battery of about 40 storage cells, and was measured by balancing the E.M.F. across the ends of a resistance included in the circuit (varying from 2000 to 30000 ohms according to the strength of the solution) against the E.M.F. of a Clark cell. The current was maintained at a constant value by adjusting the applied E.M.F. by means of a potential-divider.

After switching off the current, the pressure at K was again increased, and the separated portions of the solution stirred by means of the electrodes A. The resistance was then measured at each end, as at the beginning of the experiment.

Let R, R' be the measured resistances at the cathode before and after the passage of the current, k, k' the corresponding conductivities of the acid, and w the conductivity of the water. We then have

$$\frac{k'+w}{k+w} = \frac{R}{R'},$$

and therefore

$$\frac{\delta k}{k+w} = \frac{\delta R}{R'},$$

where

$$\delta R = R' - R, \quad \delta k = k - k'.$$

Hence, from the equation on p. 56, we obtain

$$1000p = \frac{qV\delta n}{Q}$$

$$= \frac{qV}{Q} \cdot \frac{dn}{dk} \cdot \frac{\delta R}{R'} (k + w).$$

From the numbers given by Kohlrausch, a curve was plotted, with n as abscissa and k as ordinate. This curve is a straight line over the range covered by these experiments, and we may take the value of $\frac{dn}{dk}$ to be constant. The value of k corresponding to the known concentration of the solution was read off from the curve, and the migration constant p calculated from the above equation.

The volume V of the separated liquid was determined as follows. The cell was placed in position in the tank, and filled with water up to the mark on the tube at P. It was then weighed. The separation was now carried out in the usual manner, the electrode on one side removed and dried, and the separated water extracted, the last drops being removed with filter-paper. The cell was again weighed, and the difference of weight gave the weight of water separated. The whole process was then repeated for the other side of the apparatus.

The results of the experiments are given in the following table. The concentration n of the solution is given in gram-

n	$n^{\frac{1}{3}}$	Current and time	p		
			Cathode	Anode	Mean
$1{\cdot}405 \times 10^{-3}$	·1120	$5{\cdot}72 \times 10^{-4}$ 60 min.	·1715	·1705	·171
$6{\cdot}07 \times 10^{-4}$	·0847	$2{\cdot}86 \times 10^{-4}$ 50 min.	·192	·197	·194
$5{\cdot}82 \times 10^{-4}$	·0835	$2{\cdot}87 \times 10^{-4}$ 50 min.	·195	·188	·192
$1{\cdot}633 \times 10^{-4}$	·0547	$7{\cdot}15 \times 10^{-5}$ 60 min.	·221	·204	·213
$1{\cdot}035 \times 10^{-4}$	·0469	$4{\cdot}77 \times 10^{-5}$ 60 min.	·261	·252	·256
$1{\cdot}006 \times 10^{-4}$	·0465	$4{\cdot}77 \times 10^{-5}$ 60 min.	·268	·282	·275

equivalents per thousand grams of solution, and the current in amperes. The migration constant for stronger solutions of hydrochloric acid, as measured by chemical methods, is given by Jahn* as ·167, remaining the same for dilutions varying from 31 to 151 litres per gram-equivalent. The strongest solution used in the present experiments was 712 litres per gram-equivalent.

From this table a few experiments have been omitted, in which for some accidental reason the measurements on one side of the apparatus were not trustworthy. The check given by the agreement between the results obtained from the cathode and anode was considered of great importance, for in such dilute solutions absorption of impurities from the glass or from the air was very likely to occur during the progress of an experiment. In fact with the most dilute solutions there was generally a gradual increase of the measured resistance with time, which became

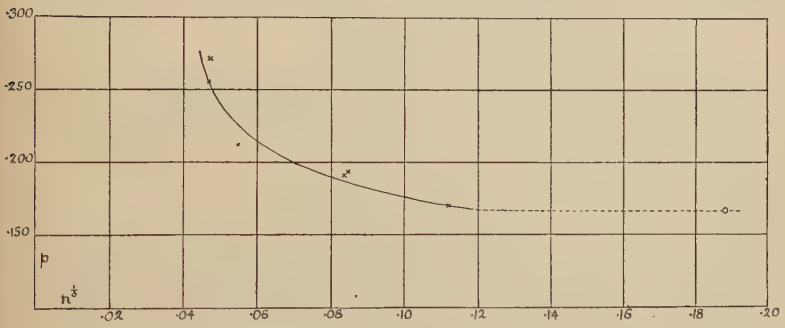


Fig. 2.

more marked when the liquid was stirred. This increase of resistance would however tend to increase the value of p deduced from the measurements at the cathode, and to diminish the value deduced from the measurements at the anode. No experiment has therefore been retained in which both results were not available.

The values of the migration constant have been plotted against the cube root of the concentration in the diagram (Fig. 2). Jahn's value for the concentration $n = \frac{1}{150}$ is also given; it is marked on the diagram by a circle.

Discussion of Results.

The change in the migration constant is much greater than that which has been observed by Whetham and Paine in the case of sulphuric acid. It has already been remarked that the numbers

* Jahn, *Zeitschr. f. Phys. Chem.* vol. LVIII. p. 641 (1907).

obtained with the most dilute solutions are not very trustworthy, mainly owing to the fact that the measured resistances did not remain constant, but generally increased with time. We can easily see however that the change is too great to be explained by a decrease in the mobility of the hydrogen ion. It has already been mentioned that the curve obtained by plotting the conductivity k of a solution of hydrochloric acid against the concentration n is a straight line which does not pass through the origin; it can however be made to do so by increasing the values of k by a constant quantity, equal to $3{\cdot}7 \times 10^{-6}$. This "corrected" curve we may then take to represent the relation between k and n which would hold if the acid were completely ionized, and the hydrogen ion possessed its maximum mobility. From the difference between the ordinates of the actual and the "corrected" curve for any given value of n, we can find what reduction in the velocity of the hydrogen ion would produce the observed diminution of conductivity. Making the calculation for a concentration 6×10^{-4} ($n^{\frac{1}{3}} = {\cdot}084$), we find that the decrease in velocity would be such as to cause an increase in the migration constant from ·167 (the normal value) to ·170. The value given by the curve (Fig. 2) for this concentration is ·191. The theory of diminished mobility of the hydrogen ion is thus seen to be insufficient to account for the observed change.

Let us now consider the alternative hypothesis, that the increase in the migration constant is an apparent one merely, and is due to the presence, in addition to the acid, of some neutral substance of which the velocities of the anion and cation are more nearly equal to one another than are those of hydrogen and chlorine. Whetham and Paine have calculated the effect of such an impurity, and have shown that if the difference of mobility of its anion and cation is small compared with the mobility of the hydrogen ion, the migration constant is increased in the ratio $(ku + k'v)/(k + k')u$, where k, k' are the partial conductivities of the hydrochloric acid and of the second substance respectively, u the mobility of the chlorine ion, and v that of the hydrogen ion. If then we calculate from this result the value of k' which would give the observed rise in the migration constant, we find $k' = 8{\cdot}7 \times 10^{-6}$.

The conductivity of the water used as solvent in the two experiments, in which the concentration was approximately 6×10^{-4} gram-equivalents, was $9{\cdot}7 \times 10^{-7}$ reciprocal ohms. It is clear therefore that this water did not contain a sufficient quantity of any neutral salt to produce the observed effect. It was preserved in vessels of Jena glass or of platinum; the pipette in which the solutions were made up, and the migration cell

itself, were however constructed of soft glass. It is however quite impossible that the conductivity of the water could have been greatly increased by solution of the glass; in one case a solution of which the strength was $1{\cdot}63 \times 10^{-4}$ gram-equivalents per litre was allowed to stand in the pipette for a period of three days, and the conductivity at the end of that time did not differ by more than 2 per cent. from Kohlrausch's value.

The observed results can probably be explained if we suppose that the principal impurity in the water was ammonia dissolved from the air. The ionization of ammonia is small, even in very dilute solution; when hydrochloric acid is added, ammonium chloride, which is highly dissociated, is produced. The partial conductivity of the ammonium chloride will then be considerably greater than the conductivity of the original ammonia, and the quantity present may be sufficient to produce the observed change in the apparent migration constant for the solution.

It is interesting to note, that Whetham and Paine concluded from their experiments that the impurities in their water probably consisted of ammonium carbonate together with an excess of carbonic acid. Goodwin and Haskell*, in an investigation of the conductivity of dilute solutions of hydrochloric and nitric acids, found that different samples of water, although their conductivities might be equal, gave solutions of which the conductivities were markedly different; the conductivity of the acid, corrected for the effect of impurities, was however the same in all cases. The large difference between the results of the present experiments and those of Whetham and Paine is therefore probably due to the specific effects of different impurities, and affords additional evidence in favour of the theory that both the change in the migration constant and the drop in the equivalent conductivity curve are due to an interaction between the acid and the impurities present in the solvent.

The author desires, in conclusion, to express his thanks to Professor Sir J. J. Thomson for his kind interest and encouragement during the progress of the work.

* Goodwin and Haskell, *Physical Review*, Dec. 1904.

On the Carriers of the Positive Charges of Electricity emitted by hot wires. By Sir J. J. Thomson, M.A., F.R.S., Cavendish Professor of Experimental Physics.

[*Read* 9 November 1908.]

A series of measurements of the values of e/m for the positively electrified particles given out by a strip of platinum wire heated to incandescence were made by the method described by the author in a paper on 'Positive Rays of Electricity,' *Phil. Mag.* Oct. 1908. When the platinum had been kept for several days in a high vacuum and heated repeatedly to incandescence, the value of e/m for the great majority of the positively charged particles was about $10^4/27$, showing that the mass of the particles was about 27 times the mass of an atom of hydrogen. The masses of molecules of CO and N_2 are 28 times that of an atom of hydrogen, and these molecules could not be distinguished by determinations of e/m. The spectroscopic examination of the gas given off by the hot wire after prolonged heating showed that the CO spectrum was bright while that of nitrogen could not be detected. For this reason I think the carriers of the positive electricity are molecules of CO and not N_2.

After the platinum by long continued heating had been brought into a state when the carriers had for the most part the mass of a molecule of CO, hydrogen was let into the vessel and the platinum foil made red-hot in an atmosphere of hydrogen, the hydrogen was then pumped out; determinations of e/m after this process had been gone through showed that the average mass of the carriers was only 8 or 9 times that of the hydrogen atom, thus the absorption of the light gas by the platinum had diminished the average weight of the carriers to about one-third of its original value; this I think shows that the carriers of the positive electricity given out by hot metals are for the most part the molecules of gas absorbed by the metal. I have much pleasure in thanking Mr G. W. C. Kaye for the assistance he has given me in making these experiments.

On the Electric Theory of Gravitation. By Sir J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics.

[*Read* 9 November 1908.]

The view that gravitational attraction is due to a slight excess of the attraction between unlike charges of electricity over the repulsion between like charges is a very old one, indeed it seems to have been regarded by some writers as almost a part of the one fluid theory of electricity. The theory is an interesting one because it involves the existence of effects which do not seem to be hopelessly too small to be tested by experiment: some of these, relating to the possible influence of the velocity of the attracting bodies on the gravitational attraction between them, have recently been considered by Lorentz. At the end of this paper I shall indicate another result of the theory which is of so special a character, that if it were established by experiment, it would follow almost as a matter of course that gravity must be due to something very closely connected with electrical action.

Before discussing this effect I will consider the theory from the point of view of stresses in a medium between the attracting bodies. In one form of electrical theory we suppose that the stress in the ether is altered by the passage through it of lines of electric force, in such a way that the tension along the lines of force and the pressure at right angles to them is increased by an amount proportional to the square of the density of the lines of force. When we endeavour to use lines of electric force to describe the state of the magnetic field or of fields through which electrical waves are passing, I think there are considerable advantages in regarding the lines of electric force from a somewhat different point of view from that usually adopted in electrostatics. In that subject it is usual to take as the lines of force due to say a positive charge e at A, and a negative charge $-e$ at B, as the lines whose directions are the resultants of the radial forces e/AP^2 and $-e/BP^2$, radiating from A and B respectively. We might however regard the lines of force as consisting of two sets, one set being straight lines radiating from A, the other straight lines radiating from B. We might, that is, regard the component fields of which the actual field is made up as having an actual physical existence, and suppose that it is the effect they produce and not their structure which is modified when several of them exist simultaneously in the same field. We can illustrate the difference between the two methods by considering the case of two parallel vertical planes A and B, A being on the left of B,

the one having a positive the other an equal negative charge. On the usual view, lines of force only exist between A and B, the lines starting from the positive charge on A and ending on the negative charge at B; the region to the left of A or the right of B being free from lines of electric force. On the other view positive lines of force start from A both to right and left, and also negative lines of force from B. In the region between A and B we have the positive lines in one direction and the negative in the opposite, both giving a contribution of the same sign to the electric force; while on the left of A and the right of B the negative and the positive lines run side by side in the same direction and, as far as the electric force goes, neutralize each other's effect.

The advantages of the second method are most felt when the electric charges are moving and there is a magnetic as well as an electrostatic field, the magnetic force at any point being regarded as due to the movement of the lines of electric force at that point, and proportional to the vector product of the number of lines of electric force and the velocity of these lines. Thus suppose the plate A is rotating while B is at rest, on the second view of the disposition of the lines of force in the field, all the positive lines of force rotate with A, thus there is motion of these lines, and therefore magnetic force throughout the whole of the field and not merely between the plates A and B: and we know from experiments that the magnetic force does exist throughout the whole region and is not localized between A and B. If we take the first view that the lines of force, due to the charges on A and B, are confined to the region between A and B, then the rotation of A would only set these lines in motion in the space between A and B, this motion would only account for the magnetic field between the plates, and we should have to introduce further hypotheses to account for the magnetic forces which as experiment shows exist in the other regions.

If we consider the differences which exist between the properties of negative and positive electricity, the negative being located on corpuscles, the positive on bodies of atomic dimensions, it does not seem by any means impossible that there may be some difference between the positive and the negative lines of force, at any rate the effects of such a difference seem a legitimate subject for discussion.

Let us take the case where all the lines of force are parallel to the axis of x, and suppose that through unit area, at right angles to x, there are P positive lines of force running in the direction in which x increases, N negative lines of force in the same direction; then on the usual theory of stress in the medium, in which we do not distinguish between the effects of positive and negative lines, these lines increase the tension along the

lines of force by $k(P-N)^2$ where k is a constant. If we discriminate between the effect of positive and negative lines we must replace this expression by

$$\alpha P^2 + \beta N^2 - 2\gamma PN,$$

where α, β, γ are not necessarily equal to each other. On this assumption let us calculate what would be the attraction between two slabs of matter A and B. Suppose that for each unit of surface of A there are P_1 units of positive electricity, N_1 of negative, while P_2 and N_2 are the corresponding quantities for B.

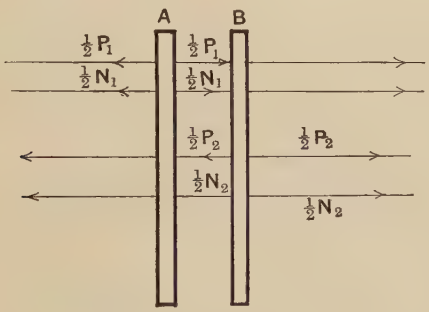


Fig. 1.

If P is the number of positive lines of electric force through unit area, N the number of negative, then we see that between the plates

$$P = \tfrac{1}{2}(P_1 - P_2), \quad N = \tfrac{1}{2}(N_1 - N_2),$$

to the right of B,

$$P = \tfrac{1}{2}(P_1 + P_2), \quad N = \tfrac{1}{2}(N_1 + N_2),$$

to the left of A,

$$P = -\tfrac{1}{2}(P_1 + P_2), \quad N = -\tfrac{1}{2}(N_1 + N_2).$$

The force on unit area of B towards A will be the tension in the region between A and B minus that in the region to the right of B, i.e. it will be equal to

$$\begin{aligned}&\{\tfrac{1}{4}\alpha(P_1 - P_2)^2 + \tfrac{1}{4}\beta(N_1 - N_2)^2 - \tfrac{1}{2}\gamma(P_1 - P_2)(N_1 - N_2)\}\\ &-\{\tfrac{1}{4}\alpha(P_1 + P_2)^2 + \tfrac{1}{4}\beta(N_1 + N_2)^2 - \tfrac{1}{2}\gamma(P_1 + P_2)(N_1 + N_2)\}\\ &= -\alpha P_1 P_2 - \beta N_1 N_2 + \gamma(P_1 N_2 + P_2 N_1) \quad \ldots\ldots(1).\end{aligned}$$

Let us take the case when both A and B are electrically neutral, i.e. when $P_1 = N_1$, $P_2 = N_2$, then this force will be equal to

$$\{2\gamma - (\alpha + \beta)\} P_1 P_2.$$

If we suppose that the atoms of the different elements are electrical systems, and that each atom contains a number of units of positive and also of negative electricity proportional to the atomic weight, then P_1 and P_2 will be proportional to m_1, m_2, the masses of unit areas of the slabs A and B, so that the attraction between the slabs will be proportional to $\{2\gamma - (\alpha + \beta)\} m_1 m_2$. Thus the attraction between two infinite slabs will be proportional to the product of their masses and independent of the distance between them, it follows from this that the attraction between finite masses separated by distances large compared with their linear dimensions, will vary inversely as the square of the distance between them.

If we take the number of positive units of electricity in the atom as equal to the atomic weight, then using the electrostatic system of units

$$\frac{P_1}{m_1} = \frac{P_2}{m_2} = 10^4 \times 3 \times 10^{10},$$

when m_1 and m_2 are measured in grammes.

Thus the attraction between the slabs

$$= \{2\gamma - (\alpha + \beta)\} 9 \times 10^2 m_1 m_2.$$

The gravitational attraction between the slabs is

$$2\pi \times 6{\cdot}6 \times 10^{-8} \times m_1 m_2,$$

hence if the attraction we are considering is the gravitational attraction

$$2\gamma - (\alpha + \beta) = \frac{2\pi \times 6{\cdot}6}{9} \times 10^{-36}.$$

But α, β, γ are all very nearly equal to $1/8\pi$, hence

$$\frac{2\gamma - (\alpha + \beta)}{\alpha} = 1{\cdot}2 \times 10^{-34}.$$

If in equation (1) we put $N_1 = N_2 = 0$, we see that the repulsion between these charges is $\alpha P_1 P_2$, if we put $P_1 = P_2 = 0$, the repulsion is $\beta N_1 N_2$, and if we put $N_1 = 0$, $P_2 = 0$, the attraction is $\gamma P_1 N_2$, thus α is proportional to the force between two unit positive charges, β to the force between two unit negative charges and γ to the attraction between a unit positive and a unit negative charge. It is to be noted that unless $\alpha = \beta = \gamma$, we have to amend the definition of unit charges usually given, as in this case, if the force between two units of positive charge at unit distance is the unit force, the force between two units of negative charge will not be so. If we recognise these differences between positive and negative electricity the definition of an unelectrified body is a matter

of some delicacy unless we have recourse to the atomic theory of electricity, and define an unelectrified body as one which contains equal numbers of positive and negative units of charge.

The weight of a body on this view depends only on the charges of electricity it contains, thus if we suppose an atom of hydrogen to contain one positive and one negative charge, the weight of an atom of hydrogen would only be twice that of a corpuscle which contains one negative unit, but the mass of the hydrogen atom is 1700 times that of the corpuscle, hence the acceleration of the corpuscle under gravity would be 850 times that of an atom of hydrogen, or 850×981; if the atom of hydrogen contained n units of positive and n of negative electricity the acceleration of the corpuscle would be $\frac{1}{n}\, 850 \times 981$, hence on this view we should expect the acceleration of the corpuscle under gravity to be very much greater than that of ordinary matter. At present the detection of an acceleration of the corpuscle as great even as 850×981 would seem to be beyond the powers of known methods, but not so much beyond as to preclude the hope that with such improvement as we may reasonably expect in the manipulation of slow cathode rays it may ultimately be capable of investigation.

On the Distribution of Electric Force along the Striated Discharge. By Sir J. J. Thomson, M.A., F.R.S., Cavendish Professor of Experimental Physics.

[*Read* 9 November 1908.]

A Wehnelt hot lime cathode was used to produce the discharge as it was found that at low pressures the striations produced in this way were remarkably steady and bright and in consequence made accurate measurements of the distribution of electric force much easier than with the ordinary discharge. It was found that just in front of the bright surface of a striation towards the cathode there was a reversal of the electric force. This reversal causes a great accumulation of ions in the part of the striation nearest the cathode, the recombination of the ions in this region will therefore be much greater than elsewhere and it is shown that a very simple explanation of the formation and behaviour of striations was given by the hypothesis that the recombination of the ions was the source of the luminosity in the striations.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On a configuration of twenty-seven hyper-planes in four-dimensional space. By Professor W. BURNSIDE, F.R.S.

[*Received* 23 January 1909.]

[*Read* 9 February 1909.]

Clifford has shewn the existence of a plane figure consisting of 2^n circles and 2^n points such that each circle passes through $n+1$ of the points and each point lies on $n+1$ of the circles*. If this figure be inverted with respect to a point outside its plane, the circles become plane sections of the sphere into which the original plane inverts. The configuration may then be specified as one of 2^n planes and 2^n points, such that in each plane lie $n+1$ of the points and through each point pass $n+1$ of the planes. The set of points is, however, restricted to lie on a sphere, or, if the configuration is modified by a projective transformation, on a quadric.

That such a configuration exists, apart from the restriction of the points to lie on a quadric, is true; and I believe a proof of the fact has been published though I cannot give a reference to it.

In Mr Grace's memoir a proof is given of the existence of some very remarkable configurations of spheres and points in three-dimensional space (pp. 182—188). If one of these, regarded as existing in four-dimensional space, be inverted with respect to a point, not in the three-dimensional space of the configuration,

* Clifford, *Collected Papers*, pp. 51, 52. See also J. H. Grace, "On circles, spheres and linear complexes" (*Camb. Phil. Trans.* Vol. XVI. pp. 153–190), with which memoir this note is more directly connected.

there results a configuration of points and hyper-planes; a certain number of hyper-planes passing through each point and a certain number of points lying in each hyper-plane; with the restriction that the whole of the points lie on a four-dimensional quadric. As in the previous case this restriction is not a necessary condition for the existence of the configuration. The object of the present note is partly to prove this result in a particularly interesting case, and partly to bring out the tactical analogy (which the numbers suggest) of the configuration of 27 hyper-planes with the configuration of 27 lines on a cubic surface.

To avoid the continual use of the word hyper-plane, a flat manifold in four-dimensional space, determined by four points, is called simply a plane; and when it is necessary to refer to one determined by three points it is called a three-dimensional plane.

A quadric in four-dimensional space is determined by fifteen points. Eleven points therefore determine four linearly independent quadrics. Every one of a system of quadrics through eleven points must therefore have five other common points. This and the fact that four points determine a plane and four planes determine a point forms the basis of the reasoning.

1. In a four-dimensional space, consider a base-point O, and five planes,

$$1,\quad 2,\quad 3,\quad 4,\quad 5,$$

passing through it. On the line of intersection of each set of three planes, mark a point distinct from O; and denote the point on the line of intersection of 1, 2 and 3 by 123. (In this symbol the sequence of the figures is immaterial.) Denote the plane which passes through 123, 124, 134, and 234 by 1234. (In this symbol again the sequence of the figures is immaterial.) Then the five pairs of planes

$$1{,}2345;\quad 2{,}1345;\quad 3{,}1245;\quad 4{,}1235;\quad 5{,}1234,$$

are a set of quadrics through 11 points, viz. O and the 10 points 123, etc. Hence they determine a set of 5 further points, which lie on all the quadrics, so that 8 of the 16 lie in each plane. Of these 5 then, one must lie on 1 and the other four on 2345. That one of the five which lies on 1 cannot also lie on 2; and a suitable notation for the five points in what follows will be

$$16,\quad 26,\quad 36,\quad 46,\quad 56,$$

(here the sequence of the figures is essential), where 16 lies in the five planes

$$1,\quad 1345,\quad 1245,\quad 1235,\quad 1234.$$

The figure thus constructed is a complete one in the sense that each point (and each plane) bears the same relation to the figure

as any other point (or any other plane). Through each point five planes pass, as given by the table

$$\begin{aligned} O &\ldots\ldots 1,\ 2,\ 3,\ 4,\ 5; \\ 123 &\ldots\ldots 1,\ 2,\ 3,\ 1234,\ 1235; \\ 16 &\ldots\ldots 1,\ 1345,\ 1245,\ 1235;\ 1234; \end{aligned}$$

and the points and planes admit simultaneous groups of permutations for which the tactical relations of this table are unaltered. The order of this group is $2^4 . 5!$; and as it affects the planes it is generated by

$$(1,\ 2345)(2,\ 1345);$$
$$(1,\ 2)(2345,\ 1345);$$
$$(1,\ 2,\ 3,\ 4,\ 5)(2345,\ 1345,\ 1245,\ 1235,\ 1234).$$

It contains an Abelian group of order 16, as a self-conjugate subgroup, and in respect of this is isomorphic with the symmetric group of degree 5.

2. Take now six planes,

$$1,\ 2,\ 3,\ 4,\ 5,\ 6,$$

all passing through the base point O; and on the 20 lines of intersection of each set of three mark arbitrary points 123, etc. With each set of five planes, out of the six, carry out the above construction, so that there arise fifteen fresh planes 1234, etc.; and thirty fresh points 16, etc. For convenience of reference the set of $(1 + 20 + 30 = 51)$ points thus arrived at may be denoted by S_0. The figure so far is obviously not complete in the sense already explained. That part of the figure which lies in the plane (three-dimensional space) 1, consists of 16 points, O, 123, ..., 156, 12, ..., 16 and 15 three-dimensional planes, viz. the intersections of 2, ..., 6, 1234, ..., 1456 with 1. This is one of the figures referred to in the introduction and is completed by noticing that

$$12,\ 13,\ 14,\ 15,\ 16$$

lie in a three-dimensional plane.

Take now 123 as a base point. Through it pass just six of the planes connected with O, viz.:

$$1,\ 2,\ 3,\ 1234,\ 1235,\ 1236.$$

On each of 19 out of the 20 lines of intersection of these planes one point (besides 123) of the set S_0 lie. Thus

$$\begin{aligned} \text{on}\quad & 1,\ 2,\ 3 && \text{there lies} && O, \\ & 1,\ 2,\ 1234 && \text{„} && 124, \\ & 1,\ 1234,\ 1235 && \text{„} && 16. \end{aligned}$$

On the line of intersection of the three planes 1234, 1235, 1236 there is no point of the set S_0. On this line mark an arbitrary point $1'2'3'$, distinct from O. With 123 as base point, the six planes through it and the twenty points (including $1'2'3'$) on their lines of intersection, complete the construction of the

beginning of this paragraph. Of the thirty additional points that arise it will be found that twenty-one belong to the set S_0, viz.:

$$\begin{array}{lllll} 145, & 245, & 345, & 45, & 54, \quad 12, \quad 21, \\ 146, & 246, & 346, & 46, & 64, \quad 13, \quad 31, \\ 156, & 256, & 356, & 56, & 65, \quad 23, \quad 32. \end{array}$$

The remaining nine, adhering to the notation already used, may be represented by

$$\begin{array}{lll} 1'2'4', & 1'2'5', & 1'2'6', \\ 1'3'4', & 1'3'5', & 1'3'6', \\ 2'3'4', & 2'3'5', & 2'3'6' \end{array}$$

(the sequence of the figures is immaterial), $1'2'4'$ being the point common to the five planes

$$1234, \quad 1245, \quad 1246,$$
$$(24, \ 25, \ 26, \ 1'2'3') \text{ and } (14, \ 15, \ 16, \ 1'2'3').$$

Since 12, 13, 14, 15, 16 lie in a three-dimensional plane, the two latter planes may be written

$$(21, \ 23, \ 24, \ 25, \ 26, \ 1'2'3'), \quad (12, \ 13, \ 14, \ 15, \ 16, \ 1'2'3').$$

Instead of 123 any of the twenty points of which it is one might be taken to make the last construction, and whichever is chosen one point must be arbitrarily chosen on a certain line before the construction can be carried out. The construction however, having led to a point $1'2'4'$ on the line of intersection of 1234, 1245, 1246, it is now possible to take 124 as a base point and, without introducing any further arbitrary element, to carry out the construction from this point. When this is done it is found that of the thirty additional points that arise twenty-one again belong to the set S_0, and the remaining nine are

$$\begin{array}{lll} \underline{1'2'3'}, & \underline{1'2'5'}, & \underline{1'2'6'}, \\ \underline{1'4'3'}, & \underline{1'4'5'}, & \underline{1'4'6'}, \\ \underline{2'4'3'}, & \underline{2'4'5'}, & \underline{2'4'6'} \end{array}$$

where $\underline{1'2'3'}$ is the point common to the five planes

$$1234, \quad 1235, \quad 1236,$$
$$(12, \ 13, \ 14, \ 15, \ 16, \ 1'2'4') \text{ and } (21, \ 23, \ 24, \ 25, \ 26, \ 1'2'4').$$

Comparing this with the specification of $1'2'4'$, it follows that $\underline{1'2'3'}$ is the same point as $1'2'3'$.

Similarly $\underline{1'2'5'}$, $\underline{1'2'6'}$, $\underline{1'4'3'}$ and $\underline{2'4'3'}$ are found to be identical with $1'2'5'$, $1'2'6'$, $1'4'3'$ and $2'4'3'$. Hence, the point $1'2'3'$ having been once chosen, when the construction of the beginning of this paragraph is carried out with each of the twenty points 123, ..., 456 in turn, besides the set S_0 a set of just twenty points (including $1'2'3'$) and no more will arise

Moreover, since

(12, 13, 14, 15, 16, 1′2′3′, 1′2′4′)

is a plane, it follows immediately that the fifteen points

12, 13, 14, 15, 16, 1′2′3′, 1′2′4′, 1′2′5′, 1′2′6′,
1′3′4′, 1′3′5′, 1′3′6′, 1′4′5′, 1′4′6′, 1′5′6′

all lie in a plane. Call this plane 1′ and the other five that arise in the same way 2′, 3′, 4′, 5′, 6′.

Through 16 pass the five planes

1′, 1345, 1245, 1235, 1234.

On the lines of intersection of 1′, 1245, 1235, 1234, there lie the points 26, 1′2′5′, 1′2′4′, 1′2′3′, and the plane through these points is the plane 2′. Hence from the configuration of § 1, the planes 1′, 2′, 3′, 4′, 5′ meet in a point through which 6′ must clearly pass. Call this point O'. There is then finally a set of 27 planes, viz.:

1, 2, 3, 4, 5, 6,
1234, 1235, ..., 3456,
1′, 2′, 3′, 4′, 5′, 6′;

and 72 points, viz.:

O,
123, 124, ..., 456,
12, 21, ..., 56, 65,
1′2′3′, 1′2′4′, ..., 4′5′6′,
O'.

Through each point pass six planes and in each plane lie sixteen points, and the configuration is complete. The relations are given by the scheme

O 1, 2, 3, 4, 5, 6;
O' ..... 1′, 2′, 3′, 4′, 5′, 6′;
123 ..... 1, 2, 3, 1234, 1235, 1236;
4′5′6′ ..... 4′, 5′, 6′, 1456, 2456, 3456;
12 ..... 1, 1′, 1456, 1356, 1346, 1345;
21 ..... 2, 2′, 2456, 2356, 2346, 2345.

A very slight modification in the notation, viz. the replacing of the symbol 1234 by (56) (the brackets will prevent confusion between the symbol for a plane and the symbol for a point) gives these relations a well-known form. In fact with Schläfli's notation for the 27 lines on a cubic surface, the table gives the 36 double-sixers that can be constituted from them. The tactical analogy between the 27 hyper-planes and the 27 lines on the surface is thus obvious; and the hyper-planes and points admit a group of 51,940 permutations for which the relations given by the table are invariant.

Note on some double fluorides of sodium. By W. A. R. Wilks, B.A., Caius College. (Communicated by Dr Fenton, F.R.S.)

[*Read* 8 February 1909.]

Many of the insoluble compounds by which elements are quantitatively estimated are, as might be expected, found in nature as minerals. Hence it was thought probable that if sodium could be estimated by means of some simple inorganic compound the latter would be found as a mineral. On considering the simple minerals containing sodium, the only one which might be suitable for the purpose appeared to be cryolite.

This compound has been artificially prepared by Deville (*An. Ch. Ph.* (3) 59, 82, 1860) and by Baud. The former saturated hydrofluoric acid by sodium carbonate and alumina in the proportion of three molecules of carbonate to one of alumina, evaporated and obtained the substance $3NaF.AlF_3$. The latter precipitated a solution of aluminium fluoride by sodium fluoride and obtained the hydrate of cryolite $6NaF.2AlF_3.7H_2O$. This hydrate is gelatinous and is soluble in water to the extent of ·352 gms. per 100 c.c. at 16° C. Natural cryolite which is $3NaF.AlF_3$ has the solubility ·034 gms. in 100 c.c.'s at 15°.

In order to find whether sodium could be detected as double fluoride, aluminium was dissolved in dilute hydrofluoric acid in a platinum dish until the acid was exhausted. On adding this solution to a solution of sodium chloride a gelatinous precipitate was obtained whose composition has not yet been determined. If the sodium chloride was dilute, however, the precipitate did not appear. It was further observed that boiling the solution facilitated the precipitation and that in this case the precipitate was no longer gelatinous but crystalline. It was also found that acids had a marked solvent action on the precipitate. After these preliminary observations another mode of preparation of the precipitant was resorted to.

Precipitated aluminium hydroxide which has been simply dried by a filter pump was added to a small quantity of strong hydrofluoric acid in a platinum dish. The hydroxide seems to exhaust the acid much better than does aluminium itself, and in the process of solution the moist hydroxide furnishes sufficient water to form a solution of convenient strength. In order to get the best results the hydrofluoric acid is allowed to stand for at least two days in contact with excess of hydroxide. The complex

hydrofluoaluminic acid formed is then so stable that hydrofluoric acid does not seem to be split off even on boiling. The solution was in fact boiled in a test tube for half an hour without any apparent action on the glass.

To diminish the concentration of hydrogen ions in the resulting solution, an acetate must be added. The acetates of nickel or copper can be used for this purpose as neither of these metals give precipitates of double fluorides. Hence to the hydrofluoaluminic acid solution an equal bulk of a cold saturated solution of copper acetate is added. This solution is then boiled and any slight precipitate filtered off. To this solution an equal volume of a 50 per cent. solution of alcohol is added. This makes the reaction still more sensitive. Care must be taken however not to add too much alcohol, otherwise a precipitate will be formed which appears to be redissolved only with difficulty on adding more water. If this precipitant has been carefully made the test for sodium can be carried out in test tubes.

The test is carried out as follows. About 5 c.c. of the precipitant is boiled in a test tube and then the sodium solution added. If the latter is strong there will be an immediate precipitate. If it is very weak, however, it will be required to be boiled for some time before the precipitate appears. To make quite certain that the precipitation was not due to action on the glass a blank experiment was carried out at the same time in another tube without adding sodium chloride. In this case there was no precipitate. The test appears to be very delicate, one part of sodium chloride in 20,000 parts of solution being readily detected.

The precipitate in these cases has not the composition of cryolite. Analysed by heating with strong sulphuric acid and weighing the sodium sulphate and alumina it seems that sodium and aluminium are present in the atomic proportions of 1 . 1 : 1. Further analyses are however being carried out.

Pure potassium and ammonium salts are not precipitated under the conditions stated above. The metals other than sodium which give precipitates are silver, magnesium, calcium, strontium, barium and lead.

Attempts are now being made to determine whether sodium can be quantitatively estimated in this way. It has already been proved that precipitation is almost if not quite complete.

It was considered possible that the elements related to aluminium treated in the same way would give precipitates with sodium. It was found that ferric hydroxide dissolved in hydrofluoric acid gave with sodium a light brown precipitate which has not yet been examined.

Chromium and beryllium, however, fail to give a precipitate even with concentrated solutions of sodium chloride.

An experiment on ionisation with γ rays. By L. Vegard, Cand. real. Universitetsstipendiat of the University of Christiania. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 8 March 1909.]

1. When a gas is ionised by Röntgen or γ rays, we are led to consider the action of ether waves or pulses upon the molecules of a gas. In order to explain the small amount of ionisation it is necessary to assume that the various molecules are in a different position relative to the pulse. Several possibilities are discussed by Sir J. J. Thomson in his book, *Conduction of Electricity through Gases*. As a way in which the molecules can have a different position relative to the pulse, he mentions the possibility that the wave-front has a structure. There is, however, another manner in which a different relative position could arise even when we assume a continuous wave-front, namely if the molecule had certain directions of ionisation.

The directions of the electric forces in a set of pulses travelling in the same direction must lie within narrow limits, e.g., all nearly parallel or perpendicular to a certain line. Now for a molecule to become ionised it might be required that the electric force should act in a certain direction relatively to some axis in the molecule, or it might require a series of pulses along this same direction.

If such conditions were necessary for ionisation, it would naturally cut down to a great extent the number of molecules ionised by the rays. It might possibly not be the only condition, but if in general n conditions, 1, 2, 3, ..., n, were necessary for ionisation, and if the probability for each of them were $p_1, p_2, \ldots, p_n$, the ionisation would be proportional to the product $p_1, p_2, p_3, \ldots, p_n$. We see from this, that if the probability for the occurrence of one of the conditions is altered a certain number of per cent., the total amount of ionisation would be altered in the same proportion. Thus if the action of the pulse within a certain angle relative to some axis of the molecule were a necessary condition, the change in the probability of the occurrence of this condition would affect the total amount of ionisation proportionally.

If now the rays had a continuous wave-front, we might be able to effect a detectable change in the probability of this condition. Suppose a cylindrical ionisation chamber is traversed by a beam of parallel rays perpendicular to the axis of the cylinder. If now we add another beam of rays traversing the same space of air

inside the cylinder, we should get a different amount of ionisation according to the different angle between the two beams. Another way of putting it is to say, that the ionisation with such rays with *a continuous wave-front* would not be an additive property if the *molecules had some axis of ionisation.*

For, as the wave-trains from one source in rapid succession traverse the chamber, these wave-trains would ionise those molecules that had a certain direction relative to the pulse, and if now a second source was applied with waves mainly in the same direction, a number of those molecules that otherwise would have been ionised by the second source are already picked up by the first one.

2. It was from these considerations that I was led to undertake the following experiment, the object of which was to find whether the ionisation by γ rays was strictly an additive property.

Experiments on the additivity of ionisation have earlier been made by T. Noda*. In his experiments, however, the two sources gave rays of a different kind, and further one of his sources was radium giving out a mixture of rays, so that these experiments were of no use for deciding this question.

Description of experiment.

3. The ionising chamber had the form of a flat cylinder (length 4·9 cm., diam. 15·5 cm.). The ends of the cylinder were made of aluminium plates, the tube itself of a thin aluminium sheet. A circular aluminium plate was fixed inside the cylinder perpendicular to its axis by means of an aluminium rod connected to an electroscope of the Wilson type. The rod was surrounded by a guard-ring which was connected to earth, insulated from the rod by means of sulphur, and fixed to the chamber by a plug of ebonite. The wires leading to the electroscope were surrounded in the usual way by conductors connected to earth.

The ionising chamber was placed with its axis vertical upon a small wooden bench fastened to the table. The γ radiation came from two sources of radium; one of them could be placed in a fixed position, the other on a moveable arm capable of rotating about a vertical axis coinciding with the axis of the cylinder. Thus the angle between the two bundles of rays could be altered. Each piece of radium was placed behind two slits formed by large lead blocks, in such a way that the radiation from each source was mainly restricted to the space between two parallel horizontal planes cutting the ionisation chamber at about equal distances (1 cm.) above and below the inner plate. The distances from the

* T. Noda, *Proc. of Camb. Ph. Soc.* Vol. XIII. 1906, p. 356.

sources to the centre of the cylinder were about 35 cm. The α and β radiation was cut off by lead plates 3 mm. thick. In all other directions the radiation was stopped by piles of large lead and iron blocks forming a layer about 5 inches thick around the radium.

The ionisation was measured by the saturation current between the inner plate and the chamber, the latter being given a potential sufficient for saturation when the inner plate had a potential nearly equal to that of the earth. The procedure was as follows:

The air in the chamber was first exposed to the radiation from one source (a) alone, and the saturation current measured; the second source (b) was then placed behind the second slit, while (b) was left unaltered, and the current measured; then (a) was removed and the measurements repeated. This operation was performed for two different positions of the moveable source corresponding to the angles, 90° and 180°, between the mean direction of the ray bundles.

The saturation current is inversely proportional to the time t required for the gold leaf to fall between the same two marks on the scale of the electroscope. This, however, is only true when there is no leak and the zero point corresponding to zero potential of the leaf remains constant. The leak observed when all radium was removed from the room was too large to be disregarded. The velocity of the gold leaf due to the leak was found for different positions of the leaf. From this the time T required for the leaf to move through the scale interval on account of the leak could be found. If the time actually measured is t', and if $\frac{t'}{T}$ is a small quantity, the time t, corrected for leak, will be $\frac{1}{t} = \frac{1}{t'} + \frac{1}{T}$. The leak might, however, be altered by the presence of radium in the room; by cutting off the rays that went directly to the chamber the leak could be estimated, and was found to be very little altered by the presence of radium. It is, however, not necessary to know this part of the leak, for we can assume that it is an additive property.

The condition for additivity can then be written:

$$\frac{1}{t_a'} + \frac{1}{t_b'} + \frac{1}{T} - \frac{1}{t_{ab}'} = 0.$$

In general this expression may be equal to some quantity ϵ. The quantity $\delta = t_{ab}'\epsilon$ will then give a measure of the departure from additivity.

The results of the measurements are given in the following table:

Part I. Angle 180°.

Interval	$\frac{1}{t_a'}$	$\frac{1}{t_b'}$	$\frac{1}{T}$	$\frac{1}{t_a'}+\frac{1}{t_b'}+\frac{1}{T}$	$\frac{1}{t_{ab}'}$	δ
50 divisions	0,1723	0,1394	0,0051	0,3168	0,3167	0,000
50 „	0,1739	0,1382	0,0041	0,3162	0,3122	+0,013
50 „	0,1828	0,1451	0,0040	0,3319	0,3315	+0,001
40 „	0,2231	0,1663	0,0065	0,3959	0,3965	−0,002
50 „	0,1885	0,1450	0,0040	0,3374	0,3396	−0,007
50 „	0,1885	0,1452	0,0039	0,3376	0,3389	−0,004

Mean value $\delta = +0{,}0002$.

Part II. Angle 90°.

Interval	$\frac{1}{t_a'}$	$\frac{1}{t_b'}$	$\frac{1}{T}$	$\frac{1}{t_a'}+\frac{1}{t_b'}+\frac{1}{T}$	$\frac{1}{t_{ab}'}$	δ
40 divisions	0,2237	0,1636	0,0052	0,3925	0,3922	+0,001
30 „	0,2916	0,2393	0,0057	0,5366	0,5379	−0,002
30 „	0,2917	0,2389	0,0048	0,5354	0,5397	−0,008

Mean value $\delta = -0{,}003$.

The time is given in minutes.

The table shows that within the limits of experimental error the ionisation of air with γ rays is strictly an additive property, and that, as far as the effect sought for is concerned, the experiment has given a negative result. In view of the previous considerations the result only shows that at least one of the two conditions necessary for the effect must be wanting; but apart from its connection with the previous considerations the result is of interest in itself, as giving a property characteristic of ionisation with γ rays.

The absorption spectra of solid tetramethylpicene and of its solutions. By J. E. Purvis, M.A., St John's College, and Miss A. Homer, Fellow of Newnham College.

[*Read* 8 February 1909.]

From the products of the action of aluminium chloride on naphthalene investigated by one of us (Homer, *Trans. Chem. Soc.* 1907, vol. xci. p. 1103) there was isolated a new hydrocarbon whose empirical formula was $C_{26}H_{22}$. It was suggested that this substance was an alkyl derivative, probably tetramethyl, of dinaphthanthracene, $C_{22}H_{14}$.

In a later paper by the present authors (Homer and Purvis, *Trans. Chem. Soc.* 1908, vol. xciii, p. 1319), it was thought that further evidence as to the constitution of this hydrocarbon might be obtained from a comparative study of the absorption spectra of its solutions with the spectra of solutions of the supposed parent substance, dinaphthanthracene, and of picene the isomeride of dinaphthanthracene. Benzene solutions of these hydrocarbons were compared because picene and dinaphthanthracene are practically insoluble in alcohol. As a result of the investigation it was found that the absorption curve of the hydrocarbon $C_{26}H_{22}$ exhibited the same type of curve as picene, and therefore the substance was considered to be an alkyl derivative of picene.

Some comparative experiments were conducted about the same time, both with solutions in alcohol and in benzene, with the solid and with the vapour of the hydrocarbon $C_{26}H_{22}$. With regard to the solutions, $N/1000$ solutions in benzene and in alcohol were taken and the absorption curves plotted in the same way as is described in the previous paper (*loc. cit.*). The solid hydrocarbon $C_{26}H_{22}$ liquefies at 49—50° C., so that it was quite easy to melt a little of the substance on microscopic glass coverslips, by which a very thin layer was uniformly spread over the surface of the glass: the layer was quite translucent. The glass was then clamped before the slit of a spectroscope and the light of a Nernst lamp, of an iron arc, and of a condensed iron spark were used as the sources of light. Photographs were taken of the

resultant absorption spectrum and they were compared with the absorption spectra of the benzene and alcoholic solutions. It was found that in the three series of experiments the substance showed three absorption bands, but that the relative position of the bands varied. The bands of the solid were moved more towards the red end of the spectrum than those of the alcoholic solution, whilst their positions in the benzene solution were between those of the solid and the alcoholic solution. In order to compare the results the following table gives the numbers of the three bands in terms of the mean oscillation frequencies:

$C_{26}H_{22}$,	solid	2264	2405	2537
$N/1000$	benzene solution	2290	2424	2584
$N/1000$	alcoholic „	2304	2441	2590

The numbers for the benzene solution are extracted from the before-mentioned paper by Homer and Purvis (*loc. cit.*). And comparing the positions where general absorption begins the numbers are:

$C_{26}H_{22}$,	solid	2182
	benzene solution	2212
	alcoholic „	2247

It is evident from these numbers that there was a shift in the position of the bands and of the general absorption towards the red end of the spectrum according to the density of the medium, whilst in the solid state the shift was more marked still.

The rate of vibration of the molecules of the solute must be affected by the molecules of the solvent, that is to say the more dense the medium the greater damping effect will it have on the rate of vibration of the dissolved molecules.

Now in the solid state the mean free path of the molecules is more restricted than when they are distributed throughout some solvent, therefore the rate of vibration of the molecules of the substance in the solid state should be slower than when in solution.

Assuming that the general absorption is due to the vibrations of the molecule and that the selective absorption is caused by intra-molecular vibrations of the atoms which are also affected by the molecular vibrations, then conditions which tend to damp the rate of the molecular vibrations, that is, which cause a shift in the general absorption will also cause a corresponding shift in the selective absorption of the substance.

The results obtained are in accordance with this view, for a comparison of the positions of the general and selective absorption of the hydrocarbon in alcoholic and benzene solutions and in the solid state shows that for benzene solutions there is a shift towards

the red end, and that this shift is more marked still for the absorption bands of the solid hydrocarbon.

Endeavours were also made to observe the absorption spectrum of the vapour of the hydrocarbon. For this purpose a small portion of the solid was placed in a thick hard glass bulb of about 50 mm. diameter. The bulb was then exhausted of air and placed in a special apparatus designed to heat the bulb equally in all directions so that the density of the vapour would be the same at every point. The light of an iron arc or of a condensed iron spark was used for different observations, but it was found that the hydrocarbon decomposed so rapidly that no results were obtained. The vapour of the hydrocarbon exhibited a most beautiful blue fluorescence, very intense at first but rapidly becoming weaker. The decrease in fluorescence seemed to be proportional to the decomposition of the hydrocarbon, as evidenced by the conversion of the substance to a brownish tarry residue. The decomposition may have been started either by the action of the small amount of air left in the bulb or by the contact action of the heated glass.

We hope to continue the work on the absorption and fluorescent spectra of the vapour of this and of other hydrocarbons.

The absorption spectra of concentrated and diluted solutions of chlorophyll. By J. E. Purvis, M.A., St John's College.

[*Read* 8 February 1909.]

(Plates I—III.)

The general phenomena of the absorption spectra of solutions of chlorophyll have been described by various observers, and references to such work up to 1908 are included in Kayser's *Handbuch der Spectroscopie*, vol. IV. But, so far as the author knows, there are no recorded observations on the comparative spectra of equivalent quantities of chlorophyll in strong and dilute solutions: and the aim of this paper is to describe a series of observations which have been made in this direction.

The well-dried leaves of fresh parsley were ground to a powder, and the chlorophyll extracted with rectified spirit, filtering the insoluble portions. A portion of the strong solution was diluted 719·8 times with rectified spirit, and the strong solution was placed in a glass cell of 5 mm. internal width, whilst the diluted solution was introduced into a glass tube, with glass ends, 3599 mm. long. The ratio of the dilutions was, therefore, as 1 : 719·8, and there was as much chlorophyll in the one solution as in the other. The two solutions were exposed to the same source of light for equal periods of time. The only alteration in the conditions was that made necessary by the change of the small cell with the strong solution for the long tube with the dilute solution; with this exception, all the conditions were fixed and exactly the same throughout the whole series of observations. The apparatus and photographic spectroscope used for the observations have been fully described in the *Proc. Camb. Phil. Soc.* vol. XII. pt. III. p. 206, in some experiments by the author on the absorption spectra of concentrated and diluted solutions of didymium and erbium salts.

Description of photographic plates.

Unfortunately, it has not been found possible to reproduce the phenomena observed on the original photographs with any close degree of precision. The fault is particularly noticeable in the weak band λ 565, which is unmistakeably clear on the original photographs. The gradual changes, however, can be traced fairly closely by comparing the changes of the bands λ 538 and λ 508 in the reproductions 1 to 9, and which are described in the corresponding paragraphs 1 to 9.

1. These photographs were taken immediately after the solutions were made. In both solutions the bands at λ 538 and λ 565 are of the same width and intensity. The λ 565 band is very weak in the reproductions, but it is quite visible in the original photographs. The general absorption in both begins at about λ 510.

2. The second series of photographs were taken after the solutions had been standing for some hours. The band λ 538 in the dilute solution is more diffuse and weaker than the band in the strong solution; otherwise there is no change in the two solutions.

3. After standing twelve hours longer, the bands λ 538 and λ 565 appear to be very like the last photographs. But in the strong solution a faint band λ 508 has appeared, which is not visible in the dilute solution.

4. After standing twelve hours longer, the concentrated solution shows that the bands λ 538 and λ 565 are very like those in the last observations: whilst the band λ 508 is more clearly marked. In the diluted solution, the bands λ 538 and λ 565 are still weaker and more diffuse than the corresponding bands in the strong solution, and λ 538 is particularly noticeable in this respect. There is no band at λ 508 in the diluted solution corresponding to that in the strong solution.

5. After standing twelve hours longer, the general appearance of the bands is much the same as in the last series of observations. But in the strong solution there is a gradual increase in the amount of light coming through on the more refrangible side of the band λ 508; and there is no such appearance in the diluted solution.

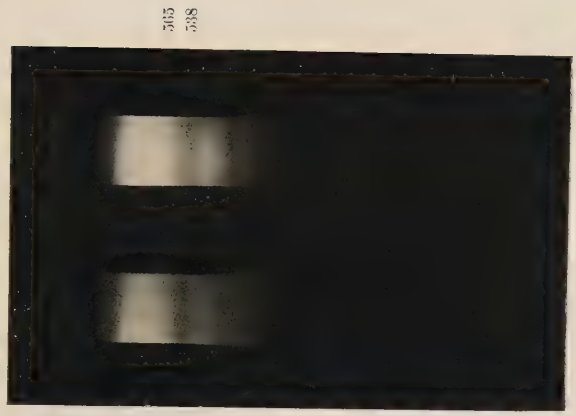
6. The solutions were allowed to stand four days longer. In the strong solution the bands λ 565, λ 538, and λ 508 are like those in the last series of observations. In the dilute solution the band λ 538 is weaker and more diffuse than the corresponding band in the strong solution. The latter band is as well marked as at the beginning of the observations. In the strong solution the band λ 508 is now well marked, and is absent in the dilute solution. Besides that, there is less general absorption of light by the strong solution than the dilute solution.

7. The solutions were allowed to stand two days longer. The general appearance of the phenomena of the two solutions is very similar to that in the last series of experiments.

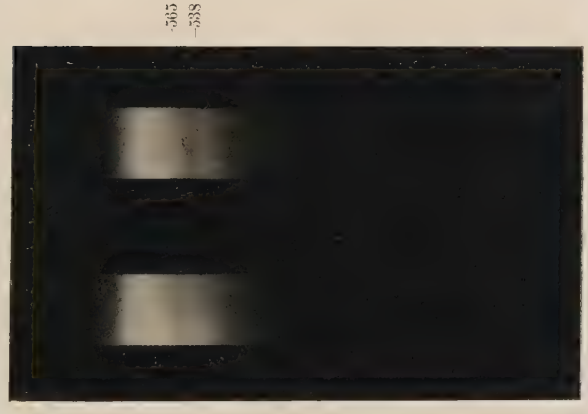
8. After standing four days longer, photographs were again taken, and they showed that the bands were very similar to those in the last series of experiments; but the general absorption in the strong solution was less than before, whilst that in the dilute solution does not appear to have altered.

9. Finally, after standing fourteen days longer, the photographs show that in the strong solutions the bands λ 538 and λ 508

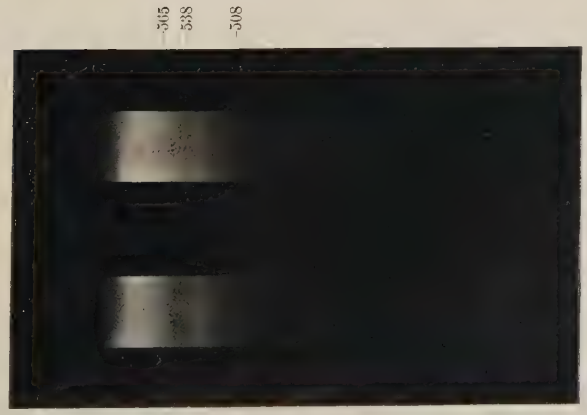
1


·565
·538
Dilute solution
Strong solution

2


·565
·538
Strong solution
Dilute solution

3


·565
·538
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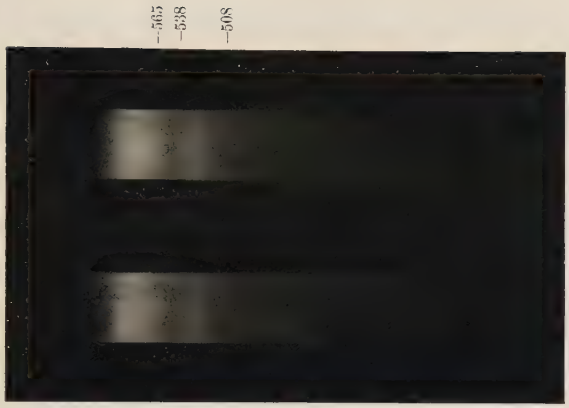
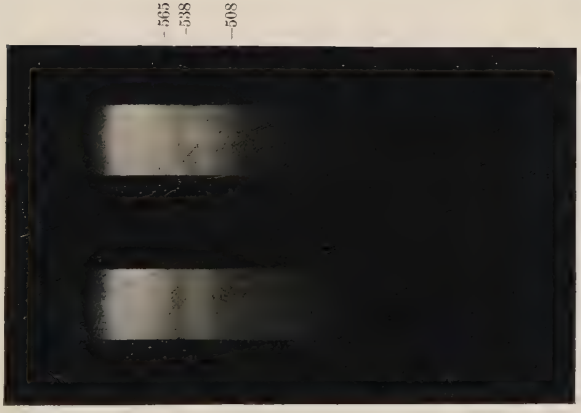
Dilute solution
Strong solution

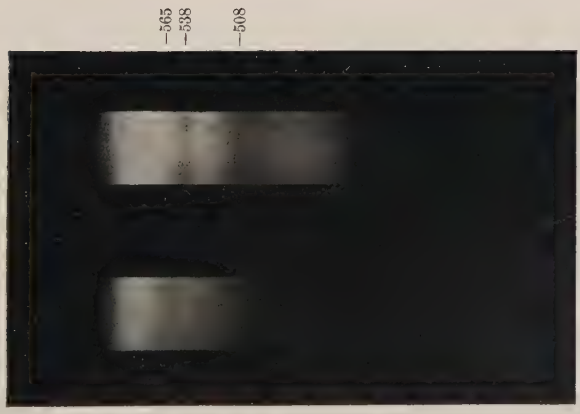
4


—565
—538
—508
Strong solution
Dilute solution

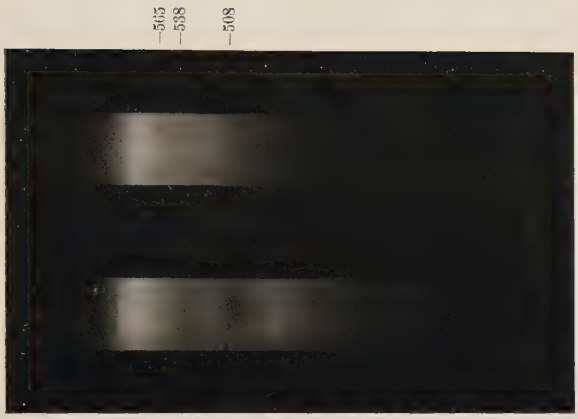
5


—565
—538
—508
Dilute solution
Strong solution

6


—565
—538
—508
Strong solution
Dilute solution

7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Dilute solution
Strong solution

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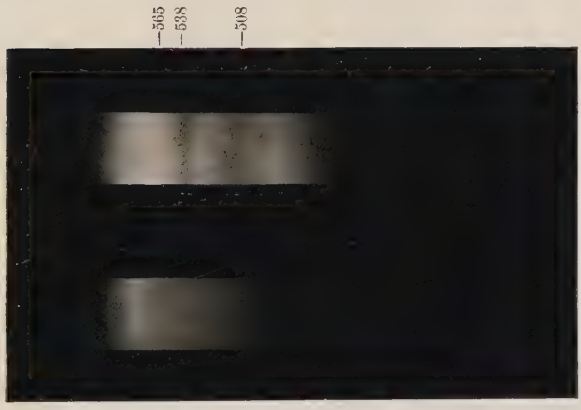
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Dilute solution

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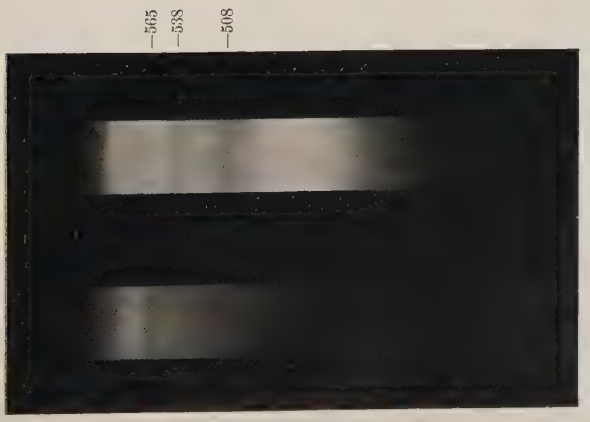

565
538
508
Strong solution
Dilute solution

are quite well marked: there are very faint traces of the band λ 565, and the general absorption has decreased very considerably. In the dilute solution the band λ 565 is stronger, and the band λ 538 is much weaker and more diffuse than the corresponding band in the strong solution, and, in fact, it has almost disappeared. In the dilute solution no band at λ 508 appeared. There is also a gradual creeping in of light on the more refrangible side in the dilute solution when compared with the last series of observations. The difference between the general absorption at the beginning (1), and at the end of the observations (9), in the dilute solutions is very small, whilst that of the strong solution is very striking.

Discussion of the results.

As no effort was made to purify the chlorophyll, it is probable that some vegetable acid was present: and, by the continued use of the alcoholic solutions, a little of the spirit may have been oxidised to acetic acid: for, at the end of the experiments, the solutions were found to give a weak acid reaction with litmus paper. In any case, the first series of photographs showed absorption spectra very similar to those described by Russell and Lapraik (*Jour. Chem. Soc.* vol. XLI. (1882), p. 334). Their solutions were obtained by the action of very dilute acids on fairly pure chlorophyll. Assuming, therefore, that the solutions used in these experiments show absorption bands characteristic of chlorophyll solutions in the presence of a very small amount of acid, some explanation is necessary to account for the gradual changes in the bands and the general absorption of two solutions containing equal amounts of chlorophyll, one of which was diluted 719 times that of the other.

(1) The greater volume of the diluted solution might have contained sufficient dissolved oxygen to oxidise the small quantity of the dissolved chlorophyll. Against this argument is the fact that although both solutions stood for several weeks in a well-lighted room and with easy access of air, the changes in the strong solution were not in the same direction as those in the dilute solution. It was the dilute solution which showed more general absorption at the end of the experiments. If the primary cause of the changes had been oxygen, the dilute solution might have been expected to have undergone a greater change.

(2) The changes might have been caused by the action of a very weak acid analogous to that which occurs by the decomposition of sugars and glucosides, whereby complex molecules are broken down into simpler ones; but the same objection may be brought against this explanation as against the last.

(3) It is more probable that the changes may have been produced by the differentiating effect of enzymes, like the oxydases, dissolved from the parsley with the chlorophyll, some of which are known to be very soluble in rectified spirit. In the concentrated solution the enzymes would be in more intimate contact with the chlorophyll than in the dilute solution, and the consequent break down of the chlorophyll molecule would be sooner effected. This suggestion would explain the less general absorption of the strong solution than of the dilute solution after they had been standing for some time, as well as the changes in the appearance and position of the bands, although the substances corresponding to the bands λ 538 and λ 508 do not appear to have suffered much change in the strong solution.

(4) It is hardly likely that the dissociating force of the solvent was a factor in the changes. The system was too complex to explain them by this theory, for there was not only chlorophyll in solution, but also, most probably, small quantities of other substances.

The dissociating effect of the solvent on some dyes may be mentioned in connection with the above-mentioned changes in the chlorophyll solutions. For example, strong aqueous solutions of dyes like eocine and magenta, when diluted in the same way as chlorophyll, gradually and slowly lose their brilliant colour whilst the strong solutions retain it. But the colours of the dilute solutions are at once restored by a single drop of an acid like hydrochloric acid, and there is no such reversion in the dilute chlorophyll solution; so that the changes produced by dilution of the dyes are not comparable with those of the chlorophyll solutions.

The absorption spectra of mesitylene and trichloromesitylene. By J. E. Purvis, M.A., St John's College.

[*Read* 8 February 1909.]

The absorption spectrum of mesitylene in the ultra violet region has been described by Hartley and Huntington (*Phil. Trans.* vol. CLXX. (1879), pp. 257—274). They noted a band between λ 275—λ 245 in strong solutions. By diluting this solution in the ratio of 1 : 2000, the strong band is resolved into three small sharp bands, which remain visible in a solution of 1 : 3000.

Drossbach (*Ber. Chem. Ges.* vol. XXXV. (1902), pp. 1486—1489, noted that general absorption occurred at λ 336, but this occurs only in a very thick layer. He did not appear to have noticed any bands.

Coblentz (*Astrophy. Jour.* vol. XX (1904), pp. 207—223), has observed the presence of a considerable number of bands and lines in the ultra red.

The author has compared the absorption spectrum of trichloromesitylene with that of mesitylene, when equivalent quantities of substances were in alcoholic solutions. He noticed that the bands and the general absorption were shifted in the trichloromesitylene more towards the red end of the spectrum when compared with their positions in the mesitylene solution. For example, through a thickness of 40 mm. of solution and $N/1000$ concentration the strong band in mesitylene lay between λ 269—λ 254, whilst that of the trichloromesitylene was between λ 287—λ 263. Also, a most important difference was the greatly increased persistence of the absorption curve of the trichloromesitylene as compared with that of the mesitylene. The loading of the ring by the chlorine atoms influenced the vibration of the molecule so that the absorption band was shifted towards the red end of the spectrum and agrees with the effect previously observed by Hartley in the case of substituted compounds, and, at the same time, the persistence of the band was greatly increased.

It has been shown by Baker and Baly (*Jour. Chem. Soc.* vols. XCI. and XCII. p. 1122), that, in the case of pyridine, the introduction of chlorine atoms into the nucleus not only shifted the bands towards the red, but also greatly increased their persistence,

and the explanation was that such an introduction reduced the residual affinity of the nitrogen atom, and thereby allowed greater freedom of vibration or pulsation of the ring.

The author has pointed out in previous communications (*Proc. Camb. Phil. Soc.* vol. XIV. (1908), pp. 381, 435 and 568) that the persistency of the band in pyridine compounds is conditioned to a considerable extent by the relative or spatial positions of the introduced chlorine atoms as well as by their introduction either in the nucleus or the side chains.

Now the increased persistence of this strong band of trichloromesitylene is very similar to that of the chlorine compounds of pyridine and several of its methyl derivatives, where the chlorine atoms had entered the nucleus, and, unless we assume some changing valency of the atoms of the benzene ring, it does not seem easy to correlate the two series of phenomena.

But before any definite conclusion can be drawn from these observations on mesitylene and trichloromesitylene, it will be necessary to examine the absorption spectra of other chlorine compounds of the former substance, both when the chlorine atom or atoms are in the nucleus and when they are in the side chains.

All that can be said at the present stage of the investigation is that (1) the strong absorption band of the trichloromesitylene is shifted more towards the red end of the spectrum than the corresponding band of mesitylene, and that (2) the persistence of the band of the former compound is much greater than that of the mesitylene.

Mr W. H. Foster, of St John's College, was good enough to make up the solutions of the two substances examined.

On a so-called "sexual" method of forming spores in Bacteria.

By C. C. Dobell, B.A., Trinity College.

[*Read* 22 February 1909.]

Two species of Bacteria—*Bacillus bütschlii* (Schaudinn) and *B. flexilis* (Dobell)—are known which differ from all other Bacteria in forming two spores instead of one in each cell. Before spore-formation, the following remarkable phenomena are to be seen:

A large individual begins to divide into two, but the division is never completed, and all signs of it are subsequently lost. The granules (? chromatin) in the cell then show streaming movements, arrange themselves in the form of an irregular spiral in the long axis of the cell, and finally heap themselves up at both ends of the organism to form the two nucleus-like spore-rudiments. Round each of these, a spore membrane is formed, so that the two spores when fully formed lie at opposite poles.

The process of incomplete division and subsequent fusion immediately preceding spore-formation were considered by Schaudinn, and also by myself, to represent a form of conjugation such as is found in some yeasts, Heliozoa, etc.—that is to say, a conjugation of sister-cells.

From further investigations upon *Bacillus spirogyra, Bacterium lunula* n.sp. and other Bacteria, it now appears to me to be certain that no "sexual" process really occurs in the disporic Bacteria. I believe that what has really happened is that the last bipartition in the life-cycle—the one immediately preceding spore-formation—has become abortive, and is not as a rule completed. The disporic individuals are therefore really double individuals—physiologically, but not morphologically.

If this interpretation of the phenomena is correct, then it has some interesting results: for instance, it has an important bearing upon the problem of the affinities of the Bacteria. One of the most important arguments in favour of the close relations of Bacteria and yeasts is removed. It has also some significance with regard to the problem of "sexuality" in the Protista and in living beings generally.

A discussion of these problems, and also of some special points (*e.g.* the nucleus of *B. spirogyra*, etc.), will be given in my full account of this matter, which I hope to publish before long in the *Quart. Journ. Microsc. Sci.*

On the alleged influence of lecithin upon the determination of sex in rabbits. By R. C. PUNNETT, M.A., Gonville and Caius College.

[*Read* 22 February 1909.]

In 1907, Professor Russo, of Catania, published an account of certain experiments* in which he claimed to have brought about a great increase in the relative number of female young produced by rabbits. The method consisted in the injection of lecithin either subcutaneously or into the abdominal cavity. In the paper referred to Russo gives particulars of 100 litters from injected does as well as of another 100 litters in which the does were not injected. The injected does produced 217 ♂♂ and 431 ♀♀, as against 400 ♂♂ and 287 ♀♀ from those not treated, and the author consequently claims to have raised the proportion of females from 41·8 % to 56·5 % of the total offspring.

Such remarkable results have naturally attracted attention and several criticisms have recently appeared. It has been pointed out† that the statistics published by the author are not complete, but that, as he himself states, only those litters from the animals treated with lecithin were recorded in which the number of females preponderated over that of males (cf. Russo, *loc. cit.* p. 367, footnote). Until the complete statistics are available, any conclusions as to the effect of lecithin upon the proportions of the sexes must necessarily be of doubtful value.

On other grounds Russo's results have been recently criticised by Heape‡. Russo claimed to have distinguished histologically between the ovarian ova destined to produce females and those destined to give rise to males. According to him fatty matter is present in the former though not in the latter, and he supposed that the injection of lecithin resulted in a higher proportion of ova containing fat, *i.e.* of female ova. Heape points out that there are good grounds for regarding Russo's male ova as ova which are in some stage of degeneration and consequently not destined to give rise to embryos of either sex.

* *Atti Acad. Lincei*, 1907, Serie 5 a, Vol. vi. pp. 313—384.
† Bateson and Punnett, *Science*, N.S., May 15, 1908.
‡ *Proc. Camb. Phil. Soc.*, Vol. xiv. p. 609.

During the past year Russo's experiments have been repeated by Basile* with negative results. From six injected does he had 117 young in 17 litters, and of these 66 were ♂♂ and 51 were ♀♀. He also bred 60 litters from nine rabbits which were not injected, and of the 440 offspring 225 were ♂♂ while 215 were ♀♀. Among the offspring of the uninjected the proportion of females was actually slightly higher than among the injected.

Being desirous of repeating Russo's experiments I wrote to him in February, 1908, asking for certain details in his method. He very kindly sent me the information for which I had asked, and at the same time informed me that he had produced similar results by feeding with lecithin in place of the injection. My experiments were carried out in accordance with the details of the feeding method of which he sent me particulars. An emulsion was prepared by shaking up 5 grams of lecithin (Merck) with 1000 c.c. of physiological salt solution (·65 °/₀). The daily dose for each rabbit was 20 c.c. of this emulsion made up into a paste with meal, and the treatment was continued for three months before the doe was put to the buck†. I also went on with it for a few days subsequent to this. No difficulty was experienced in getting the rabbits to take their daily dose, for they soon became exceedingly fond of it and ate it greedily as soon as it was offered them. During the past summer 10 does were treated in this way, and they eventually produced between them 47 young. Of these 24 were ♂♂ and 23 were ♀♀. From these and other does not treated with lecithin I have had 18 litters with 103 young, and of these 54 were ♂♂ and 49 were ♀♀. In either series of experiments the numbers are not large, but so far as they go they are entirely opposed to the view that feeding on lecithin has any influence on the relative proportion of the sexes among rabbits.

* *Atti Acad. Lincei*, 1908, Vol. xvii. 1, p. 643.

† Here I would express my thanks to my friend Mr F. A. Potts for kindly feeding the rabbits for me during a week when I was absent from Cambridge.

A coloured thio-oxalate. By H. O. Jones, M.A., Clare College, and H. S. Tasker, B.A., Emmanuel College.

[*Read* 8 February 1909.]

In investigating the action of substances on oxalyl chloride, $(COCl)_2$, in the hope that chlorine might be removed and C_2O_2 be produced, thiophenol and its metallic compounds were used, since the easy formation of diphenyl disulphide renders these substances good reducing agents.

Sodium thiophenolate reacts violently with oxalyl chloride, but in place of C_2O_2 two molecules of carbon monoxide are produced.

The action of thiophenol itself is less violent. On mixing equivalent quantities of oxalyl chloride and thiophenol a yellow colour is immediately produced and hydrogen chloride is evolved. The evolution of gas continues until the mixture finally sets to a solid mass. By crystallization from ether long sulphur-yellow prisms are obtained, melting at 119—120° C. The substance distils unchanged at atmospheric pressure.

On treatment with aqueous potash the substance dissolves, giving in solution potassium thiophenolate and potassium oxalate, the colour disappearing.

This behaviour, coupled with the mode of formation, indicates that the substance is diphenyl-dithio-oxalate, formed according to the equation

$$2C_6H_5SH + (COCl)_2 = (COSC_6H_5)_2 + 2HCl.$$

On combustion ·2857 gms. of the substance gave ·6402 gms. of CO_2 and ·0951 gms. of H_2O.

Observed percentages C 61·12 H 3·70.

Calculated for diphenyl-dithio-oxalate C 61·31 H 3·65.

The substance is the first member of the class of dithio-oxalates, and is interesting by reason of its colour (which is undiminished by heating in ethereal solution with animal charcoal or by repeated recrystallization), since diphenyl oxalate and monothioxalic esters are colourless, while glyoxal is yellow.

In order to see if C_2O_2 could be obtained as the result of decomposing this substance, it was heated in an evacuated vessel with metallic sodium. The resultant gas was collected by means of a Sprengel pump. There was a sudden rush of gas, in quantity far short of the theoretical, and some deposition of carbon. The gas was insoluble in water or potash but was soluble in cuprous chloride and burnt with a blue flame. Its vapour density was taken by weighing a known volume. It corresponded with that required for carbon monoxide.

94·24 c.c. at 654 mm. and 16° C. weighed ·1002 gms.	
Vapour density (H = 1)	14·5
Theory for CO	14

With concentrated sulphuric acid, carbon monoxide is again given off and a purple colour is given which is characteristic of the behaviour of diphenyl-disulphide under the same conditions.

Other coloured esters and several metallic salts of dithio-oxalic acid have been prepared, and their properties and derivatives are being investigated.

Observations on the changes in the Common Shore-crab caused by Sacculina. By F. A. Potts, M.A., Trinity Hall.

[*Read* 22 February 1909.]

Professor Alfred Giard, whose recent death all zoologists must deplore, established a great reputation by the brilliance and versatility of his researches. Perhaps his most noteworthy discovery was that the Rhizocephala, a family of aberrant parasitic Cirripedes, produce an extraordinary modification of the sexual characters of the Decapod crustacea they infect*. The evident results of the association are the dwindling of the reproductive glands of the host and the effacement of already existing secondary sexual characters (such as copulatory styles) and assumption of those of the other sex.

Since Giard wrote, the nature of these effects has been clearly set forth by Geoffrey Smith† for the case of the spider-crabs (*Brachyura oxyrhyncha*). Perfect sterility and the disappearance of the gonads results. The abdomen of the male becomes trough-shaped, often completely resembling the female type. The first two abdominal appendages modified as copulatory styles dwindle, and on the other segments a varying number of swimmerets, characteristic of the female, are developed. Modification in the converse sense is not observed in the female, but that of the male evidently indicates the development of a hermaphrodite condition: for male crabs, which have undergone these changes, freed from the parasites and allowed to recover, may regenerate an ovotestis with mature sexual products of both kinds.

This, then, is an example of *extreme modification*. The common shore-crab (*Carcinus moenas*) when attacked by the same parasite offers a case of *slight* or *incipient modification*. The short description we owe to Giard is vague on several important points, and the common association of the shore-crab and *Sacculina* on our own coasts‡, also renders it desirable that another account of the phenomenon should be given.

* "La Castration Parasitaire," *Bull. Sc. Dép. Nord.* Ser. 2, Tome xviii. 1887, p. 1, and Ser. 3, Tome i. 1888, p. 12.

† "Rhizocephala," Mon. 29, *Fauna u. Flora des Golfes von Neapel*, 1906.

‡ In Plymouth Sound a considerable proportion of the shore-crabs are found to harbour *Sacculina*. It is curious to note that in many localities, *e.g.* Liverpool Bay and the Isle of Man, the parasite is rarely or never found.

In the first of two papers on "La Castration Parasitaire" Giard mentions the fact that a certain alteration in the character of the abdomen can be noticed in male shore-crabs attacked by *Sacculina*. The diagrams which he gives show plainly that the infected males often possess an abdomen broader than that usually associated with this sex, yet not attaining the full trough-like development of the female. It was Giard's view that the female might approach the male type when parasitised; he does not here however refer to any modification of the female. Though no definite statement is made the reader of this paper is led to assume that atrophy of the gonads takes place.

To understand the account of the changes in the sexual characters, certain particulars of the life history need to be supplied. The elucidation of the extraordinary development of *Sacculina* is found in a brilliant paper by Delage*. The fertilised eggs develop into free swimming Nauplius larvae. These fix at the base of hairs on the carapace of crabs, and the almost disorganised mass of cells which has resulted from histolysis of the larval body passes through the gap in the carapace which the articulation of the hair affords. It wanders through the body cavity, and eventually becomes attached to the intestine in the region of the abdomen. Here it grows, absorbing nutriment from the blood, and attains to the development of the adult parasite. The internal stage of development is terminated by a moult. For if the *Sacculina* is ripe for emergence it is found that the part of the parasite destined to become external has eaten its way through the muscles and epithelium of the abdominal wall of the host and when the chitinous exoskeleton is cast, this visceral mass emerges. Connection with the internal root-system is maintained by the narrow peduncle occupying the aperture which served for escape.

In crabs (*Brachyura*) the abdomen is clothed with a fairly thick exoskeleton. It follows that when the new shell is formed after the moult described above the *Sacculina* acts as a rivet and prevents the fleshy part of the abdomen, in any subsequent attempt at moulting, being withdrawn from its chitinous case. The fact that a crab infected by *Sacculina* is prevented from further moulting and consequently from growth after the parasite becomes external is well known, but I consider it to be due to the mechanical disability, which I have pointed out above, alone. It might conceivably be a repression of growth occasioned by the drain of nutriment for the parasite or the action of some specific substance secreted by that organism. But if this were so we might expect to find a tendency to inhibit the moult which introduces the *Sacculina* to the external world, also at work. For

* *Arch. Zool. Exp.*, Ser. 2, Tome II. 1884, p. 417.

while the parasite is in its internal stage the change occurs which finds expression in the modification of the secondary sexual characters in the new carapace. If a change of this magnitude can be induced in the early stages of parasitism we might also expect that the effect on growth and moulting would be established and the *Sacculina* prevented from ever appearing at the exterior. There does not appear to be any evidence for infrequency of moults in crabs with *Sacculina interna*. It may here be pointed out that the hermit crab when attacked by *Peltogaster** (a near relative of *Sacculina*) still moults regularly, a fact we can easily trace to the investment of the abdomen by soft skin which tears easily and so detaches itself in moulting, round the base of the parasite. The degree of modification produced in this case is great, but *Peltogaster* does not mechanically prevent the shedding of the exoskeleton, and growth appears even to be favoured by the presence of the parasite.

The general nature of change in the secondary sexual characters has already been indicated. The increase in the width of the abdomen is exemplified in the following diagrams similar to those which accompany Giard's paper. It will first be noticed that

Male. Modified male. Female.

while all the segments of the female abdomen are freely moveable, in the male the third, fourth and fifth are fused together. The first evidence of modification in the infected male is the re-established segmentation of the abdomen.

The degree of modification may be actually measured by estimating the ratio of the breadth of the abdomen at the segment to the length $\left(\frac{AB}{CD}\text{ in Fig.}\right)$. The indices so given are as follows:

Uninfected male crab	·57—·58
Uninfected female crab	·90
Infected male crab which has suffered the maximum amount of modification	·77

* *Q. J. Microsc. Sc.*, Vol. L. 1906, p. 599.

The male crab with the largest index does not approximate very closely to the female type, and those so markedly modified are but a small proportion of the whole. A third of the crabs measured could be called entirely unmodified, a proportion which corresponds roughly with the quarter unaffected by the parasite in the case of the spider-crab, and of the hermit-crab.

The number of crabs measured was 59, so that the series is too small to be satisfactorily plotted in the form of a curve. Some idea of the distribution may be gained by the statement that 37 possessed an index between ·56—·66, while that of the remaining 22 lay from ·67 to ·77. In the first group there were 11 with an index of ·60, and in the second 5 with one of ·72.

One point in Giard's short account which I desired to test was his assertion that the older (or larger) crabs were not subject to modification. The average carapace breadth of the crabs of index ·56—·66 to be sure is 4·9 as compared to 4·5 of those from ·67—·77, but the figure for the index ·72 is 4·6 and for ·77 the extreme limit of modification is 4·7. Occasional crabs of exceptional size are met with, with abdomina considerably broadened, and my observations give practically no support even for the statement that younger crabs are more liable to modification than older individuals.

In the abdomen the effect of the parasite was distinctly less than is the case with the spider-crabs. With the other secondary sexual characters no alteration was experienced. There was no diminution of the copulatory styles in size and no appearance of the abdominal swimmerets which are characteristic of the female.

In the female no change was ever detected in the abdomen as a result of parasitism, and the swimmerets appeared fully as well-developed and thickly fringed with hairs as in crabs free from the *Sacculina*.

With regard lastly to the sexual glands of the infected shore-crab, here again there is little alteration. The females never bear eggs, and the ova in the ovary are as a rule small and white, and devoid of great stores of yolk. In the uninfected male the testes are lobulated organs, stretching on each side across the liver. The development varies greatly even in normal specimens. The vasa deferentiae follow, and speedily become wide tubes with a very characteristic milky white appearance, due to the multitude of spermatophores, each crammed with spermatozoa, which they contain. Posteriorly the male duct opens on the terminal joint of the last thoracic leg, at the end of a long fleshy penis.

In infected males a certain diminution in size of the testis is often noticeable, but actual atrophy never takes place. There is very little reduction in size of the vasa deferentiae, and they always

preserve the normal appearance, containing spermatophores in large number with ripe spermatozoa. The genital aperture is never blocked up with chitin as occurs in the cases of extreme modification, and the penis shows no tendency to disappear. In fact it is quite clear that the effect on the reproductive glands is only small—the most noticeable change being the failure of the female to produce large heavily-yolked eggs. The male, it might be supposed, could impregnate the female successfully were it not that the presence of the *Sacculina* prevented copulation. It would be interesting to observe whether infected males ever attempt the sexual act.

In summarising emphasis must be laid on the *low grade* of the modification. One organ only, the abdomen, is subject in the male to a change of a rather striking nature, though the testis continues to produce spermatozoa and the ducts to be filled with spermatophores. It may be pointed out that the term "castration parasitaire," as applied to the phenomenon by Giard, is somewhat misleading here as in other cases, for the modification of the secondary characters is not associated with the suppression of the gonads.

In *Pachygrapsus marmoratus*, the common shore-crab of the Bay of Naples, another of the numerous hosts of *Sacculina*, the modification of the abdomen of infected males is of the degree described above, *i.e.* about half-way between male and female types. In other respects the change is far greater than that in *Carcinus*. The female swimmerets appear on the abdomen in more or less complete sets (no mention is made of diminution of the copulatory styles), and the gonads are greatly reduced and may entirely disappear. The case of *Carcinus* is better compared with that of the "stylopised" males of bees (*Andrena*)* which undergo considerable external change, though throughout parasitism the testis of one side remains fully developed, and produces spermatozoa.

* *Perez. Mém. Soc., Bordeaux*, T. III. 1880, p. xlii.

On the secondary Röntgen radiation from air and ethyl bromide. By J. A. Crowther, M.A., St John's College.

[*Read* 25 January 1909.]

In a previous paper* I have shewn that certain gases and vapours, and notably compounds containing arsenic or bromine, give off amounts of secondary Röntgen radiation enormously greater than that given off by air under similar conditions. Thus the ionization produced in air by the secondary rays from ethyl bromide is some 540 times greater than that produced by the rays from air itself under similar circumstances. The rays from ethyl bromide are much softer than those from air, and in a recent paper Prof. Bragg† has suggested that in this fact may lie a possible explanation of what he describes as the "startling results obtained in the case of arsenic and bromine." He points out that the rays from an ordinary X-ray bulb are hard to substances of low atomic weight such as aluminium or air, but soft to the elements of higher atomic weight such as copper or iron. The secondary rays from air are of the same type as the primary rays. The secondary rays from bromine however are of a much softer character, and so, to quote Prof. Bragg's paper, "are brought within reach, so to speak, of air, which rapidly converts them into cathode rays, so that there is a very large ionization." He concludes, finally, "that the very large secondary radiations which some substances appear to give, therefore owe their magnitude largely to the fact that the air in which they are measured is sometimes ten to twenty times as favourable to them as to the primary rays which produced them."

I was, of course, not unaware of the increased ionizing power of the softer rays from ethyl bromide as compared with those from air, and in my original paper, I endeavoured to correct for this fact, on what seemed to be, in the absence of direct evidence, the reasonable assumption that the ionization produced in air by rays of differing hardness, was proportional to the amount of absorption undergone by the rays in the secondary ionization chamber. The final values given in Column III, Table 1, of my earlier paper are reduced on this assumption. With this correction the relative intensity of ethyl bromide compared with air became 217, as given in the table.

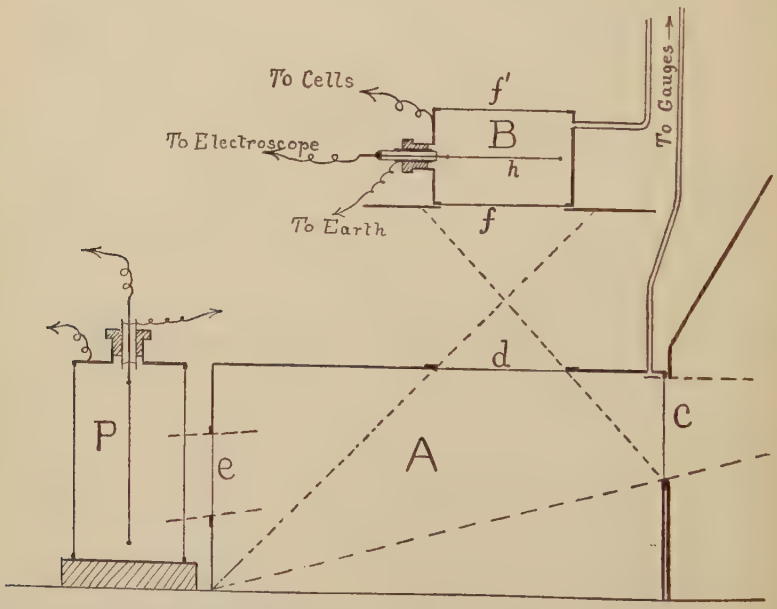
The assumption on which this calculation is made has never

* Crowther, *Phil. Mag.* [6], Vol. xiv. p. 653, 1907.

† Bragg and Glasson, *Proc. Roy. Soc. South Australia*, Vol. xxxi. Oct. 1908.

been directly verified, and in the light of Prof. Bragg's interesting suggestion, it was thought desirable to make some direct experiments on the point, and to compare the relative amounts of ionization produced by the secondary rays from ethyl bromide and from air, in some gas not so unfavourable as air, to the harder radiations.

Methyl iodide, and ethyl bromide suggested themselves as the most likely for the experiments. The ionization produced by the primary rays in both is very large, and their coefficients of absorption for the primary rays are also large. The values obtained for ethyl bromide, however, vary less rapidly with the nature of the rays than those for methyl iodide, so that comparable results are more easily obtainable with it than with the latter. On the whole, therefore, ethyl bromide was preferred; and the experiment consisted in comparing the amounts of ionization produced in ethyl bromide by the secondary rays from ethyl bromide, and from air.



The apparatus employed was a modification of that used in my previous researches* on the subject, and it will not be necessary to describe it in any great detail. The radiating gas was contained in a rectangular brass box A; the primary rays entering through an aluminium window C; while the secondary rays passed

* Crowther, *Phil. Mag.* [6], Vol. xiv. p. 653, 1907.

upwards through a second aluminium window d into the secondary ionization chamber B. A portion of the primary rays passed out through a third aluminium window e, into an ionization chamber P where its intensity could be measured.

The secondary ionization chamber B was of a different form to that employed in the previous experiments. It is very difficult to obtain the "saturation current" across a cylindrical ionization chamber, with a central wire electrode, such as was previously employed. An elementary calculation will shew that the field near the outer cylinder is only a small fraction of the average intensity between the electrodes, and so, to obtain saturation in this part of the gas it is necessary to apply a potential between the electrodes much greater than would be required in the case of a uniform field. As ethyl bromide is a somewhat difficult gas to saturate, the cylindrical form of ionization chamber was abandoned in favour of the form shewn in the figure. The central electrode is here a thin sheet of aluminium leaf stretched on a circular frame of copper wire. It is insulated by a quartz tube passing through an earthed guard ring; the latter in turn, being insulated from the case (which is charged to a potential of 480 volts) by an ebonite stopper. The ends f, f' of the chamber are of aluminium foil which is held in position by a similar device to that employed for the windows in the box A, and described in a previous paper. The various joints are made air-tight by means of sealing wax.

The chamber B is so placed with respect to A, that no secondary radiation from the walls of the latter can enter it. The currents through the two chambers B and P are measured by Wilson electroscopes in the usual way.

The ionization chamber B is filled with ethyl bromide vapour at some suitable known pressure (about 200 mm. of mercury). The ratio of the currents through B and P is then measured with the box A filled first with air and then with ethyl bromide vapour at a known pressure. From the ratio of these two quantities we can find the relative amounts of ionization produced in the ionization chamber B by the secondary rays from air and ethyl bromide. Corrections have to be applied for the absorption of the secondary rays from the gas by the aluminium windows d and f; and in the case of ethyl bromide, for the absorption of the primary and secondary rays in the gas of the gas chamber A. Finally we have to correct for the fact that the air and ethyl bromide are employed at different pressures. The various data required are known, and have been given in a previous paper on the passage of Röntgen rays through gases*, and the various corrections required can be easily performed. In this way it was found that the amounts of

* *Proc. Roy. Soc.* Vol. LXXXII. p. 103 (1909).

ionization produced in ethyl bromide by the secondary rays from ethyl bromide was 162 times that produced in ethyl bromide by the secondary rays from air under similar conditions. The corresponding value when air was used in the secondary ionization chamber was 543. It will thus be at once apparent that the ethyl bromide is much more favourable to the secondary rays from air than is air itself.

But although the coefficients of absorption for different types of rays are more nearly equal for ethyl bromide than for air, they are still not independent of the hardness of the rays. The coefficient of absorption of ethyl bromide for the primary rays used in these experiments, and therefore, also, for the secondary rays from air, was ·13; while its coefficient of absorption for its own secondary rays was ·38. Knowing the dimensions of the secondary ionization chamber *B*, and the pressure of the ethyl bromide in it, we can easily calculate the quantity of each type of ray absorbed by the gas in *B*. The width of the chamber was 3·5 cms.; the pressure of the ethyl bromide in it 210 mm. of mercury. The percentage energy absorbed in the gas was, therefore, for the soft rays from ethyl bromide 19·2 °/₀, and for the hard rays from air 12·4 °/₀. If we assume that the amount of ionization produced is proportional to the amount of energy absorbed, we thus obtain for the relative amount of secondary radiation from ethyl bromide, compared with that from air, the final value **105**; a result which is just about half that given in the original paper. Though thus considerably reduced, the secondary radiation from ethyl bromide still remains abnormally large. If we divide the relative secondary radiation by the relative density (3·78) we obtain the value 28; as against 1·0 for the lighter elements; 8·0 for stannic chloride; and 8·4 for methyl iodide.

It may be noted that, as the penetrating power of the secondary rays from stannic chloride, methyl iodide, and substances of low atomic weight is the same as that of the secondary rays from air, the values obtained for the relative amounts of secondary radiation from these substances compared with air are not subject to any correction for selective absorption by the medium in the secondary ionization chamber, and therefore need no modification as the result of the present experiments.

It has already been mentioned that in my original paper the relative ionizations produced in the air of the secondary ionization chamber by the different types of secondary rays were corrected on the assumption that the amount of ionization was proportional to the absorption. It was not possible, however, to measure the absorption of the different types of rays in air itself, the absorption by air being too small to be readily appreciable. What was done, therefore, was to measure the absorption of the rays by some

solid, and to assume that the ratio of the absorptions was the same for solid and gas. The phenomena of the selective transmission of the primary rays by air, which Bragg has pointed out in the paper already quoted, were at the time far from clear, and unfortunately tinfoil, a substance which does not shew this selective transmission of hard rays, was employed as the absorbing substance, and the values obtained for it were used in calculating the final result. The ratio of the coefficients of absorption of the secondary rays from ethyl bromide, and air, is for tinfoil approximately 2·8. The ratio for air itself is undoubtedly much higher. As mentioned above it is hardly possible to measure the absorption of air itself for these different rays, the absorption in any reasonable distance being so small; but filter paper suggests itself as being composed of elements approximating very nearly in atomic weight to air. The absorption of filter paper, therefore, has been measured for the secondary rays from ethyl bromide and air; and the ratio of the two coefficients of absorption for filter paper has been found to be 5·4. Using this corrected value for the coefficients of absorption, and assuming again that the ionization is proportional to the energy absorbed (that is to say in the case of air, where the percentage absorbed is very small, to the coefficients of absorption), we deduce from our original measurements of secondary radiation made in air the corrected value 101, for the relative amount of secondary radiation from ethyl bromide. This value agrees remarkably well with the value 105 obtained for the same quantity from the present measurements, using ethyl bromide as the absorbing gas in the secondary ionization chamber.

Summary.

The relative amounts of ionization produced in ethyl bromide vapour by the secondary Röntgen radiation from ethyl bromide and air have been measured. From this, knowing the absorption of ethyl bromide for the two kinds of secondary rays, the relative intensity of the secondary radiation from ethyl bromide has been calculated; and has been found to be 105 times that from air.

The relative absorption of the two kinds of secondary rays has been measured for a substance of low atomic weight; and the value obtained has been used to correct the original results for the secondary radiation from ethyl bromide, which were obtained from measurements made in air. With this correction, the two values for the secondary radiation from ethyl bromide, obtained by measurements made in ethyl bromide, and in air, have been shewn to be in agreement.

In conclusion I wish to express my best thanks to Prof. Sir J. J. Thomson for his kindly interest in these experiments.

A string electrometer. By T. H. LABY, B.A., Research Student, Emmanuel College, Cambridge, Joule Student of the Royal Society.

[*Read* 25 January 1909.]

The minute force, 10^{-8} dynes, that is sufficient to perceptibly deflect a stretched silvered quartz fibre, is of the same order of smallness as that acting on the gold leaf of the most sensitive of the usual electroscopes. This suggested that the fragile and irregularly shaped gold leaf might be replaced with advantage by a silvered quartz fibre.

I constructed a very simple form of string electrometer, and was testing it, when through Mr N. R. Campbell I learnt that Dr C. V. Burton had designed a model of a similar instrument. The Cambridge Scientific Instrument Company kindly lent me it to test. The results given below were obtained with this instrument, after it had been somewhat altered.

The Einthoven Galvanometer.

Einthoven has found that the movements of a stretched quartz fibre subject to a varying lateral force afford a precise and delicate means of measuring that stress. With his string galvanometer, in which a silvered fibre replaces the usual suspended coil, he has obtained photographs of the moving fibre on a falling plate using a magnification of about 500. Even with this magnification the motion of the fibre was so free from irregularities of an extraneous origin that Einthoven has been able to subject the oscillograph curves thus obtained to a minute analysis*. He has also followed vibrations with a frequency of 3000 per sec. The lightness (from 6×10^{-8} to 5×10^{-7} gm. per cm.) and excellent elastic properties of the silvered quartz make such results possible.

Description of the Instrument.

The string electrometer (see fig. 1) used in these experiments consisted of two vertical insulated plates, A, B, 15 cm. long by ·8 cm. thick with their edges parallel and at an adjustable distance apart.

* *Kon. Ak. van Weten. te Amsterdam*, VIII. p. 210, 1906.

These plates were connected to the ends of a battery, *CD*, the middle of which was earthed. The silvered quartz fibre, *EF*, is stretched parallel to and equidistant from the two plates *A*, *B*; this position of the string relative to *A* and *B* is attained by the lateral movement of its points of support, *E*, *F*; while its tension, which has to be kept very constant, is controlled by the screw *S*.

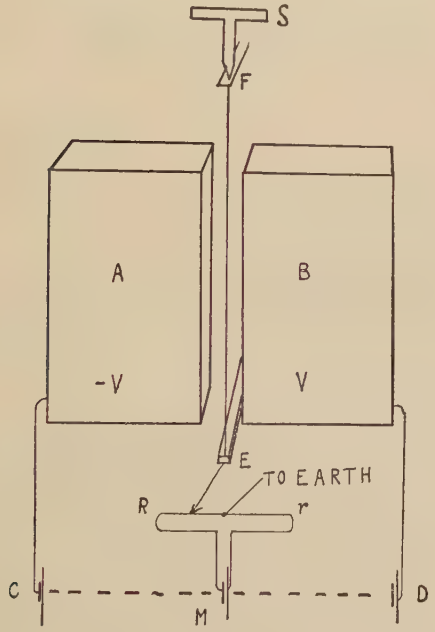


Fig. 1.

The terminals of the middle cell *M*, of the battery *C*, *D*, were connected to the potentiometer *Rr*, the middle of which was earthed, while the connection to the fibre could be moved from *R* to *r*, *i.e.* its potential varied from −1 volt to 1 volt by steps of 1/50 of a volt.

As the fibre was not highly insulated, tests could not be made with it isolated, so that the results given later, it should be pointed out, were got with the potential of the fibre maintained.

The motion of the middle of the fibre was observed by means of a microscope with a Zeiss achromatic objective A, and micrometer eye-piece No. 3. The tube length was such as to give a magnification of 100. A slight illumination gave a sharp image of the black silvered fibre on a bright ground.

Properties of a String Electrometer.

The purpose of the tests was to find how:

(1) the sensitiveness,

(2) the oscillograph powers of the electrometer depended on

(*a*) distance of the plates apart,
(*b*) their potential difference,
(*c*) the tension on the fibre.

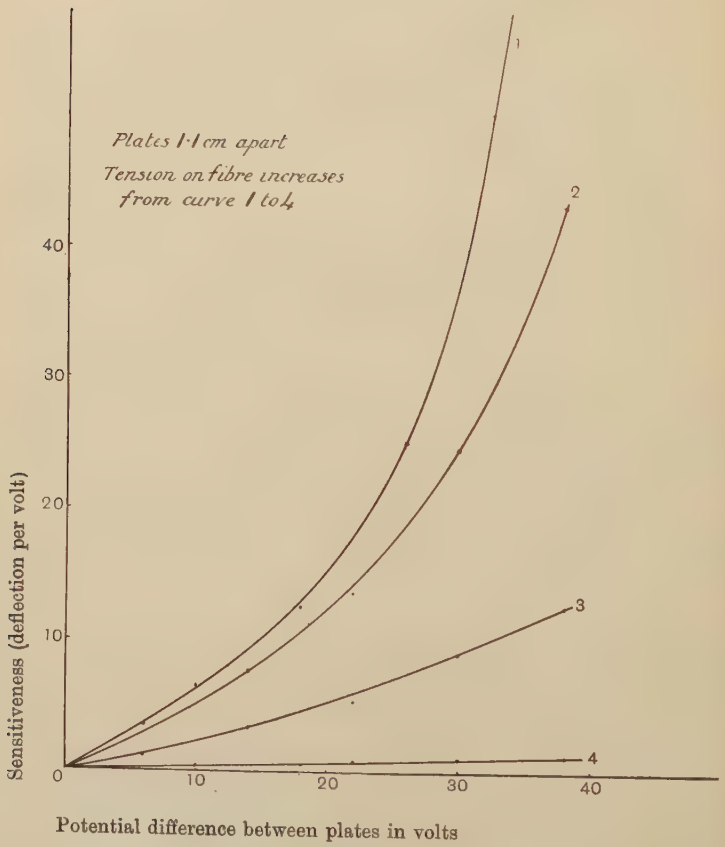


Fig. 2.

Plates far apart (1·1 *cm.*). The plates were set far apart (compared to the distance moved through by the fibre) and then

the sensitiveness (deflection per volt) was observed for a range of plate potential differences. With a new tension the same observations were repeated. In this way the equal tension potential difference-sensitiveness curves (figs. 2 and 3) were obtained. The

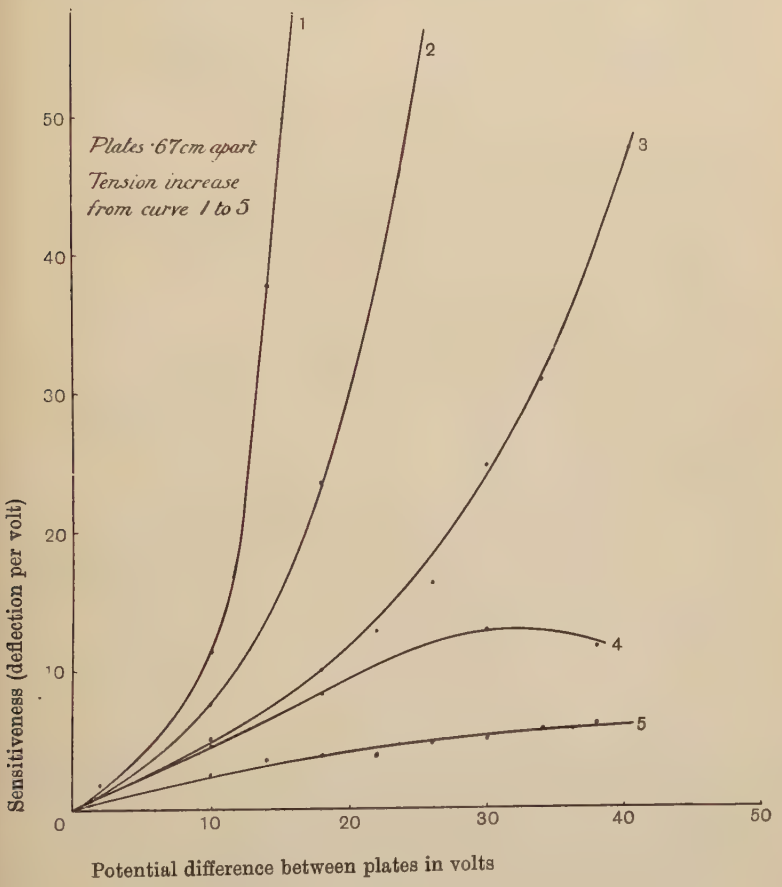


Fig. 3.

sensitiveness is expressed in eye-piece divisions per volt on the fibre: one eye-piece division = ·0012 cm. movement of the fibre itself. These curves express many of the properties of the instrument. We see from them that a high sensitiveness was obtained with the fibre not very tight (curves 1, 2, figs. 2 and 3), which increased with the P.D. between the plates to its greatest value in fig. 3 for 18 volts, when the sensitiveness was 60. There is a practical

limit, as will be explained later, to the sensitiveness that may be obtained by increasing the potential difference, or decreasing the tension. With the plates at this distance apart the position of the fibre was plotted against its voltage and the curve of fig. 4 obtained. It will be observed that one of these graphs is a straight line, so that the deflection is proportional to the voltage even for high sensitiveness, and for a wide range (for an electroscope) of voltages.

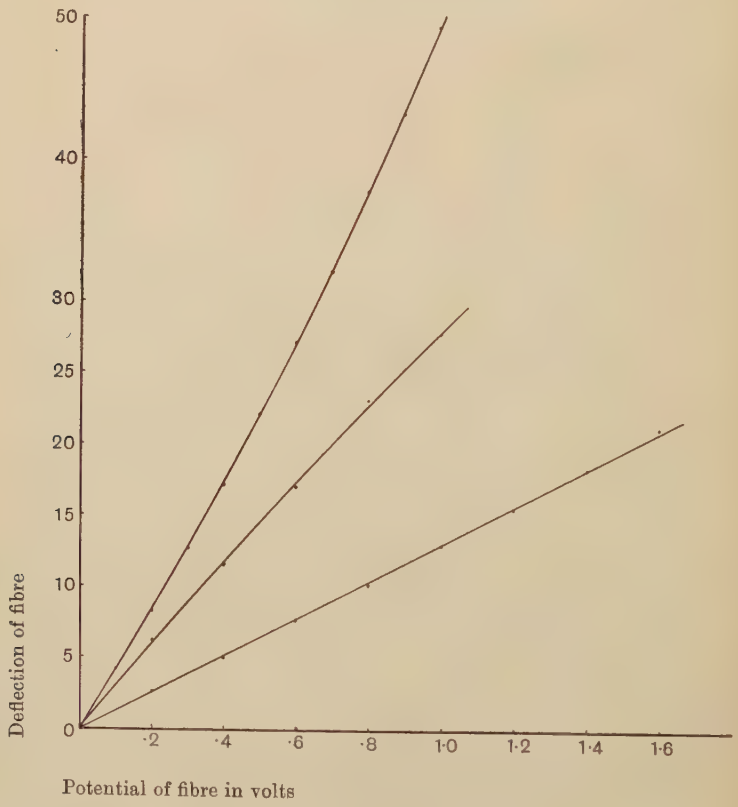


Fig. 4.

Plates near together (6·6 *mm. and* 3·3 *mm.*). When the distance between the plates was reduced to 6·6 mm. and finally to 3·3 mm. several properties of the instrument, to be seen in figs. 3 and 5, came into greater evidence.

The chief change in the potential difference-sensitiveness curves (fig. 3, plates at 6·67 mm.) is that smaller P.D.'s appear to

give a larger sensitiveness for a given tension, or as is to be expected theoretically the sensitiveness for a given fibre tension depends on V/d^2, where V is the P.D. and d the distance between the plates. This could not be fully proved as the tension of the fibre did not remain sufficiently constant. With the plates at 3·3 mm. apart the deflection of the fibre as its potential changed became markedly different. With the fibre tight its deflection was still proportional to its potential (the straight line of fig. 5), but when slackened the

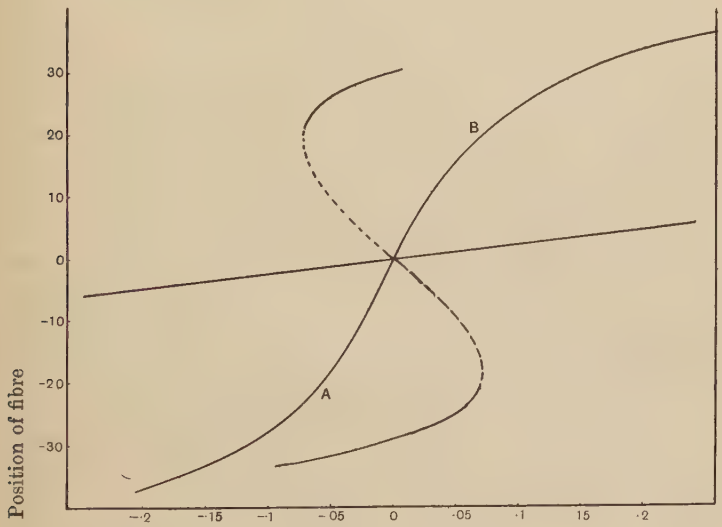


Fig. 5.

position, which the fibre took up for different potentials, is represented by the curve AB (fig. 5); on lowering the tension still further the fibre had two stable positions and apparently an unstable one not experimentally realisable. In other words, if the fibre is slackened the sensitiveness increases, but for a decreasing range of voltages: theoretically we pass through infinite sensitiveness to instability. This property of the instrument, which sets a superior limit to its useful sensitiveness, is best grasped by examining fig. 5.

In Mr C. T. R. Wilson's* tilted electroscope, as is well known, this property is used to obtain a high sensitiveness, the adjustments (plate potential, tilt and length of gold leaf) are altered

* *Proc. Camb. Phil. Soc.* XII. p. 135, 1903.

till the portion AB (fig. 5) of the deflection potential curve is realised.

Characteristics of the String Electroscope.

The fibre will stand vibration and sudden large changes of voltage without breaking. The instrument has been carried on a bicycle without damage to the fibre.

Sensitiveness. No common standard of comparison has yet been proposed for electroscopes. The unit of deflection used in this paper is one division of the micrometer eye-piece: 1/5 of this could be read with certainty. The closeness with which the observations fall on curves shews that the unit chosen is not unduly small. In fig. 4 (plates 6·6 mm. apart) a sensitiveness of 50 divisions per volt is shewn. At the same time the curve is nearly a straight line, *i.e.* the deflection is approximately proportional to the voltage, and the P.D. between the plates was less than 40 volts. The electrometer has been quite sensitive using only 20 volts P.D.; such a small P.D. could be kept constant more readily (by the use of Weston cells, say) than the larger ones usually required for electroscopes and electrometers. On the other hand, large voltages, which require to be constant, would generally still be required, for example, to saturate an ionised gas. With the plates at 3·3 mm. apart and a P.D. of 36 volts a sensitiveness of 100 divisions per volt over a range of ·4 volt was readily obtained.

Oscillograph Uses.

I do not think, however, that the value of the string electrometer will lie especially in its sensitiveness. It is not difficult to make a gold leaf electroscope highly sensitive over a small range. But there is a distinct need now for an electrometer capable of automatically recording rapid changes of voltage, in the study of discontinuous phenomena in the direction indicated by von Schweidler*. For example in the beautiful experiment of Rutherford and Geiger of counting the number of α particles, which arrive in an ionisation vessel, by the excursions of an electrometer the smaller the free period of the electrometer the better.

To test the oscillograph powers of the instrument, the plates were placed 3 mm. apart at a P.D. of 120 volts, and the tension

* *Congrès international pour l'étude de la radiologie et de l'ionisation*, Liége, 1905.

on the fibre increased; the sensitiveness was then 50 divisions per volt. Yet the motion of the fibre was dead beat, and it took up a new position in less than one-tenth of a second. Since the movements of a fibre can be readily recorded photographically the instrument seems to promise well as an oscillograph. Further experiments are being made on this use of it.

It should be recalled that all the above experiments were made with a steady source of potential. Mr H. Darwin has designed a new instrument in the light of the experience obtained with this one; when it is finished more complete tests than these will be made with it.

I am indebted to Mr Horace Darwin and Mr Whipple for having lent me the string electrometer I used, and for having had certain alterations made in it. I am grateful to Prof. Sir J. J. Thomson for the encouraging interest he took in these experiments.

Interference fringes with feeble light. By G. I. Taylor, B.A., Trinity College. (Communicated by Professor Sir J. J. Thomson, F.R.S.)

[*Read* 25 January 1909.]

The phenomena of ionisation by light and by Röntgen rays have led to a theory according to which energy is distributed unevenly over the wave-front (J. J. Thomson, *Proc. Camb. Phil. Soc.* xiv. p. 417, 1907). There are regions of maximum energy widely separated by large undisturbed areas. When the intensity of light is reduced these regions become more widely separated, but the amount of energy in any one of them does not change; that is, they are indivisible units.

So far all the evidence brought forward in support of the theory has been of an indirect nature; for all ordinary optical phenomena are average effects, and are therefore incapable of differentiating between the usual electromagnetic theory and the modification of it that we are considering. Sir J. J. Thomson however suggested that if the intensity of light in a diffraction pattern were so greatly reduced that only a few of these indivisible units of energy should occur on a Huygens zone at once the ordinary phenomena of diffraction would be modified. Photographs were taken of the shadow of a needle, the source of light being a narrow slit placed in front of a gas flame. The intensity of the light was reduced by means of smoked glass screens.

Before making any exposures it was necessary to find out what proportion of the light was cut off by these screens. A plate was exposed to direct gas light for a certain time. The gas flame was then shaded by the various screens that were to be used, and other plates of the same kind were exposed till they came out as black as the first plate on being completely developed. The times of exposure necessary to produce this result were taken as inversely proportional to the intensities. Experiments made to test the truth of this assumption shewed it to be true if the light was not very feeble.

Five diffraction photographs were then taken, the first with direct light and the others with the various screens inserted between the gas flame and the slit. The time of exposure for the first photograph was obtained by trial, a certain standard of blackness being attained by the plate when fully developed. The

remaining times of exposure were taken from the first in the inverse ratio of the corresponding intensities. The longest time was 2000 hours or about 3 months. In no case was there any diminution in the sharpness of the pattern although the plates did not all reach the standard blackness of the first photograph.

In order to get some idea of the energy of the light falling on the plates in these experiments a plate of the same kind was exposed at a distance of two metres from a standard candle till complete development brought it up to the standard of blackness. Ten seconds sufficed for this. A simple calculation will shew that the amount of energy falling on the plate during the longest exposure was the same as that due to a standard candle burning at a distance slightly exceeding a mile. Taking the value given by Drude for the energy in the visible part of the spectrum of a standard candle, the amount of energy falling on 1 square centimetre of the plate is 5×10^{-6} ergs per sec. and the amount of energy per cubic centimetre of this radiation is $1{\cdot}6 \times 10^{-16}$ ergs.

According to Sir J. J. Thomson this value sets an upper limit to the amount of energy contained in one of the indivisible units mentioned above.

On the parametric representation of the coordinates of points on a cubic surface in space of four dimensions. By H. W. Richmond, M.A., King's College.

[*Received and read* 8 March 1909.]

By a cubic surface in space of four dimensions is here understood the locus represented by a homogeneous equation of the third degree in five variables, and the problem considered is a method of expressing the ratios of the five variables as algebraic functions of three parameters. Should the cubic surface possess a double point, projection with that point as vertex leads at once to a solution.

If however the surface has no double point, take any straight line L which lies wholly on the surface; six such lines pass through each point and twenty-seven lie in every space of three dimensions, at least three of the latter being real. Through L and any point P of the surface can be drawn an S_2, whose intersection with the surface will consist of the line L and a conic, which must intersect L in two points, Q_1 and Q_2. Conversely, if any two points Q_1 and Q_2 be chosen upon L, the tangent S_3's at Q_1 and Q_2 have in common an S_2 which contains L and a conic passing through Q_1 and Q_2. Thus if points of the line L are determined by a parameter, ϕ, then to any two values of it ϕ_1 and ϕ_2 correspond two points Q_1 and Q_2, and consequently a conic passing through Q_1 and Q_2. Another parameter ψ will define each point of the conic, and thus the coordinates of each point of the surface will be algebraic functions of ϕ_1, ϕ_2 and ψ.

In fact the equation

$$u^2x + 2uvy + v^2z + 2u\lambda + 2v\mu + \rho = 0 \ldots\ldots\ldots\ldots (1),$$

in which λ and μ are quadratic functions and ρ a cubic function of x, y, z, represents a quite general cubic surface on which the line L, $x = y = z = 0$ lies. A point on L is determined by a parameter ϕ if $v = u \,.\, \phi$; and the tangent S_3 at the point is

$$x + 2y\phi + z\phi^2 = 0.$$

Thus on the S_2 common to the tangent S_3's at points where ϕ has the values ϕ_1 and ϕ_2,

$$x : y : z :: \phi_1\phi_2 : -\tfrac{1}{2}(\phi_1 + \phi_2) : 1;$$

and if in (1) we substitute $x = z \,.\, \phi_1\phi_2$, $y = -\tfrac{1}{2}z(\phi_1 + \phi_2)$ and then write $u\phi_1 - v = z \,.\, \psi$, the problem is solved.

The study of discontinuous phenomena. By NORMAN CAMPBELL, M.A., Fellow of Trinity College, Cambridge.

[*Read* 22 February 1909.]

§ 1. The application by Kohlrausch* by Meyer and Regener† and by Geiger‡ of von Schweidler's theory§ of the discontinuities in the emission of rays by a radioactive substance to the measurement of the charge carried by an α particle opens up a new and most important field of physical research. The ingenuity of the method must always give great interest to their work, but it has lost much of its immediate importance since Rutherford and Geiger‖ have measured the same quantity by the more direct and probably essentially more accurate method of counting the number of particles, one at a time. The justification of the lengthy discussion of von Schweidler's method which is given in the following pages is twofold. In the first place, it is believed that the discussion by the four authors first named of the principles according to which their observations should be interpreted is neither exhaustive nor completely devoid of error, and that a more thorough discussion will enable the results obtained by von Schweidler's theory to rival in accuracy those attained by Rutherford and Geiger. In the second place, it should be remembered that radioactivity is not the only discontinuous process which we study. The trend of modern theory is everywhere to replace by discontinuity the continuity which was the basis of the science of the last century. Any method which is especially applicable to discontinuous processes is certain to be fruitful of results in every department of investigation, and any considerations which can be advanced in the elucidation of such a method are not devoid of value; at the present time I am engaged in an attempt to apply the method to a totally different form of ionisation current.

§ 2. It will be desirable first to put von Schweidler's theory in a slightly more general and a somewhat more accurate form. The author of that theory used in his calculations Bernoulli's integral of probability. Now the use of the integral calculus in this case is open to the objection that it assumes that the number of possible cases, the probability of which is considered, is so large that it may be regarded as infinite. But the essential feature

* *Wien. Ber.* 1906, p. 673.
‡ *Phil. Mag.* April, 1908, p. 539.
‖ *Proc. Roy. Soc.* A. 81, p. 191.
† *An. d. Phys.* xxv. p. 757.
§ See Kohlrausch, *loc. cit.*

of the phenomena that we propose to investigate is that they are discontinuous, that is to say, that they are to be regarded as made up of a finite number of events and not of an infinite number: in some cases to which we may have to apply our theory it is by no means certain that the number is even very large. By using the integral calculus we assume in our mathematical considerations the proposition which we deny in our physical considerations. Accordingly we must confine ourselves throughout to finite quantities.

Consider a series of s trials in each of which one of the two mutually exclusive events A and B must happen. Let the probability that A happens be p and the probability that B happens be q. Then it can be shown easily that if A happens m times and B $m-s$ times, the most probable value of m is ps: and further that the probability that A happens $ps-x$ times is

$$\frac{s!}{(ps-x)!\,(qs+x)!}\,p^{ps-x}\,q^{qs+x}.$$

We shall require the mean value of the 'deviation' x for a very large number (σ) of sets of s trials. There are three chief forms of mean value, $\overline{x^2}$, $\bar{x}$, and $|\overline{x}$. Remembering the identities

$$c\,(p+q)^{c-1}\,.\,q = \sum_n nA_n p^{c-n} q^n$$

and $$c\,(c-1)\,(p+q)^{c-2}q^2 + c\,(p+q)^{c-1}\,q = \sum_n n^2 A_n p^{c-n} q^n,$$

where $$A_n = \frac{c!}{(c-n)!\,n!}$$

we can easily show that, since $p+q=1$,

$$\overline{x^2} = \sum_{x=ps}^{x=-qs} x^2 \frac{s!}{(ps-x)!\,(qs+x)!}\,p^{ps-x}\,q^{qs+x}$$

$$= spq$$

$$\bar{x} = 0$$

and that $$|\overline{x}| = \sqrt{\frac{\overline{spq}}{2\pi}}, \text{ if } s \text{ is large}^*.$$

The mean values x^2 and x are the same as those given by von Schweidler for the case when s is very large: but we see that their form is quite independent of any assumptions as to the order of magnitude of s, p, q. But the form of $|\overline{x}|$ is not so independent, and no further use will be made of this mean.

* See Bertrand, *Calcul des probabilités*, chap. IV.

In calculating these means we have assumed that a 'very large number' of sets of s trials are taken. It is desirable to inquire how large the number must be in order to reduce the probable error within any desired limits. The problem is very similar to that of determining the probable error of the calculated probable error of a set of observations. If s is very large, it is well known that the expression given for the probability of a 'deviation' x may be reduced by Stirling's theorem to an exponential form, so that the probability that the value of x lies between x_1 and $x_1 + dx$ is

$$\frac{h}{\sqrt{\pi}} e^{-h^2 x_1^2}\, dx, \quad \text{where} \quad h = \frac{1}{\sqrt{2pqs}}.$$

Accordingly the probability of a given set of σ values of x, x_1, x_2, x_3 ... such that $x_1^2 + x_2^2 + x_3^2 + \ldots x_\sigma^2 = \sigma a^2$ is

$$\left(\frac{h}{\sqrt{\pi}}\, dx\right)^n e^{-h^2 \cdot \sigma a^2}.$$

We shall find that our observations give us the value of $\sigma a^2 = \Sigma x^2$. h is then chosen so as to make the probability of the occurrence of this value of Σx^2 a maximum: we find

$$h^2 = \frac{\sigma}{2\Sigma x^2} \quad \text{or} \quad \overline{x^2} = \frac{1}{2h} = spq, \text{ as before.}$$

Accepting these values, we can find the probability that h has the value $h + uh$: this probability is

$$\left(\frac{h + uh}{\sqrt{\pi}}\, dx\right)^n e^{-(h+uh)^2 \sigma a^2} = Ce^{-nu^2},$$

where

$$C = \left(\frac{h}{\sqrt{\pi}}\, dx\right)^n e^{-h^2 \cdot \sigma a^2}.$$

Hence the 'measure of precision' of h is $\sqrt{n}$. In order that the probable error may be less than 1 %, $\sqrt{n}$ must be greater than 100 or n greater than 10^4. In order to attain this degree of accuracy we must take 10^4 sets of the series of s trials. We have assumed that s is large, but since the expression for the probability will be of nearly the same form whatever the value of s and since we require only a rough value for the measure of precision, we may apply the result to all values of s such as are likely to occur.

§ 3. Let us now apply this theorem to such cases as are likely to occur in physical investigation. Suppose that a very large number of events NT happen in a time T. If we fix our attention on a period τ within T and small compared to it, the chance that any one of these events will happen within τ is $\frac{\tau}{T}$: it must be noted

that the probability is the same whether the parts of τ are adjacent, so that τ is continuous, or whether they are scattered at random.

Now the occurrence of the NT events correspond to the s trials, and the events A and B correspond to the falling of one of the NT events respectively within or without τ. Hence

$$NT = s, \quad p = \frac{\tau}{T}, \quad q = 1 - \frac{\tau}{T}.$$

Then if observations are taken over σ periods, each of length T, and if $N\tau + x_1$, $N\tau + x_2$, ... $N\tau + x_\sigma$ are the number of the events which happen within the period τ is the 1st, 2nd, ... σth observations, we have proved that

$$\overline{x^2} = NT \cdot \frac{\tau}{T} \cdot \left(1 - \frac{\tau}{T}\right).$$

If we make τ small compared to T, we have

$$\overline{x^2} = N\tau \quad \text{.....(1).}$$

In the particular case of radioactivity, T is the time over which the activity of the substance suffers no appreciable decay: the NT events are the breaking up of NT atoms. If, then, we had some instrument which would indicate the number of atoms which break up during any period τ short compared to T, we might discover the value of N by taking a sufficient number of observations and equating to $N\tau$ the sums of the squares of their deviations from the means or 'fluctuations.' An electrometer connected to an ionisation vessel which could be exposed for a known time to the action of the rays from the substance would be such an instrument. But it must be noted that, though the absolute value of the mean 'fluctuation' $\sqrt{\overline{x^2}} = \sqrt{N\tau}$ can be made as large as we please by increasing the period τ, the ratio of that value to the value of the whole number of rays emitted during the period τ is equal to $\frac{\sqrt{N\tau}}{N\tau}$ and decreases with an increase of τ. Numerical calculation shows that (unless Rutherford and Geiger's method of magnifying the effect due to a single ray is employed) no value for τ can be found for which the mean fluctuation exceeds the probable error in the measurement of the value of the total current.

Accordingly all who have attempted to apply von Schweidler's theory have employed some balancing method by which the mean total current is reduced to zero, so that the fluctuations become fluctuations about the zero and only their absolute magnitude need be taken into account. These may be made as large as is desired by an increase in N and τ. Geiger and, before him,

Kohlrausch, balanced against each other currents due to two independent radioactive sources. Since some doubt has been thrown on Kohlrausch's results by later workers no further reference will be made to them. Meyer and Regener balanced the current from one radioactive source by means of a Bronson resistance, to the terminals of which a compensating potential difference was applied by a potentiometer arrangement. And these two arrangements are examples of the only two principles which seem possible in any measurements of this kind.

But it must be noted that, if a reading of the instrument is taken while the rays are acting, this reading will not indicate the value of the fluctuation at the moment of observation. The instrument has inertia, and its indication at any time is a function of the fluctuations during some finite period preceding the moment of observation. In Geiger's method it might be possible to get rid of the inertia of the needle by removing the sources of rays after a definite period of action and allowing the instrument to take up a steady position before the reading is taken. But we shall see that there are grave practical difficulties in such a procedure: and the method is not applicable to Meyer and Regener's observations by reason of the 'inertia' of the Bronson resistance. Accordingly the problem before us is to determine the relation between the observed fluctuations, which depend on the constants of the instrument, and the real fluctuations which depend only on the nature of the source. We will consider first the method of Meyer and Regener.

Meyer and Regener's method.

§ 4. Let a charge E be given to the electrode system when the electrometer is at zero and at rest. If the capacity of the electrode system be C, the resulting potential will be E/C. If the current through the Bronson resistance is proportional to the potential difference between its terminals, this potential difference will diminish exponentially with the time, so that the potential difference acting on the electrometer needle at a time t after the charge has been communicated is

$$E/C\, e^{-pt}$$

where p is a constant depending only on the nature of the resistance.

Let I be the moment of inertia of the needle, μ the coefficient of damping, k the coefficient of torsion and KV the couple acting on the needle when the P.D. between the quadrants is V. (It is assumed that the deflection θ is proportional to the steady P.D.

applied.) Then the equation of motion of the needle subsequent to the communication of a charge E is

$$I\frac{d^2\theta}{dt^2}+\mu\frac{d\theta}{dt}+k\theta-K.E/C.e^{-pt}=0 \quad \ldots\ldots\ldots\ldots(2).$$

Taking into account the initial conditions the solution is:

$$\theta=Ae^{-\alpha t}+Be^{-\beta t}+Pe^{-pt} \quad \ldots\ldots\ldots\ldots\ldots(3),$$

where

$$\alpha=\frac{\mu-\sqrt{\mu^2-4Ik}}{2I}, \qquad P=\frac{KE}{C(Ip^2-\mu p+k)}, \qquad A=P\frac{\beta-p}{\alpha-\beta},$$

$$\beta=\frac{\mu+\sqrt{\mu^2-4Ik}}{2I}, \qquad B=P\frac{p-\alpha}{\alpha-\beta}.$$

All these quantities can be found by suitable experiments on the electrometer and the resistance. Accurate determinations of them would require some ingenuity and labour, but they can be found with accuracy sufficient for the purpose of this research by methods which will be obvious to everyone. (See § 10.)

It is easy to show that, subject to the assumptions made, the effects of charges communicated at any times, $\tau_1, \tau_2, \ldots \tau_m$ are the same as if the needle had been at zero and at rest at the moments of communication. Hence the deflection at a time T, which is subsequent to the communication of all the m charges, will be given by

$$\theta_T=\Sigma_{r=1}^{r=m}(Ae^{-\alpha(T-\tau_r)}+Be^{-\beta(T-\tau_r)}+Pe^{-p(T-\tau_r)}).....(4).$$

Let us now change the origin of time to the time T, which is the time of observation. Writing t_r' for the time before the moment of observation at which the rth particle was emitted, we have

$$\theta_T=\Sigma_{r=1}^{r=m}(Ae^{-\alpha t_r'}+Be^{-\beta t_r'}+Pe^{-pt_r'}) \quad \ldots\ldots\ldots\ldots(5)$$

$$=\Sigma_{r=1}^{r=m}f(t_r') \text{ (say)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6).$$

θ_T is a function both of the number of particles emitted and the times at which they were emitted. In the case which we are considering we do not know these times and hence cannot determine the number directly from a single observation. But I shall proceed to show that we can determine it from the average of a large number of observed values of θ^2_T. (It will appear later why we have to take the average of θ^2_T and not of $|\theta_T|$.) For the main principle of the argument I am indebted to Mr G. H. Hardy, Fellow and Mathematical Lecturer of Trinity College, Cambridge, who points out that though no elaborate analysis is necessary for

the solution of the problem, interesting questions are raised in a branch of mathematics which has received little attention.

§ 5. Let the period of observation be divided up into m periods $\tau = \frac{T}{m}$, such that $f(t_r')$ may be considered sensibly constant during any one period τ. The total length of the period of observation T (that is the time from the moment of insulating the electrode to that of taking the observation) will be considered to be the same for all observations: but from the form of $f(t_r')$ it is clear that all periods T may be considered equal so long as αT, βT and pT are all large compared with unity.

Let y_r be the number of particles emitted during the rth period τ. Then, during any one period of observation, y_r is a function of r. In this statement the word 'function' is used in the widest possible sense, and not in the restricted sense often employed in analysis. Further, to each value of r during any one period T, corresponds one and only one value of y_r. Let us write

$$y_r = \phi(r) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7).$$

The function $\phi(r)$ has different forms in different periods T. Let there be ν periods of observation and let the form of $\phi(r)$ in the ρth period be $\phi_\rho(r)$. Since $\phi(r)$ can have a large number of forms, a definite meaning can be attached to the expression 'the probability that $\phi(r)$ has the form $\phi_\rho(r)$': let Φ_ρ be this probability.

Let $N\tau + x_1$, $N\tau + x_2$, ... $Nt + x_n$ be all the possible values of y: then $\phi_\rho(r)$ has m of these values during the m periods into which T is divided. The probability of the occurrence of any value x_r is, by § 2,

$$\frac{(NT)!}{(N\tau + x_r)!\,(NT - N\tau - x_r)!}\left(\frac{\tau}{T}\right)^{N\tau - x_r}\left(1 - \frac{\tau}{T}\right)^{NT - N\tau + x_r} = \chi_r \text{ (say)}.$$

Hence the probability that $\phi(r)$ has a definite m of the possible values of y, that is, the probability that $\phi(r)$ has a definite form, is

$$\chi_1 \cdot \chi_2 \cdot \ldots \chi_m = \Phi_\rho \ \ldots\ldots\ldots\ldots\ldots\ldots (8).$$

Now it is important to notice, for this is the essential step in the argument, that the value of Φ_ρ is not affected by the order in time in which the definite values of x_r occur. Φ_ρ is the same if the values of x_r are the same: it is not altered if $x_r = a_u$ when $r = u$ and a_v when $r = v$, instead of a_v when $r = u$ and a_u when $r = v$. That is to say, Φ_ρ is wholly independent of r.

Now if $\phi(r)$ has the form $\phi_\rho(r)$, the corresponding value of θ^2_T will be

$$\theta^2_T = [\Sigma_{r=1}^{r=m}\, \phi_\rho(r)\, f(r\tau)]^2 \ \ldots\ldots\ldots\ldots\ldots\ldots (9).$$

The probability of this value of θ^2_T is Φ_ρ: hence the sum of the values of θ^2_T for the ν periods of observation is given by

$$\Sigma_\nu \theta^2_T = \nu \Sigma_{\rho=1}^{\rho=\nu} \Phi_\rho [\Sigma_{r=1}^{r=m} \phi_\rho(r) f(r\tau)]^2 \quad \ldots\ldots\ldots(10),$$

where the first sign of summation denotes summation with respect to all the forms which $\phi(r)$ takes in the ν periods.

§ 6. Now summation with respect to a large number of forms of a function is not a process of which the methods have been elaborated. The value of the right-hand side of (10) cannot be found by a direct method, but it can be found by means of the following artifice.

Expanding the squared factor in brackets we obtain

$$\Sigma_\nu \theta^2_T = \nu \Sigma_{\rho=1}^{\rho=\nu} \Phi_\rho \Sigma_{r=1}^{r=m} \Sigma_{s=1}^{s=m} [N^2\tau^2 + N\tau(x_r + x_s) + x_r x_s] f(r\tau) f(s\tau) \quad \ldots\ldots\ldots\ldots\ldots(11).$$

Since Φ_ρ is independent of r and s, we may reverse the order of summation and write

$$\Sigma_\nu \theta^2_T = \nu \Sigma_{r=1}^{r=m} \Sigma_{s=1}^{s=m} \Sigma_{\rho=1}^{\rho=\nu} \Phi_\rho [N^2\tau^2 f(r\tau) f(s\tau) + N\tau(x_r + x_s) f(r\tau) f(s\tau) + x_r x_s f(r\tau) f(s\tau)] \ldots\ldots(12),$$

where the inner summation denotes that we take the average of the terms in square brackets over all periods of observation before summing them for the periods into which a single period of observation is divided. Now $N\tau$ is a constant and, as has been emphasized already, the average value of x_r or x_s is independent of r or s. The value of x_r in any one period T depends on the value of r, but the probability that it will have that value is independent of r. Hence the average value of $(x_r + x_s)$ or $x_r x_s$ is independent of $f(r\tau)\ f(s\tau)$ and we may take the latter factor outside the summation with respect to ρ. Further we may notice that the average value of x_r or x_s is zero by § 2: hence the average of $(x_r + x_s)$ is zero and the average value of $x_r x_s$ is zero, except when $r = s$. (It must be remembered that we are not taking the average for different values of r and s, but the average for the same values in different periods, so that no x_s is a member of Σx_r.) Hence our equation finally reduces to

$$\Sigma_\nu \theta^2_T = \nu \{\Sigma_{r=1}^{r=m} \Sigma_{s=1}^{s=m} N^2\tau^2 f(r\tau) f(s\tau) + \Sigma_{r=1}^{r=m} f^2(r\tau) \Sigma_{\rho=1}^{\rho=\nu} \Phi_\rho x_r^2\} \quad \ldots\ldots\ldots\ldots\ldots(13).$$

But $\Sigma_{\rho=1}^{\rho=\nu} \Phi_\rho x_r^2$ is simply the average value of x_r^2 for all periods T, that is, the average of x_r^2 for a very large number (ν) of cases which are perfectly independent. By equation (1) above, the value of this average is $N\tau$ and hence

$$\Sigma_\nu \theta^2_T = \nu \{\Sigma_{r=1}^{r=m} \Sigma_{s=1}^{s=m} N^2\tau^2 f(r\tau) f(s\tau) + \Sigma_{r=1}^{r=m} N\tau . f^2(r\tau)\} \ldots(14).$$

Since $f(r\tau)$ is a continuous function of r, we may, with certain reservations to be considered later, replace the summations by integrations. Putting $\tau = dt$ we find

$$\frac{1}{\nu}\Sigma_r \theta^2_T = \overline{\theta^2_T} = N^2 \int_{t=0}^{t=T} dt\, f(t) \int_{t=0}^{t=T} dt\, f(t) + N \int_{t=0}^{t=T} dt\, f^2(t) \ldots (15),$$

or, remembering that

$$e^{-\alpha T} = e^{-\beta T} = e^{-pT} = 0,$$

$$\overline{\theta^2_T} = N^2 \left(\frac{A}{\alpha} + \frac{B}{\beta} + \frac{P}{p}\right)^2 + N\left\{\frac{A^2}{2\alpha} + \frac{B^2}{2\beta} + \frac{P^2}{2p} + \frac{2AB}{\alpha+\beta} + \frac{2BP}{\beta+p} + \frac{2PA}{p+\alpha}\right\} \ldots\ldots\ldots\ldots\ldots (16).$$

§ 7. θ_T is the deflection of the needle from its zero position, but it will be found more convenient to express (16) in terms of the deflections of the needle from its mean position $\overline{\theta_T}$. If θ'_T denote the deflection from this mean position

$$\theta^2_T = \{(\overline{\theta_T}) + \theta'_T\}^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17),$$

or since $\Sigma\theta'_T = 0$

$$\overline{\theta^2_T} = (\overline{\theta_T})^2 + \overline{\theta'^2_T} \ldots\ldots\ldots\ldots\ldots\ldots (18).$$

But, by an argument precisely similar to that given above, it can be shown that

$$\overline{\theta_T} = N\left(\frac{A}{\alpha} + \frac{B}{\beta} + \frac{P}{p}\right) \ldots\ldots\ldots\ldots\ldots\ldots (19).$$

Hence

$$\overline{\theta'^2_T} = N\left\{\frac{A^2}{2\alpha} + \frac{B^2}{2\beta} + \frac{P^2}{2p} + \frac{2AB}{\alpha+\beta} + \frac{2BP}{\beta+p} + \frac{2PA}{p+\alpha}\right\} \ldots (20).$$

We have found a relation between the unknown N (the number of particles emitted per second) and the quantities $\overline{\theta'^2_T}$, A, B, P, α, β, p, which are known, except so far as the charge E is concerned. E may be eliminated by the device of Meyer and Regener, who, by the use of their compensation method, measured not $\overline{\theta'^2_T}$ but $\left(\frac{\theta'_T}{\overline{\theta_T}}\right)^2 = \Delta^2$. The formula applicable to their experiments is

$$\Delta^2 = \frac{1}{N} \cdot \frac{\dfrac{A^2}{2\alpha} + \dfrac{B^2}{2\beta} + \dfrac{P^2}{2p} + \dfrac{2AB}{\alpha+\beta} + \dfrac{2BP}{\beta+p} + \dfrac{2PA}{p+\alpha}}{\left(\dfrac{A}{\alpha} + \dfrac{B}{\beta} + \dfrac{P}{p}\right)^2} \ldots\ldots (21).$$

Since E is a factor of A, B, and P, it disappears in the ratio on the right hand of (21).

In passing it may be noted that the argument cannot be applied to the determination of $|\overline{\theta_T}|$. For, in the place of the square of the expression in brackets in (10) we should have the modulus of that expression. The modulus cannot be expanded in powers of the variables concerned and, accordingly, the order of the summations cannot be reversed. But on this reversal hangs the whole argument. In any case it would be advisable to avoid making use of $|\overline{\theta_T}|$, for we have seen in § 2 that its value can be found with accuracy only if $N\tau$ is very large.

§ 8. Before proceeding to discuss in detail the application of this theory to experiment we must consider how far the assumptions that have been made are likely to be fulfilled in experimental conditions.

(1) We have assumed that τ is small compared with T. Since we have seen that the theory is true whatever the value of $N\tau$, it is clear that this assumption will cause no difficulty. We have already, in replacing the summations of the last paragraph by integrals, made τ infinitesimal in comparison with the time constants of the instrument. But T is the whole period over which the physical conditions are constant, and it is clear that the experiment will be arranged so that these are constant over a period long compared with the time of observation.

(2) (1) is the only condition which is essential to the theory, but we have seen that in order to get a probable error less than $1\,{}^{\circ}\!/_{\circ}$, we must take a very large number of observations. The quantity ν corresponds to the σ of § 2 and hence, for this degree of accuracy must be not less than 10^4. Now it is clear that 10^4 observations cannot be made by any process of looking at the instrument and writing the observation down on paper: on a favourable estimate the process would need a month's continuous observation, day and night. Accordingly we must start our discussion of what instruments are to be used with considering how this large number of observations is to be taken.

§ 9. The obvious method of taking this large number of observations is to record photographically the fluctuations of the instrument and to deduce the value of $\overline{\theta'^2}_T$ from the resulting trace by finding the value of $\frac{1}{T}\int_0^T \theta'^2_T dt$. It would not be hard to devise a mechanical integrator which would give the value of $\int y^2 dx$ for any irregular curve, and, if this instrument were sensitive, its indications would correspond to an average taken over a very large number of cases: in fact, if dx is the smallest interval over which the instrument can be expected to distinguish between different values of y, then an integration over a length x would

be equivalent to $\frac{x}{dx}$ observations. But it might appear at first sight that this method was unjustifiable because all the periods T would not be independent. But the only dependence between them is expressed by

$$\Sigma_{\rho=\rho_0}^{\rho=\rho_0+a} \phi_\rho(r_0) f(r_0) = \Sigma_{r=0}^{r=a} \phi_{\rho_0+r_0}(r) f(r) \ldots\ldots\ldots(22),$$

a relation which affects in no way the argument that the probable value of $\phi_\rho(r_0)$ is independent of r_0: it is independent of everything except $N\tau$ and, as was shown in § 3, $N\tau$ is independent of the way τ is selected. Or, if that proof seems inconclusive, we may remember that the average value of θ'^2_T for successive independent periods T must be the same at whatever moment (distant by an amount greater than T from the moment of insulation) the first period is dated. The proposed method consists of nothing more than averaging the averages of the same number of independent periods starting from different moments.

Again, it may be argued that this method adds nothing in accuracy since the same observations are used over and over again. It is quite true that the method does nothing to diminish errors due to faulty observations of θ'_T, but it does diminish errors due to the application of a theory in which we have assumed $\nu = \infty$ to experiments in which ν is finite.

We may note also that photography adds to the accuracy in the observation of the position of a moving object. The only objection to the method is that greater weight is assigned to observations in the middle of a series than to those at the ends, but this objection does not seem to be serious; if observations are really of equal weight no harm is done by attributing a greater weight to a selection taken at random.

§ 10. Let us now consider the instruments which it will be desirable to use.

Firstly, we must be able to determine their constants with accuracy. The determination will doubtless be carried out by raising the electrode system (including the resistance) to a known potential ν and watching the return of the needle to zero. (See § 16, below.) The equation of motion of the needle in this experiment will be

$$\theta = A'e^{-\alpha t} + B'e^{-\beta t} + P'e^{-pt} \ldots\ldots\ldots\ldots(23),$$

where α, β, p are as before and

$$P' = \frac{KV}{(Ip^2 - \mu p + k)}, \qquad A' = \frac{P(\beta - p) - V\beta}{\alpha - \beta},$$

$$B' = -\frac{P(\alpha - p) - V\alpha}{\alpha - \beta}.$$

If the quantities α, β, p are all different, it will be no easy matter to find their separate values: the analysis of a curve into three component exponentials involves great labour and the result is not likely to attain a high degree of accuracy. But most electrometers and electroscopes are either just periodic or just aperiodic, and they can be adjusted to the boundary condition between these two states without great difficulty. But in this boundary state

$$\sqrt{\mu^2 - 4Ik} = 0 \quad \text{and} \quad \alpha = \beta.$$

Let us suppose, then, that $\beta = \alpha + \gamma$, where γ is small compared to α. Taking into account the values of the constants, A, B, A', B', our equations undergo the following simplifications. (23) becomes

$$\theta = (V - P')e^{-\alpha t} + P'e^{-pt} - \gamma t B' e^{-\alpha t} \ldots\ldots\ldots\ldots (23'),$$

$f(t)$ (equation (6)) reduces to

$$f(t) = P\,(e^{-pt} - e^{-\alpha t}) - \gamma t B e^{-\alpha t} \ldots\ldots\ldots\ldots\ldots (6'),$$

and (21) becomes

$$\Delta^2 = \frac{1}{N} \frac{P^2\left(\frac{1}{2p} + \frac{1}{2\alpha} - \frac{2}{\alpha + p}\right) + P\left(\frac{\gamma\beta}{2\alpha^2} - \frac{\gamma\beta}{(\alpha + p)^2}\right) + \frac{\gamma^2\beta^2}{4\alpha^3}}{\left\{P\left(\frac{1}{p} - \frac{1}{\alpha}\right) - \frac{\gamma\beta}{\alpha^2}\right\}^2} \ldots (21').$$

From (6′) and (21′) it is evident that nearly equal values of p and α are to be avoided: for, if these two quantities are equal, the two equations contain only terms involving the small quantity γ, which is hard to determine. Accordingly we must make one of the two quantities p and α very large and the other very small. There are three reasons why α should be made large and p small.

(1) A small value of p corresponds to a high value of the resistance of the Bronson cell. But the greater this resistance, the greater is the deflection of the indicating instrument for a given current passing through the cell. Since it is easier to measure variations of the same proportional amount in a quantity, when that quantity is large, than when it is small, it is desirable that the steady deflection of the indicator, corresponding to the mean current, should be as great as possible.

(2) A large value of α makes the terms involving the unknown small quantity γ negligible. Any slight deviation from the boundary state between periodicity and aperiodicity will be unimportant.

(3) If α is large the required constants can be determined more easily from the equation (23′). Since p and α are very different, the value of the terms involving $e^{-\alpha t}$ can be found from

the beginning or end of the observed decay curve, and those involving e^{-pt} from the end or beginning, according as α is greater or less than p. If α is large, t will be small for the part of the curve over which α is determined, and the term in γ will be of very little importance.

All the considerations advanced indicate that a Bronson cell should be chosen with as high a resistance as possible, and an indicating instrument with as short a period as possible. It seems desirable, therefore, to employ a gold-leaf or quartz-fibre electroscope in place of a Dolezalek electrometer. The only objection to the use of such an instrument is that it will be somewhat difficult to determine with accuracy the value of its time-constant $\frac{1}{\alpha}$, which will be of the order of one second. But it must be remembered that, if α is large compared to p, the terms in (21′) involving the former quantity are small compared to those involving the latter. Moreover, if the movement of the instrument is recorded by photography, the value of this quantity may be found by a method similar to that employed for measuring the constants of an Einthoven string galvanometer. The use of a Dolezalek electrometer seems prohibited, since, if it is aperiodic, its time-constant will be so great that it will be difficult to make the Bronson resistance so large as to avoid an approximate equality between α and p.

§ 11. This elaborate discussion of experimental methods will be worthless unless there is some reason for believing that the method is not affected by some inevitable source of inaccuracy. There seems no reason why the observations should not be taken with very considerable accuracy, and the theory that has been given is, so far as I can see, complete and trustworthy. Let us consider the sources of systematic error which cannot be eliminated by the most careful experimentalist.

Firstly, there is the Bronson resistance: the current through this instrument is subject to fluctuations which are added to those due to the source observed. These fluctuations can be diminished by using β rays in place of α rays, as the agent of ionisation in the resistance: for though a much larger number of β particles will be required and the absolute error, measured in number of β particles, will be increased, the number of ions produced by a β particle is so much less than the number produced by an α particle that the absolute error, measured in current, is less. But, against the use of β particles there is the objection that they cannot, with any convenience, be absorbed completely in the air of the resistance and hence chance fluctuations may arise from changes of temperature, etc. But in any case, the error due to the resistance can be eliminated by calculating the value of the

mean fluctuations by means of an approximate value of the electronic charge. (It may be noted that it might be possible to use some other form of high resistance, but probably all conductivity is ultimately discontinuous and we know far more about the conductivity of a gas than about that of a solid or liquid.)

Secondly, there is a source of error in the fluctuations of other parts of the apparatus during the period of observation: the cells used for maintaining the potential are the most likely source of trouble on this account. Perhaps such errors might be estimated and eliminated by means of carefully devised 'blank' experiments.

Meyer and Regener seem to suggest that an error may be introduced by the finitude of the time required for a charge received by the electrode system to attain a steady distribution on that system. But it can be shown readily by a little arithmetic that error from this source is negligible. The error could only enter by fluctuations of the electrometer due to the vibration of the charge about its final position. Now, if R is the resistance of the vibrating circuit (in ohms), L its self-inductance (in henries), C its capacity (in farads), the period of vibration of the charge is

$$\frac{4\pi L}{\sqrt{\frac{4L}{C} - R^2}}.$$

Now taking the values of R, L, C, which will give the greatest possible value for the period, we may put $R = 1$, $C = 10^{-9}$, $L = 10^{-9}$ (remembering that the self-inductance in henries of a solenoid of 100 turns and 1 cm.^{2} area is 10^{-6}). Then the period is about 10^{-8} seconds. Now in the experiments of Geiger and Meyer and Regener the number of α particles coming off per second was never greater than 10^6: hence the period of oscillation of the charge is small not only compared with that of the instrument, but also with the average interval between two charges.

§ 12. It may be desirable to comment upon the work of Meyer and Regener in the light of the considerations which have been advanced.

Meyer and Regener did not make use of the actual theory which has been elaborated, but they made use of its principle. They assumed that $\overline{\Delta^2}$ was proportional to $1/N$ and that the factor of proportionality depended on the instrumental constants only: this assumption, as we have seen, is justified. Accordingly they were justified in their expectation that, if the same instrumental arrangements are used throughout, $\overline{\Delta^2}$ should be proportional to $1/N$ for different values of N. Since they only took some 100 observations of $\overline{\Delta^2}$ they could only hope for an agreement

between theory and experiment of about 10 °/₀, in place of the 1 °/₀ at which we have aimed.

The best series of their results shows such an agreement as might have been anticipated, but in other series the value of $\overline{\Delta^2}$ for large values of N was too large compared with that for the small values. In the first series an aperiodic electrometer was used, in the second a periodic instrument. It is not obvious from the theory that has been given why this distinction should be found between different experimental arrangements: for the only difference made in equation (21) is that α and β are complex with a periodic needle and real when the needle is aperiodic. But Meyer and Regener's method of observing the fluctuations is open to exception: they took a reading of θ'_T only when the needle reversed its motion at a peak in the fluctuation curve. Now there is no reason that I can see for choosing the peaks for observation in preference to any other points on the curve: the effect of the various particles emitted is integrated by the instruments at the peaks in a manner no less complex than when the needle is moving constantly in the same direction. The only justification for choosing the peaks is that they are easy to observe (since the needle is at rest) and that they are points chosen at random. Now it seems likely that there might be a tendency to overlook peaks in the curve near the zero position of the needle, when the total range of the fluctuation is small, and to observe all the peaks which are more distant from the zero position. This tendency would be the more marked, the greater the range of the fluctuations and the swifter the motion of the needle. For the latter reason it would be more marked with a periodic instrument than with an aperiodic, and for the former reason it would be more marked when the total current was large. Accordingly we might expect to find that, in the case of the periodic needle, the observed values of $\overline{\Delta^2}$ would be too large in the case of large values of N, relatively to those values for smaller values of N. This differentiation is what is shown in Meyer and Regener's tables.

On the whole, Meyer and Regener's attempt to prove the form of the relation between $\overline{\Delta^2}$ and $\frac{1}{N}$ seems to have been fairly satisfactory: but criticism can be directed against their attempt to deduce from $\overline{\Delta^2}$ an absolute value for $\frac{1}{N}$. In the first place, they seem to neglect altogether the effect of the constant of the Bronson resistance and practically write

$$\overline{\Delta^2} = \frac{1}{N} \cdot \alpha.$$

If their absolute value accorded well with that deduced from other work it can only be because p was small compared with α: but, for all they say, p might have been less than α, or, still worse, equal to α.

Meyer and Regener also introduce a curious correction by extrapolation for the capacity of their electrode system. I should have thought that it was obvious on general grounds that the value of $\overline{\Delta^2}$ was independent of the capacity as shown by (21), for both $\overline{\theta_T}$ and $(\overline{\theta_T{}^2})^{\frac{1}{2}}$ are inversely proportional to the capacity. They extrapolate for zero capacity: but, surely, if the capacity is zero the fluctuations must be infinite and the value of $\frac{1}{N}$ infinite. If extrapolation is to be used at all, it would appear to be more reasonable to extrapolate for infinite capacity: such a process reduces the term in p (equation (20)), which they neglect, to zero —but unfortunately it reduces all other terms in the same ratio.

Lastly, Meyer and Regener, in estimating the average value of Δ^2, divide $\Sigma\Delta^2$ by $n-1$, and not by n, where n is the total number of observations. There seems to me a confusion here with the determination of the probable error of a variable given by a set of dependent equations from the 'residuals' of those equations after the probable value of the variable is substituted. But there seems to be no quantity analogous to 'residuals' in the case we are considering: the mean fluctuation is simply proportional to the mean 'error' in N.

Geiger's method.

§ 13. We will now proceed to the discussion of Geiger's method, in which no Bronson resistance was used, but the current due to one α ray source balanced by another current, opposite in sign and, on the average, equal in modulus, due to another source. The deflection of the needle is the algebraic sum of the deflections due to the two sources.

If we consider only one source, the equations corresponding to (3) and (5), giving the deflection of the needle at all times after one or m particles respectively have been emitted from that source, can be found directly from those equations by putting $p=0$. Hence, using the same notation as before,

$$\theta = A_1 e^{-\alpha t} + B_1 e^{-\beta t} + P_1 \quad \text{.....(24)},$$

where $$P_1 = \frac{KE}{Ck}\, A_1 = P_1 \frac{\beta}{\alpha-\beta}, \qquad B_1 = -P_1 \frac{\alpha}{\alpha-\beta},$$

and $$\theta_T = \Sigma_{r=0}^{r=m} (A_1 e^{-\alpha t_r'} + B_1 e^{-\beta t_r'} + P_1) \quad \text{.....(25)}$$

$$= \Sigma_{r=0}^{r=m} F(t_r') \text{ (say)} \quad \text{.....(26)}.$$

§ 14. Let the suffix + denote in all cases quantities corresponding to one source, and the suffix − denote quantities corresponding to the other source. Then, it is clear that (10) may be replaced by

$$\Sigma_\nu(\theta_{+T}+\theta_{-T})^2=\nu\Sigma_{\rho=1}^{\rho=\nu}\Phi_{+\rho}\Phi_{-\rho}[\Sigma_{r=1}^{r=m}(\phi_{+\rho}(r)+\phi_{-\rho}(r))F(r\tau)]^2\ldots(27),$$

where $\phi_{+\rho}(r)$ and $\phi_{-\rho}(r)$ may have any of the possible values

$N\tau+x_{+1},\ldots, N\tau+x_{+n}$, or $-N\tau+x_{-1},\ldots, -N\tau+x_{+n}$ respectively,

for we must remember that $N_+=-N_-$.

Reasoning in precisely the same way as before, we arrive at the following equation corresponding to (13),

$$\Sigma_\nu(\theta_{+T}+\theta_{-T})^2=\Sigma_{r=1}^{r=m}F^2(r\tau)\,\Sigma_{\rho=1}^{\rho=\nu}\Phi_{+\rho}\,\Phi_{-\rho}(x_{+r}+x_{-r})^2\ldots\ldots(28).$$

Now $\Sigma_{\rho=1}^{\rho=\nu}\Phi_{+\rho}\,\Phi_{-\rho}(x_{+r}+x_{-r})^2$ is the average value of $(x_{+r}-x_{-r})^2$ for a very large number of values of that quantity. But by a well-known theorem in probability, the average value of $(a\pm b)^2$ is $\overline{a}^2+\overline{b}^2$ if a and b are independent. But the average of x_{+r}^2 and of x_{-r}^2 is the same and equal, by (1), to $2N\tau$. Hence, we get in place of (15),

$$\frac{1}{\nu}\Sigma_\nu(\theta_{+T}+\theta_{-T})^2=2N\int_{t=0}^{t=T}dt\,F^2(t)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

$$=2N\left\{\frac{A_1^2}{2\alpha}+\frac{B_1^2}{2\beta}+P_1^2T+\frac{2A_1P_1}{\alpha}+\frac{2P_1B_1}{\beta}+\frac{2A_1B_1}{\alpha+\beta}\right\}\ldots(30),$$

or $$\overline{\Delta'^2}=2N(uT+v)\text{ (say)}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(31).$$

Now, since $\overline{\Delta'^2}$ is dependent on T, we cannot compare directly values of Δ' for different values of T. That is to say, if we record photographically, as suggested in § 10, the values of Δ'^2 for all values of T, we cannot put $2N\infty$ (average of all these values of Δ'^2). We must compare values of Δ'^2 for different values of T by dividing each value of Δ'^2 by the appropriate value of $(uT+v)$, and then take the average of $\dfrac{\Delta'^2}{uT+v}$. We cannot even, as might appear at first sight, take the difference of values of Δ' for a series of times differing by T and equate the average of the squares of these differences to $2N(uT+v)$: for such a procedure would involve the false proposition that

$$|\overline{\Delta_1'-\Delta_2'}|^2=\overline{\Delta_1'^2}-\overline{\Delta'^2}_2.$$

Accordingly the labour of deducing the value of N from the observed fluctuations will be very much greater than in Meyer and Regener's method, for it would be difficult to construct a

mechanical integrator which should give the value of $\int \frac{y^2}{(ax+b)} dx$ for a wholly irregular curve.

§ 15. As regards the choice of instruments for this method, the conclusions reached in considering Meyer and Regener's method are still valid. Since p is zero there is no need for considering the relation between the periods of the electrometer and the resistance, but it is still desirable that α should be very large. For there is some uncertainty in the determination of the value of this quantity: since the importance of the terms involving α in comparison with the term $P_1^2 T$, which does not involve α, decreases as α increases, it is clearly desirable to make α as large as possible, that is, to use an electroscope with as short a period as possible.

On general grounds Geiger's method seems less satisfactory than that of Meyer and Regener for observations on phenomena for which both methods are, in principle, equally applicable. But it is easy to imagine circumstances in which Geiger's method would, and Meyer and Regener's would not, be applicable: for instance, the object of the experiment might be to detect discontinuity in the difference between two sources, and indeed I am engaged at present in endeavouring to conduct such experiments. It is desirable, therefore, to consider Geiger's method a little more closely.

§ 16. It must be pointed out that the theory of Geiger's method which has just been elaborated does not represent any practicable experimental conditions. It is impossible to reduce the quantity p accurately to zero. In the first place, there is always an 'insulation leak,' which acts like a Bronson resistance and sets a limit to the deflection which the indicating instrument will attain when acted upon for an infinite time by a finite current. But this leak can (though with great difficulty) be made so small that values of T can be chosen such that αT is very large and pT is very small. In such a case Geiger's method could be applied according to the theory just given. But, in most of the cases to which the method is likely to be applied, there is also an 'apparent insulation leak,' which is a far more serious matter. It must be remembered that no ionisation current is really perfectly 'saturated.' A condition can often be attained in which an increase of the acting potential difference from 1000 to 2000 volts does not increase the current by more than 0·001 of its value, but this does not prove that an increase of 1 volt in the P.D. may not cause an increase of one millionth in the current. And a change of one millionth may be extremely important when we are concerned with the minute differences in the magnitudes of two

balanced currents very nearly equal. The effect of a lack of perfect saturation is to diminish the current due to either source when the potential of the electrode is of the sign corresponding to a preponderance of the current due to that source: it sets a finite limit to the potential which the electrode can attain, diminishes the magnitude of the fluctuations and, in every manner, acts like an insulation leak.

Errors from this source, even if their possibility had occurred to me, would have been rejected as unimportant, if I had not had actual experience of them. I have found the false insulation leak a very serious trouble in dealing with currents for which saturation is likely to be far easier of attainment than in the case of α ray currents; these currents are those due to the photoelectric effect in a very high vacuum. In dealing with these currents, the saturation, tested by the ordinary method, is perfect at 50 volts: tested by balancing two currents and then watching the decay of a potential difference applied to the common electrode, saturation is not perfect even with a P.D. of 500 volts, though it is much more nearly perfect than at 50 volts.

The existence of this false insulation current leak reduces Geiger's method to that of Meyer and Regener. p has a finite value which can be found by watching the decay of a P.D. given to the electrode when the two currents are balanced. It should be pointed out that this apparent insulation leak affects Meyer and Regener's experiments also: in that case too the value of p must be found when the current to be measured is flowing through the Bronson resistance and the compensating P.D. applied.

§ 17. A few remarks on Geiger's work are all that remains.

(1) In the first place we may note that by taking some 100 observations only of the fluctuations Geiger, like Meyer and Regener, introduced a probable error of some 10 %. Since the agreement which he found between theory and experiment was of this order of accuracy, it might seem that in other respects his work was satisfactory. But

(2) It seems to me that Geiger employed a totally inadequate and inaccurate theory in the interpretation of his observations. He assumes, like Meyer and Regener, that the value of his fluctuations depends only on the constants of his instruments, but equation (33) shows that it is also a function of the time of observation. Only the most general consideration is necessary to show that in Geiger's method (assuming the absence of an important insulation leak) the effects of the particles which came off at a distant time are as important as the effects of those which came off only just before the moment of observation: the mean value of the fluctuations over any short period must depend upon the distance of this period from the time when the electrode was

insulated. But Geiger adds all his fluctuations together, whatever the time of observation, as if they were comparable.

Accordingly it appears to me that Geiger's expectation of obtaining a result even in approximate accordance with that predicted by other work was justified only if he had a large false insulation leak.' If there is such a leak, then Geiger's method is reduced to that of Meyer and Regener, and all observations at more than a certain distance from the moment of insulation are comparable. The existence of an insulation leak would also explain that the 'time of swing of the needle' ($5\frac{1}{2}$ secs.) is so much smaller than that which is found with most Dolezalek electrometers: for the period between two peaks might then depend, not on the vibrations of the needle, but on the period of decay of his quasi-'Bronson-resistance.' But, if this suggestion is correct, he neglected totally the real time of swing of his needle.

(3) As a minor point we may note that Geiger, attempting to use von Schweidler's theory, puts $\overline{|\Delta'|} = \sqrt{\overline{Nt}}$. Correctly $\overline{|\Delta'|} = \sqrt{\frac{2\overline{Nt}}{\pi}}$ for one source of rays and $2\sqrt{\frac{\overline{Nt}}{\pi}}$ for 2 independent sources (error about 20 %). The quantity which should be equal to $\sqrt{\overline{Nt}}$ is $\sqrt{\overline{\Delta'^2}}$. As I have pointed out, it seems impossible to find a complete expression for the value of $\overline{|\Delta'|}$ in terms of N and the instrumental constants.

In view of these sources of error, it is difficult to avoid the conclusion that the close agreement found by Geiger between prediction and experiment was mainly fortuitous.

In conclusion I must offer my best thanks to Mr G. H. Hardy for his invaluable help in §§ 5—8: the main principle of the argument is his rather than mine; to Sir J. J. Thomson, who has pointed out several errors in the original calculations; and also to Mr T. H. Laby, whose acute criticism first turned my attention to the subject and aided greatly in the attainment of clear ideas.

Summary.

§§ 1—3. A slight modification and generalisation of von Schweidler's theory of the fluctuations in the emission of rays by a radioactive substance.

§§ 4—7. A consideration of the theory of Meyer and Regener's experiment based on von Schweidler's work.

§§ 8—12. Application of this theory to experimental arrangements.

§§ 13—17. The theory of Geiger's experiment in the same direction.

On the nature of anthocyanin. By Miss M. Wheldale, Newnham College. (Communicated by Professor Bateson, F.R.S.)

[*Read* 8 March 1909.]

Many investigators have suggested, from time to time, that a relationship exists between the widely distributed tannins in plants and the equally widely distributed pigment known as "anthocyanin." Curtel[1], Dennert[2], Gautier[3], Ichimura[4], Mirande[5], Newbigin[6], Wiesner[7], Wigand[8] and Zopf[9], among others, mention the occurrence of a colourless chromogen, of the nature of a tannin, in tissues, which eventually may become coloured with anthocyanin; the colouration arises under the influence of various agents, such as nutrition, temperature, light, and injury, either mechanical or such as is brought about by attacks of fungi, etc. The process of transition, moreover, from chromogen to pigment has been regarded as one of oxidation.

Not only has the above relationship been established, but the view has also been held that anthocyanin itself is some compound of a tannic acid.

Overton[10], for instance, from the increased production of anthocyanin brought about by artificial feeding of plants with carbohydrates, arrived at the conclusion that this pigment may very possibly be a sugar compound of a tannic acid, that is a glucoside.

Gautier[11] also isolated various colouring matters from the red leaves of the Vine and he regards these pigments as coloured tannic acids.

Pfeffer[12], in a general review of the subject, states that the red and blue pigments dissolved in the cell-sap, such as anthocyanin, seem to be tannins or compounds allied to phenols, though in his view they may also be derived from other substances since they often occur in plants which contain no tannin.

Heise[13], in addition, isolated two pigments from the Bilberry and showed one to be the glucoside of the other. Both substances appear to be aromatic compounds though they have not directly been proved to be tannins.

The word tannin is a somewhat difficult term to define; it is used in general for a class of substances very widely distributed in plants and having certain properties in common.

Their characteristics, as given in most text-books, are an astringent taste, the production of a blue or a green colour with iron salts, tanning properties with animal membranes, power to precipitate albumin and gelatine, etc., etc. The exact constitution of the tannins is also rather indefinite though it is obvious that they are aromatic compounds of the benzene series. Broadly speaking tannins include numerous tannic acids, such as gallic, gallotannic, catechinic, caffeïc, protocatechuic, diprotocatechuic, ellagic, quercitannic, etc., usually existing in plants in the form of glucosides, *i.e.* ethereal compounds with various sugars or with the trihydric phenol, phloroglucin.

It has been suggested that tannins may be divisible into two groups, derived respectively from a dihydric phenol, such as pyrocatechol, and a trihydric phenol, such as pyrogallol. The kind of relationship might be illustrated as follows:—

$C_6H_4(OH)_2$ pyrocatechol	$C_6H_3(OH)_2 . COOH$ protocatechuic acid	$C_6H_3(OH)_2 . CO . O . C_6H_3 . OH . COOH$ diprotocatechuic acid
$C_6H_3(OH)_3$ pyrogallol	$C_6H_2(OH)_3 . COOH$ gallic acid	$C_6H_2(OH)_3 . CO . O . C_6H_2(OH)_2 . COOH$ tannic acid or digallic acid

The tannins included in the first group give as a rule an iron-greening reaction (though this is by no means invariably the case), produce pyrocatechol on dry distillation and protocatechuic acid when fused with caustic alkali. Those of the second group give an iron-blueing reaction, produce pyrogallol on dry distillation and gallic acid on fusion with caustic alkali.

Waage[14] and also Nickel[15] are of the opinion that another series of tannic acids arises from the oxidation of phloroglucin, the isomer of pyrogallol:—

$C_6H_3(OH)_3$ phloroglucin	$C_6H_2(OH)_3 . COOH$ phloroglucin carbonsäure (carboxylic acid)	$C_6H_2(OH)_3 . CO . O . C_6H_2(OH)_2 . COOH$ phloroglucin gerbsäure (tannic acid)

Hence there may well be a complexity of tannins of different natures.

With a view to finding out whether, and if so, in what way anthocyanin is related to tannin, the extracts from flowers of numerous natural orders, both with and without anthocyanin, were tested for tannin with various reagents. Although the behaviour of the pigments has been diverse in many ways, a certain amount of classification can be based on the results obtained.

It must at the same time be borne in mind that some of the reagents used in these tests, react not only with the tannins but also with many other bodies, such as phenols and their numerous derivatives.

The reagents* employed were as follows:—

(1) Strong bases, such as solutions of caustic potash or soda and of ammonia.

(2) Lime water. A blue, brown or red colour or precipitate shows tannins.

(3) Strong acids, such as sulphuric, hydrochloric and nitric acids. The reactions of BRISSEMORET[16] are interesting in this connection though they have not been employed to any great extent.

(4) Iron salts. Ferric chloride solution was used and also a solution of ferrous sulphate, in which some oxidation to the ferric salt has taken place. A blue-black or dull green colouration shows tannins.

(5) Precipitation by basic lead acetate which precipitates most tannins and glucosides.

(6) Potassium ferricyanide solution and ammonia. A reddish-brown colouration changing to brown shows tannins.

(7) Uranium acetate solution. A brown precipitate or reddish-brown or brown colour shows tannins.

(8) Ammonium molybdate solution, both alone and with the addition of a saturated solution of ammonium chloride. According to Gardiner† alkaline molybdate gives a red colour with tannin. If excess of ammonium chloride is added, a voluminous yellow precipitate is produced. Moreover by means of this reagent, according to the above author, tannin may be readily separated from tannic, *i.e.* digallic acid, for whereas the compound of tannin and molybdate (?) is insoluble in ammonium chloride, that with tannic acid is soluble. The reagent therefore separates the glucoside tannins from tannic acid.

As a preliminary statement it may be said that tannins, like that of the oak gall, giving an intense blue-black colour with iron salts and also reactions 2, 6, and 7 are rare among herbaceous plants. There are however widely distributed among the latter, certain colourless or slightly yellow substances, giving an iron-greening reaction though they are not typically tannins, since they do not as a rule give reactions 2, 6, 7, and 8. These substances are characterised in addition by the bright yellow colour produced when they are acted upon by strong bases and by lime-water. They give the same colour also with strong mineral acids, such as sulphuric and hydrochloric, and they are precipitated by basic lead acetate as canary-yellow precipitates.

These reactions are best seen when extracts of white flowers are used or those of yellow flowers, in which the colour is due to plastid pigment only. Flowers giving these reactions are arranged in the following scheme.

* Darwin and Acton, *Physiology of Plants.*

† Gardiner[17].

I. White flowers, of which an extract gives a bright yellow colour with alkalis and acids, a bright yellow precipitate with basic lead acetate and a reaction with iron salts.

(*a*) Those giving a green colouration with iron salts: *Mesembryanthemum Haworthii, Colchicum sp., Nerine flexuosa,* paper white variety of the polyanthus *Narcissus,* white berries of *Symphoricarpus racemosus,* white variety of *Chrysanthemum carinatum,* white variety of autumnal cultivated *Chrysanthemum,* ivory white variety of *Viola tricolor,* white variety of *Abutilon striatum, Erica Bowieana* and white variety of *Primula sinensis.*

(*b*) Those giving a blue colouration with iron salts: None.

II. Yellow flowers (albino as regards anthocyanin, but with yellow plastid pigments), of which the extract gives the same reactions as those in I.

(*a*) Those giving a green colouration with iron salts: Ivory variety of *Helianthemum vulgare, Calendula officinalis,* yellow variety of autumnal cultivated *Chrysanthemum, Erigeron Canadensis, Gaillardia sp., Gazania splendens, Helenium nudiflorum, Helianthus annuus, Picris pauciflorus, Senecio Jacobaea,* pale and deep yellow varieties of *Zinnia elegans, Hypericum Hookerianum, Spartium junceum, Linum flavum, Bartonia aurea,* yellow variety of *Abutilon striatum, Jasminum nudiflorum, Argemone grandiflora, Eschscholtzia Californica, Primula Kewensis, Potentilla fruticosa, Hyoscyamus chloranthus,* yellow variety of *Salpiglossis grandiflora,* calyx of *Physalis alkekengi,* pale and deep yellow varieties of *Tropaeolum majus, Tropaeolum canariense,* and pale and deep yellow varieties of *Viola tricolor.*

(*b*) Those giving a blue colouration with iron salts: *Alyssum maritimum.*

Exceptions were the following, which give a yellow precipitate with basic lead acetate, a yellow colour with alkalis but not the above reactions with iron salts: *Taraxacum officinale*, Brassica sinapis*, Cucurbita Pepo**, and fruit of *Lycopersicum esculentum.*

It might be noted that these exceptions are chiefly plants which do not normally produce anthocyanin.

The xantheïc pigments of many yellow flowers seem to be very similar in nature to the substances present in the above Classes I. and II. They differ in that the yellow extract usually deepens to orange-yellow or red with alkalis and acids and the precipitates with basic lead acetate are of corresponding shades. The iron reaction is usually green. Class III. contains genera whose flowers give these reactions.

* Tested only with ferric chloride which does not always give a satisfactory result.

III. Yellow flowers containing a so-called xantheïc pigment (sometimes in addition a yellow plastid pigment), of which the extract gives a colouration with iron salts; a yellow, orange-yellow, orange or red precipitate with basic lead acetate and similar colourations with acids and alkalis.

(*a*) Those giving a green colouration with iron salts: *Mesembryanthemum pomeridianum* (yellow)*, *Centaurea eriophora* (orange-yellow), *Centaurea scabiosa* (orange-yellow), yellow variety of *Chrysanthemum carinatum* (orange), *Corydalis lutea* (orange-yellow), yellow variety of *Dahlia variabilis* (red), yellow variety of *Helichrysum bracteatum* (orange), *Tagetes signata* (orange), *Coronilla glauca* (orange), *Mirabilis Jalapa* (yellow), *Primula acaulis* (orange), *Calceolaria sp.* (orange), and *Nemesia strumosa* (yellow).

(*b*) Those giving a blue colouration with iron salts: *Linaria multipunctata* (orange-red) and *Linaria vulgaris* (orange-red). The following are exceptions in that they do not give the above reactions with iron salts: *Coreopsis Drummondii*† (red) and *Verbascum Lychinitis* (yellow).

When anthocyanin was present in the genera examined, the reactions given by the usual reagents were diverse, but the more common type of reaction is that given in Class IV., *i.e.* a green colouration with alkalis, a green precipitate with basic lead acetate and a green or blue colouration with iron salts. It is doubtful whether the blue colouration given by the anthocyanin of many herbaceous genera is in any way similar to the true iron-blueing tannin reaction. Extracts from the flowers of some shrubs, such as red varieties of *Rosa*, give a true iron-blueing reaction like that of the oak-gall, and this result may be due to admixture of tannin present in the plant. I am inclined to believe that the iron-blueing reaction of the flower extract of herbaceous plants may only be a specific reaction of anthocyanin itself and not an indication of a true tannin nature of this substance. Yet this point of view by no means vitiates the suggestion that this pigment may be an aromatic substance closely allied to the tannins.

The anthocyanin of Class IV. is of the purple or purplish-red type. Though the colours of the flowers, from which it is obtained, often differ among themselves, the alcoholic extracts have the common property of being purple or purplish-red in colour.

IV. Red, purple or blue flowers containing anthocyanin (sometimes in addition plastid pigment), of which the extract gives a green colour with alkalis and a green precipitate with basic lead acetate.

* Denotes colour given with alkalis and basic lead acetate.
† Tested only with ferric chloride.

(*a*) Those giving a green colouration with iron salts: Blue *Vinca major* and blue *Campanula sp.*, carmine pericarp of *Euonymus europaeus*, blue *Tradescantia virginiana*, blue *Aster tripolium*, blue and magenta varieties of *Cineraria sp.*, crimson, mauve, and purple varieties of autumn *Chrysanthemum*, crimson *Hieracium rubrum*, magenta variety of *Helichrysum bracteatum*, pink and scarlet varieties of *Zinnia elegans*, mauve *Aubrietia graeca*, brown *Cheiranthus Cheiri*, pink *Epacris pulchella*, carmine *Salvia dulcis*, purple *Salvia Horminum*, purplish-red *Salvia involucrata*, blue *Anemone coronaria*, carmine *Tropaeolum speciosum*, purple varieties of *Viola tricolor*.

(*b*) Those giving a blue colouration with iron salts: Orange *Alstroemeria aurantiaca*, magenta *Impatiens episcopi*, crimson *Helianthemum vulgare*, pink *Echeveria retusa*, magenta *Azalea amoena*, floral bracts of *Euphorbia splendens*, carmine leaf bracts of *Euphorbia pulcherrima*, crimson *Coronilla viminalis*, orange *Lilium Tigrinum*, crimson *Abutilon striatum*, purple *Fuchsia sp.*, black berries of *Rosa pimpinellifolia*, black berries of *Atropa Belladonna*, magenta *Petunia violacea*.

From the above classification we may conclude that among herbaceous plants aromatic substances giving an iron-greening reaction are common. An iron-blueing reaction however is rare when anthocyanin is absent, but more common when this pigment is present, though it is probable that the reaction in the latter case does not indicate a true tannin nature for this substance.

When a species is in type anthocyanic with an albino variety (as regards this pigment), for example *Chrysanthemum*, *Viola*, *Abutilon*, *Helianthemum*, *Zinnia* and *Helichrysum*, then with alkalis the albino extract is coloured yellow and that of the anthocyanic type blue with a small amount of alkali but green with excess. Similarly the albino extract gives a yellow precipitate with basic lead acetate, and that of the type a green precipitate. The iron reaction is usually the same for both type and variety, though in the case of *Helianthemum* and *Abutilon*, the iron reaction is green for the non-anthocyanic varieties but blue for the anthocyanic type.

A possible explanation for the reactions observed might be as follows:—That many plants free from anthocyanin, and also albino varieties of anthocyanic types, contain colourless aromatic chromogens, probably often in combination with sugars as glucosides as suggested by Overton[10]. These chromogens appear to have acid properties and usually give an iron-greening reaction; as present in the plant in the undissociated state they are almost colourless, but on addition of strong alkali a dissociable salt is formed, of which the anion is bright yellow. The yellow colour disappears again on neutralisation with acid, so that the reaction

resembles that of an acid-alkali indicator. With heavy metals, such as copper and lead, insoluble coloured salts are formed.

There appears to be evidence, as will be discussed later, that the production of pigment in anthocyanic species is brought about through the agency of an oxydase so that anthocyanin may be regarded as a coloured oxidised product of an aromatic chromogen and moreover, like the unaltered chromogen, as an acid capable of producing salts. In the acid state the molecule of anthocyanin is red but the addition of alkali produces a blue salt.

We may suppose that there is usually unaltered chromogen present in the plant in addition to anthocyanin, and the simultaneous existence of these two bodies would account for the green colour given by alkalis and the green precipitate by basic lead acetate, *i.e.* a mixture due to the blue salt of the oxidised product and the yellow salt of the chromogen. The stronger—more oxidised—acid would probably react first with bases; hence the blue colouration with a slight amount of alkali.

On this supposition, the green precipitates prepared and analysed by HEISE[13] and GAUTIER[11] represents a mixture of the salts of both anthocyanic acid and its chromogen.

An alternative explanation is that offered by OVERTON[10], who regards anthocyanin as a di- or poly-basic acid, the blue colour given by a small amount of alkali being due to the acid salt, the green colour with excess of alkali to the neutral salt.

GRAFE[18], too, is of the opinion that one of the pigments isolated by him from *Althaea rosea* is a di-basic acid; he regards the green colour with alkalis as a specific reaction of anthocyanin and not one due to admixture with any other substances giving a yellow reaction.

Evidence for the assumption that the aromatic chromogen forms a component of anthocyanin has been arrived at through experiments in cross-breeding*. It appears, from this source, that the components of anthocyanin can be represented by various Mendelian factors, the loss of which gives rise to derivative varieties. In *Antirrhinum majus*, an ivory-white variety exists, which is free from anthocyanin, but contains a chromogen such as those described above. The magenta type contains anthocyanin. It has been proved experimentally that the factor representing the chromogen is essential to the constitution of the magenta type.

Cross-breeding with *Antirrhinum* shows moreover that the so-called xantheïc pigment of the yellow variety is essential to the constitution of the ivory-white and is hypostatic to it. In fact xantheïn appears to be a yellow aromatic chromogen in a

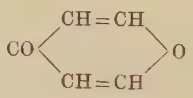
* Wheldale[19].

lower stage of formation than the ivory-white, yet capable, like the latter, of being acted upon by the oxydase with the productions of anthocyanin, which in this case is mixed with unaltered yellow chromogen and consequently produces the crimson colour of the crimson variety.

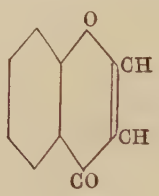
Antirrhinum is the type of a number of genera in which the colourless chromogen of the ivory-white variety gives rise, probably through loss of some factor, to a xantheïc pigment.

Definite knowledge as to the nature and constitution of these aromatic chromogens, from the oxidation of which anthocyanin may arise, is a difficult chemical problem. I am inclined to believe that they belong to the flavone and xanthone classes of natural colouring matters*, which they closely resemble in properties and reactions. The flavones and xanthones are chiefly present in plants as glucosides or rhamnosides, most have fifteen carbon atoms in their empirical formulae and on fusion with caustic alkali many give protocatechuic acid and phloroglucin.

The nucleus of these substances is γ-pyrone, which may be regarded as an anhydride of a diolefine-dioxyketone :—



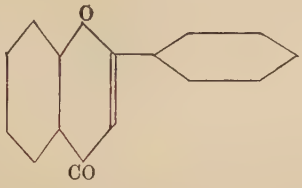
Xanthones and flavones are derived from pheno-γ-pyrone :—



Xanthone being a dibenzopyrone :—

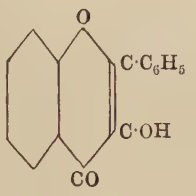
* See Czapek, *Biochemie der Pflanzen.*

and flavone a β-phenyl-benzo-γ-pyrone:—



Among those of other investigators, the very valuable researches of PERKIN[21–40] and his colleagues on the yellow colouring matters of plants in connection with their dyeing properties, have thrown much light on the constitution of these bodies, and since they are of great importance in connection with this point I propose giving a short account of the properties and constitution of the chief members of the group, as well as their distribution among various natural orders. The xanthones appear to be less widely distributed than the flavones and it is to the latter that most importance may be attached.

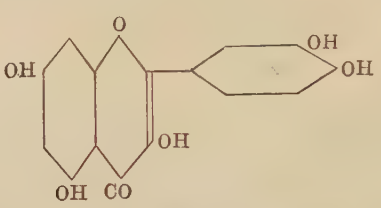
Most of the colouring matters may be regarded as arising from a hydroxy-derivative of flavone:—



Generally speaking they are yellow crystalline bodies, not very soluble in water in the free state, more soluble probably as glucosides. They are soluble in acids giving yellow, orange or red solutions, and in the absence of water many form crystalline compounds with a molecule of the acid, these same compounds being again decomposed into acid and colouring matter on addition of water. At the same time they appear to have basic properties; in alkalis they are soluble, giving again yellow, orange or red solutions. Many, moreover, are able, in the absence of water, to withdraw potassium from potassium acetate, with the formation of a mono-potassium salt. They are precipitated by lead acetate as yellow, orange or red precipitates and with ferric chloride solution they usually give a dull green colouration, occasionally reddish-brown. The decomposition products on fusion with alkali are frequently phloroglucin and protocatechuic acid, though sometimes resorcinol, resorcylic or hydroxybenzoic acids are formed

according to variations in the constitution of the colouring matter. With aniline or toluidine nitrate and potassium nitrite they give a cinnabar-red precipitate. In the plant the flavones mostly occur as glucosides, the sugar being rhamnose or glucose.

Quercetin $C_{15}H_{10}O_7$ is very widely distributed. It may be regarded as a pentahydroxy flavone:—

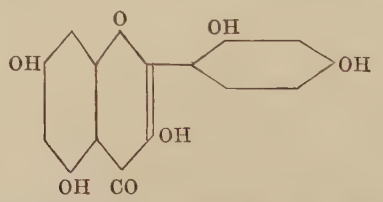


Quercetin occurs free, that is not as a glucoside, in many plants. The following are interesting examples:—its occurrence in flowers of *Prunus spinosa, Crataegus oxyacantha, Delphinium Zalil*, and *Cheiranthus Cheiri*; in leaves of *Rhus rhodanthema, R. metopium, Ailanthus glandulosa, Coriaria myrtifolia, Calluna vulgaris*; it exists also in the outer bulb scales of *Allium cepa* and in many other plants.

On fusion with caustic alkali, quercetin gives protocatechuic acid and phloroglucin derived from the catechol and phloroglucinol nuclei respectively.

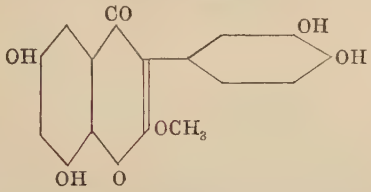
In many plants on the other hand it is combined with glucose or rhamnose as glucosides. It occurs as '*osyritrin*'; a glucose glucoside, in leaves of *Osyris compressa*; '*osyritrin*' is also identical with the '*viola quercitrin*' of flowers of *Viola tricolor variensis* and with '*myrticolorin*' in leaves of *Eucalyptus macrorhyncha*. Again with one molecule of rhamnose it occurs as the glucoside '*quercitrin*' in the bark of *Quercus tinctoria* and with two molecules of rhamnose as '*rutin*' in *Ruta graveolens*. The flowers of *Viola odorata* and *Trifolium repens* also contain quercetin in the form of a glucoside.

Morin, $C_{15}H_{10}O_7$, occurring in the wood of *Morus tinctoria* and *Artocarpus integrifolia*, may be regarded as an isomer of quercetin containing the resorcinol instead of the catechol nucleus:—

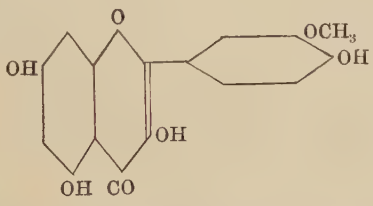


On fusion with alkali it gives β-resorcylic acid and phloroglucin.

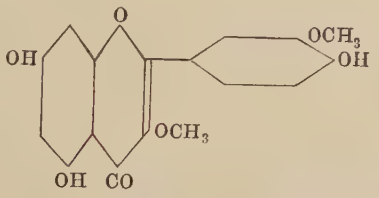
Rhamnetin, $C_{16}H_{12}O_7$, a monomethyl ether of quercetin is found as a rhamnose glucoside—'*xanthorhamnin*'*—in fruits of *Rhamnus spp.* Its constitution may be represented as :—



Isorhamnetin, $C_{16}H_{12}O_7$, an isomerous monomethyl ether, occurs in the flowers of *Cheiranthus cheiri* and of *Delphinium Zalil*: it may be represented as :—

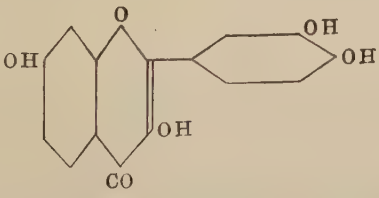


Rhamnazin, $C_{17}H_{14}O_7$, a dimethyl ether of quercetin, occurs as a glucoside in addition to xanthorhamnin in the fruits of *Rhamnus spp.*; it may be represented as :—



Fisetin, $C_{15}H_{10}O_6$, occurs in the wood of *Rhus cotinus, R. coriaria* and *R. rhodanthema.*

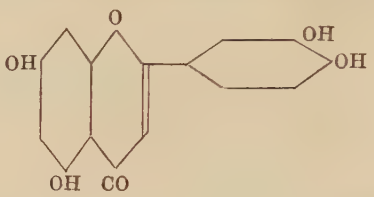
It differs from quercetin in containing a resorcinol instead of a phloroglucinol nucleus and may be regarded as a tetrahydroxy flavone :—



Hence on fusion with caustic alkali, the decomposition products are protocatechuic acid and resorcinol.

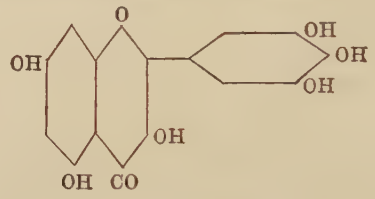
* Cp. "On some points in the Histology and Physiology of the Fruits and Seeds of *Rhamnus*." H. Marshall-Ward and J. Dunlop, *Ann. Bot.* Vol. I. 1887–8.

Luteolin, $C_{15}H_{10}O_6$, is found in *Reseda luteola* and *Genista tinctoria*. It is an isomer of fisetin and may be represented as :—



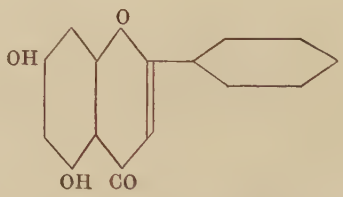
On fusion with alkali, phloroglucin and protocatechuic acid are formed.

Myricetin, $C_{15}H_{10}O_8$, is found in the bark of *Myrica nagi*, in the leaves of *Rhus coriaria* and *R. cotinus*, *R. metopium*, *Pistacia lentiscus* and others. It may be regarded as a hydroxyquercetin containing a pyrogallol instead of a catechol nucleus :—

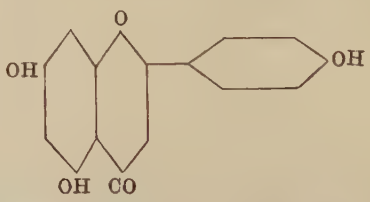


Hence on fusion with caustic alkali it gives gallic acid and phloroglucin.

Chrysin, $C_{15}H_{10}O_4$, is found in buds of *Populus sp.* It is a dihydroxy flavone :—

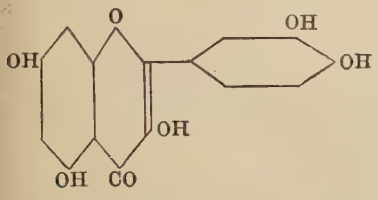


Apigenin, $C_{15}H_{10}O_5$, occurs in the form of a glucoside—'*apün*'—in the leaves, stem and fruit of Parsley, *Apium petroselium*. It may be regarded as a hydroxychrysin or trihydroxy flavone :—

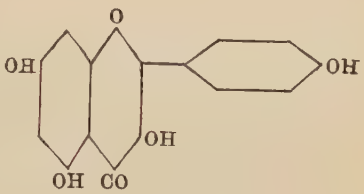


On fusion with caustic alkali, the decomposition products are phloroglucin and parahydroxybenzoic acid.

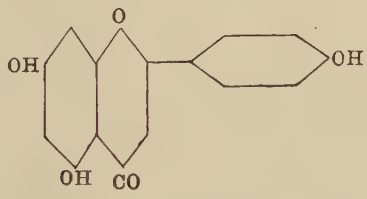
Kampherol, $C_{15}H_{10}O_6$, occurs as a rhamnose glucoside '*robinin*' in the flowers of *Robinia pseudacacia*, and also as another glucoside in the flowers of *Delphinium consolida* and *Prunus spinosa*. It may be regarded as a connecting link between apigenin and quercetin :—



Quercetin



Kampherol



Apigenin

On fusion with caustic alkali it gives parahydroxybenzoic acid and phloroglucin.

The above account conveys some idea of the wide distribution of the flavone series of colouring matters. Some fifty to sixty genera have been examined by PERKIN and his colleagues and these genera are members of many different natural orders. Several of the flavones are common to genera by no means closely allied and again others, as far as his investigations have gone, are apparently specific. The constitution of these bodies, as given above, shows that there is great scope for slight variations and many isomeric forms may exist and may be specific for different plants. At the same time the general nature and tendencies of these bodies are similar, and it is not difficult to conceive of their acting as chromogens in the formation of the various kinds of red pigments found in plants.

I have repeated the method of extraction of PERKIN and HUMMEL[27] for quercetin from onion skins and I find this body very similar in its properties and reactions to the aromatic chromogens present in flowers. Sometimes these bodies crystallise out from the flower extract as in *Narcissus Tazetta**, *Eschscholtzia*, and *Viola*† in which case the substance is more easily prepared in its pure state. The yellow crystalline pigment of *Viola* is without doubt the 'viola quercitrin' of PERKIN.

* Bidgood[41]. † Wheldale[20].

The flowers of the original type in *Narcissus Tazetta* owe their colour to the presence of several pigments; the lemon yellow of the perianth segments is due to plastids containing xanthin, and the orange yellow of the corona to plastids containing both xanthin and carotin. In addition, all parts of the flower contain a yellow flavone pigment which crystallises from the extract in clusters of yellow needles. To ascertain something of its nature, this yellow crystalline pigment was fused with caustic alkali, the melt dissolved in water, neutralised and precipitated with basic lead acetate. The precipitate on decomposition with sulphuretted hydrogen and extraction with ether, gave a crystalline body having the properties of protocatechuic acid. The ether extract also gave evidence of the presence of phloroglucin. Hence there is little doubt that the *Narcissus* pigment may be placed in the flavone series.

A white variety of the same *Narcissus* has no plastid pigments but a pale yellow non-crystallisable flavone having the usual characteristics. On fusion with caustic alkali, phloroglucin and parahydroxybenzoic (?) acid were detected as decomposition products. Hence the flavone of the variety must differ in constitution slightly from the crystalline flavone of the type.

A second variety has the crystallisable flavone only in the perianth, while the corona contains carotin.

A third variety contains a non-crystallisable flavone and xanthin in addition in the corona.

In a similar way the non-crystallisable chromogen from the white flowers of *Primula stellata* and *P. sinensis* was isolated and fused with alkali. The ether extract contained phloroglucin and another body, very probably parahydroxybenzoic acid, since it gave a red precipitate with aqueous iodine and caustic potash, which became yellow on addition of acid.

The work of Naylor and Chappel[12] on *Rosa gallica* forms an additional confirmation of the above suggestion as to the nature of the aromatic chromogens. These authors isolated from the petals of the above species of *Rosa* a yellow crystalline colouring matter, giving an orange-red precipitate with lead acetate and a brownish-black colouration with ferric chloride. It was soluble in caustic potash to a yellow or orange brown solution and in sulphuric acid to an orange-yellow liquid.

Fused with caustic potash it gave phloroglucin and protocatechuic acid.

Heise[13] also found protocatechuic acid as a decomposition product from the fusion with alkali of one of the anthocyanic pigments isolated by him from the Bilberry.

If the existence of a colourless chromogen as a precursor of 'anthocyanin' be accepted, it is possible that the recent work

of PALLADIN[43] may throw light upon the method of production of the pigment from its chromogen. The above author maintains, on the evidence of BERTRAND[44], that oxydases can only oxidise aromatic compounds of a certain constitution, and since the products of oxidation may be coloured PALLADIN terms such oxydases 'pigment-building.' He further maintains that chromogens are present in many plants; for he found that if the extracted sap of such plants be heated sufficiently to destroy the enzymes existing in the plants themselves, then the addition to the extracted sap of a small quantity of the peroxydase of CHODAT and BACH[45 and 46] together with a few drops of hydrogen peroxide, produces a purple, red or reddish-brown colouration.

As to the constitution of the chromogen, PALLADIN[43] suggests that the evidence of OVERTON[10] points to the chromogen as being an aromatic substance combined with sugar and of this nature are glucosides. To quote PALLADIN:—"Die vorstehend beschriebenen Versuche und ebenfalls diejenigen OVERTON ergaben, dass verschiedene Zuckerarten ein Material darstellen, aus welchem verschiedenartige Atmungschromogene gebildet werden. Auch Glukoside können wahrscheinlich als Material für die Bildung der Chromogene dienen, wie es auch OVERTON vermutete. Zugunsten dieser Voraussetzung spricht der Umstand, dass die meisten pflanzlichen Glukoside Verbindungen verschiedener Zuckerarten mit aromatischen Stoffen sind: nach den gegenwärtig bekannten Tatsachen sind auch Atmungschromogene nichts anderes als aromatische Verbindungen. Die aromatischen Spaltungsprodukte der Glukoside liefern bei der Oxydation verschiedene Pigmente. Es ist wohl möglich, dass die bei der enzymatischen Spaltung der Glukoside in den Pflanzen entstehenden einfacheren aromatischen Verbindungen direkt als Atmungschromogene fungieren."

The peroxydase of CHODAT and BACH employed by PALLADIN is prepared from the root of the horse-radish. Methods of preparation are given by DE STOECKLIN[47] and are based upon the extraction of the ground root with water or dilute alcohol and subsequent precipitation with absolute alcohol. The ferment separates out as a white powder, which may be purified by solution and reprecipitation. The activity of the ferment can be tested by its power to produce purpurogallin when added to pyrogallol solution in presence of hydrogen peroxide.

I have prepared the peroxydase by the method of DE STOECKLIN and have tested its action upon an extract of flowers of *Primula sinensis*. Flowers of a white variety slightly tinged with purple were gently heated in water and subsequently pounded and filtered. To the colourless extract a little of the peroxydase solution was added, together with a few drops of hydrogen peroxide. With

care the formation of a purple pigment can be detected as the oxydase is added, though it rapidly vanishes leaving a brown colouration only. The same restoration of the pigment is brought about sometimes by the addition to the extract of hydrogen peroxide only, in which case evidently some of the *Primula* oxydase is still active. It seems probable that the rapid disappearance of the restored pigment is due to some unfavourable condition of the experiment.

An attempt to isolate an oxydase from the young flowers and buds of coloured varieties of *Primula sinensis* has resulted only in the production of a solution which gave a brownish-red colour with pyrogallol in presence of hydrogen peroxide but which was without effect upon the colourless extract of *Primula* itself.

Although no positive result has been obtained with *Primula* oxydase on its own chromogen, further attempts will be made to isolate a peroxydase from other genera, such as *Antirrhinum, Lathyrus,* etc., and it is possible that these may be more successful.

The theories of PALLADIN and OVERTON however, taken into consideration with results derived from work in genetics and from the researches of PERKIN, seem to provide evidence sufficiently strong to justify the assumption, that anthocyanin is a product of ferment oxidation of the glucosides of members of the flavone series of colouring matters, or of substances very closely allied to these.

If we regard anthocyanin as the product of oxidation of a colourless aromatic chromogen, then the following table of genera having many varieties, shows several anomalies:—

In *Antirrhinum, Dianthus, Matthiola* and *Verbena,* the chromogen gives a brown colouration with iron salts, though the reactions with basic lead acetate, with alkalis and with acids would indicate that it is of a similar nature to those already described. In pink—rose dorée—*Antirrhinum,* red *Matthiola,* scarlet *Dianthus* and scarlet *Verbena,* the addition of the reddening factor produces no change in the iron-reaction but only the ultimate addition of a blueing factor, which then produces a green iron-reaction—*Antirrhinum, Verbena*—or a blue iron-reaction—*Matthiola.* In pink *Dianthus,* the reddening factor gives a green, in other varieties the blueing factor, a blue iron-reaction.

Most frequently, as in *Phlox, Lathyrus, Dianthus* and *Matthiola* (exceptions *Antirrhinum* and *Verbena*) as well as in *Helianthemum* and *Abutilon,* the introduction of a blueing factor appears to be connected with the production of an iron-blueing reaction.

As regards basic lead acetate, the chromogen in *Dianthus* and *Matthiola* does not give the usual reaction, so that the reds and purples become bluer on addition of this reagent but no green colour is produced.

	Basic lead acetate					Alkalis					Iron salts		
	yellow	orange	red	green	blue	yellow	orange	red	green	blue	green	blue	red or brown
Phlox Drummondi													
White	...	...	....	...	...	...	...	no colour					
Yellow	...	orange	...	...	...	...	orange	...	...	...	green		
Ivory	...	orange	...	...	...	...	orange	...	...	...	green		
Salmon rose pink	...	...	reddish	...	...	...	...	reddish	...	...	green		
Various shades of crimson, violet & magenta	...	...	...	green	...	...	...	...	green	...	...	purple	
Lathyrus odoratus													
Red	...	...	...	green	...	...	...	...	green	...	green		
Purple	...	...	...	green	...	...	...	...	green	...	...	blue	
Dianthus Caryophyllus													
Yellow	...	...	unaltered	...	...	...	orange	...	...	...	...	...	brown
Pink	...	...	reddish	...	...	...	...	reddish	...	...	green		
Scarlet	...	...	bluish red	...	...	...	...	...	green-ish	...	...	...	brown
Deep crimson	...	...	...	...	blue	...	...	...	green	...	...	blue	
Matthiola sp.													
White and cream	...	...	unaffected	...	...	yellow	...	...	...	...	...	...	brown
Red	...	...	bluish red	...	...	...	...	...	green	...	...	...	brown-ish red
Purple	...	...	...	...	blue	...	...	...	green	...	...	blue	
Antirrhinum majus													
White	...	...	unaltered	...	...	...	...	unaltered	...	...	...	unaltered	
Yellow	...	...	red	...	...	...	...	red	...	...	...	...	brown
Ivory	...	...	...	...	...	yellow	...	...	...	...	...	...	brown
Rose Dorée Pink	...	...	reddish	...	...	...	...	reddish	...	...	...	...	brown
Magenta	...	...	...	green	...	...	...	...	green	...	green		
Verbena sp.													
Salmon rose pink	...	...	reddish	...	...	...	...	reddish	...	...	...	...	brown
Scarlet	...	...	...	green	...	...	...	...	green	...	...	...	brown
Violet	...	...	...	green	...	...	...	...	green	...	green		

All forms of anthocyanin so far considered (except the pinks of *Phlox*, *Dianthus*, *Antirrhinum*, the pink and scarlet of *Verbena* and the red of *Matthiola*) have been of the type which may be termed purple and purplish-red, that is, though the flower-colours may be dissimilar, the extracted solution has a purplish tinge and the colouration with alkalis is green.

The following pink and scarlet colours resemble the red varieties of *Dianthus* and *Matthiola* in giving a red or bluish-red colouration or precipitate with basic lead acetate and a full green or some green colour with alkalis:—

Pink *Impatiens sultani* (violet)*.
„ *Begonia-Glorie de Lorraine* (greenish).
Berries of *Berberis vulgaris* (unaffected).
Red *Delphinium cardinale* „
Scarlet variety of *Pentstemon sp.* (brownish-red).

From the above examples it is apparent that a difference in the reactions with iron salts and with basic lead acetate is frequently associated with the production of an anthocyanin differently coloured, *i.e.* pink or scarlet, from the more frequent purplish-red form.

This difference is even more obvious in the following cases of scarlet flowers, with the extract of which both alkalis and basic lead acetate give a red, purplish-red or purple colour:—

Scarlet *Impatiens Holstii* (reddish)*.
„ *Lobelia cardinalis* (unaffected).
„ *Salvia splendens* (red).
„ *Phaseolus multiflorus* (unaffected).
„ *Papaver Rhoeas* (brown-red).
„ *Anagallis grandiflora* (red).
„ *Alonsoa miniata coccinea* (unaffected).

These colour differences may be regarded as further evidence in favour of the suggestion that purple, purplish-red and some pink forms of anthocyanin are derived from iron-greening chromogens. When the latter are absent from the plant the flower colour is of a strikingly different shade.

The significance of the change from a green to a blue iron-reaction, which so often accompanies a blueing of the anthocyanin, is as yet inexplicable. It is possible that as the oxidation of the anthocyanin progresses, less of the chromogen remains and hence the green iron-reaction of the latter is hidden by the iron-reaction of anthocyanin itself. This suggestion, however, hardly fits such cases as those in which the chromogen is present in a quantity sufficient to produce a green colouration with basic lead acetate and with alkalis, though at the same time the iron-reaction is blue. Probably examination of a larger number of genera may

* Denotes colour with iron-salts.

throw more light upon this point. In connection with the statement, that in genera showing various reds and purples, *i.e. Phlox, Lathyrus, Dianthus* and *Matthiola*, the ultimate purple variety contains an anthocyanin giving a blue iron-reaction, it is of interest to mention the fact that this ultimate purple anthocyanin can often be differentiated into a redder portion (though not identical with the anthocyanin of the reds), soluble in alcohol, and a bluer portion insoluble in alcohol, but readily soluble in water.

All cases have not been thoroughly investigated, but as far as they go the results point to the conclusion that the bluer portion gives a blue precipitate with basic lead acetate, a blue colouration with alkalis, and a blue colour with iron salts.

The bluer portion predominates in berries of *Sambucus nigra* and *Ligustrum vulgare*, which give a blue colour with all the above reagents. A more careful investigation was made of the berries of *Ligustrum*. For this purpose the dried berries were pounded and extracted with absolute alcohol. The alcohol extract contained, in addition to chlorophyll, a crimson colouring matter, which may be termed (A). When the alcohol ceased to be coloured, the residue was dried again and then treated with water. The filtrate was deep purple red and contained the greater part of the colouring matter of the berries. The purple-red pigment may be termed (B). The reactions of the two portions were respectively as follows:

	Basic lead acetate	Alkalis	Iron salts
(A)	Green precipitate	Green colouration	Dull green colouration
(B)	Blue ,,	Blue ,,	Blue ,,

This result is comparable to those obtained by the following authors:—

Grafe[18] isolated a colouring matter—Malvenfarbstoff—from the flowers of *Althaea rosea* by precipitation of the extract with lead acetate and subsequent decomposition with sulphuretted hydrogen. The pigment was separated into two portions differing in their solubilities, *i.e.*:—

Colouring matter (A)	Colouring matter (B)
Soluble in alcohol	Insoluble in alcohol but soluble in water
$C_{14}H_{16}O_6$	$C_{20}H_{30}O_{13}$

The author regards (B) as a di-basic acid; the relationship between the two pigments is shown in the following equations:—

$$C_{20}H_{30}O_{13} + H_2O = \underbrace{C_6H_{12}O_6 + C_{14}H_{18}O_8}_{\text{glucoside}},$$

$$2C_{14}H_{18}O_8 - 2H_2O - O_2 = 2C_{14}H_{16}O_6.$$

From this we see that the pigment insoluble in alcohol is an oxidation product and a glucoside of the pigment soluble in alcohol.

HEISE[13] obtained a pigment—Heidelbeerfarbstoff—from the Bilberry which was differentiated into two portions:—

Colouring matter (A)	Colouring matter (B)
Insoluble in cold water, soluble in alcohol	Soluble in water and alcohol
$C_{14}H_{14}O_7$	$C_{20}H_{24}O_{12}$

The relationship is shown as:—

$$C_{20}H_{24}O_{12} + H_2O = \underbrace{C_6H_{12}O_6}_{\text{glucoside}} + C_{14}H_{14}O_7.$$

GAUTIER[11] also extracted two pigments from the red leaves of the Vine—Verfärbungroth der Rebblätter—and termed them respectively:—

α-ampelochroïc acid	β-ampelochroïc acid
Insoluble in cold water, soluble in hot water	Soluble in cold water
Green precipitate with basic lead acetate	Green precipitate with basic lead acetate
Dark green with iron salts	Dark violet with iron salts
$C_{19}H_{16}O_{10}$	$C_{26}H_{24}O_{15}$

The results obtained from *Ligustrum*, *Althaea* and *Vitis* seem to point to purple anthocyanin as being sometimes a mixture of a red and bluer portion, the former giving a green-, the latter a blue-iron reaction. The bluer portion is more soluble in water and is generally present in greater quantity; this may explain the iron-blueing reaction of some purple forms of anthocyanin.

The analyses made by GRAFE and by GAUTIER suggest that the bluer pigment is more highly oxidised, and the change from red to blue anthocyanin may possibly be one of oxidation resembling the change which causes the colourless chromogen to become red.

The separation of purple pigments into red and bluer constituents is most obvious in the anthocyanin of certain allied orders, *i.e. Amarantaceae*, *Nyctaginaceae*, *Phytolaccaceae*, and *Portulacaceae*. The pigment of these genera differs in its reactions from the usual type of anthocyanin* and is at the same time more readily separable into two portions distinguished by means of their respective solubilities in alcohol.

Pigment soluble in both alcohol and water	Basic lead acetate	Alkalis	Iron salts
Amaranthus sp.	yellow precipitate	yellow	green
Mirabilis Jalapa	orange-yellow precipitate	,,	,,
Bougainvillea glabra	orange ,,	green	,,
Phytolacca decandra	orange-red ,,	yellow	,,
Portulaca grandiflora	orange ,,	,,	,,

* Wheldale[20].

Pigment soluble in water only	Basic lead acetate	Alkalis	Iron salts
Amaranthus sp.	orange-red...	yellow (purple with ammonia)...	reddish
Mirabilis Jalapa	,, ...	,, ,, ,, ...	,,
Bougainvillea glabra ...	red ...	purplish-green ...	,,
Phytolacca decandra ...	orange-red...	yellow ...	,,
Portulaca grandiflora ...	,, ...	,, ...	,,

There is evidently a tendency for the iron-greening reaction to disappear as the pigment becomes more oxidised.

Collectively the results so far obtained as to the nature of 'anthocyanin' may be brought to bear upon the phenomena observed in the genetics of flower-colour. In *Lathyrus* and *Matthiola* (Bateson, Saunders and Punnett*) red colour is produced by the meeting of two factors *C* and *R*, of which *C* has been regarded by the authors as possibly representing a chromogen, and *R* the presence of an enzyme acting upon this chromogen with the production of red pigment. If either the chromogen or the enzyme be absent, then the plant has lost the power of producing red pigment and constitutes an albino. Moreover an additional factor *B* can be carried by the albino and *B* co-existing with *C* and *R* can produce a blueing of the red pigment. It seems possible that *B* may again be an enzyme acting upon the red pigment to produce a bluer oxidation product as we have seen in *Althaea*; in which, according to Grafe, there are two pigments, one the oxidation product of the other.

In *Antirrhinum* the albino (as regards anthocyanin) carrying *C*, the chromogen, is distinguishable to the eye as the ivory-white. In the true albino no chromogen is present and no aromatic compound can be detected, but since the mating of an ivory-white with an albino carrying *R* produces a magenta cross-bred, the true albino probably carries the oxidising enzyme.

In the yellow variety of *Antirrhinum* a similar yellow aromatic substance is present as a derivative of that in the ivory; the oxidation process in this case gives crimson as a result either of the production of a different pigment or of red colouration in the presence of unaltered yellow chromogen.

In *Lathyrus* and *Matthiola*, among the offspring produced by the selfing of an individual heterozygous in *C*, *R* and in the white plastid factor which is epistatic to cream, we find the proportion of anthocyanic, *i.e.* purple and red, to non-anthocyanic, *i.e.* white and cream, is as 9 : 7. Moreover among the non-anthocyanic, the proportion of whites and creams carrying *C* to those carrying *R* is as 1 : 1.

Similarly in *Antirrhinum*, among the offspring from the selfing of an individual heterozygous in the yellow chromogen

* Reports to the Evolution Committee of the Royal Society, I—IV.

factor, in the factor modifying this to ivory and in *R*, the reddening factor, we find the proportion of anthocyanic, *i.e.* magenta and crimson individuals, to the non-anthocyanic, *i.e.* ivory, yellow and white, is again as 9 : 7.

Of the non-anthocyanic, the proportion of those carrying *C*, *i.e.* ivories and yellows, to those carrying *R*, *i.e.* true albinos, is again as 1 : 1.

Through the kindness of Professor Bateson and Mr R. P. Gregory, who have been working for some time upon the inheritance of flower-colour in *Primula sinensis*, I have lately been able to examine a number of varieties of this genus. The flowers of white varieties contain, as stated on p. 151, a chromogen apparently of the nature of a flavone. Capability to produce red pigment—anthocyanin—is probably due to the presence of an oxydase in the plant; hence a white variety has either lost the power to produce the oxydase or contains a positive factor—an inhibitor or reductase—which prevents the formation of the pigment.

It is possible, on the one hand, that the loss of the oxydase gives rise to certain white-flowered varieties with green stems. These varieties are known as 'recessive whites': mated with a full-coloured variety, such as deep crimson or magenta, they give a full-coloured F_1. On the other hand, the presence of an inhibitor in the *flower* only (as suggested by Bateson and Gregory), may be the explanation of the existence of certain white-flowered varieties having red stems; these varieties are termed 'dominant whites' and mated with a full-coloured variety give an F_1 which is only tinted.

When anthocyanin is present, two series of varieties are distinguishable, a magenta series and a crimson series, the former being epistatic to the latter. As previously stated, the same two series, magenta and crimson, occur in *Antirrhinum* and are probably due to the action of an oxydase on the pale yellow flavone of the ivory and the deep yellow flavone (xantheïn) of the yellow varieties respectively. In *Primula* no yellow (xantheïc) variety arises from the crimson but loss of oxydase from crimson and from magenta gives whites indistinguishable to the eye, though those thrown by magenta contain of course, the magenta factor.

I have examined the magenta and crimson pigments and also the whites derived from each series. Whites from magenta appear to carry a chromogen giving a more intense yellow with alkalis than those from crimson; hence magentas give, on the whole, a green colour with alkali, due to the blueing of the anthocyanin and the yellowing of the unaltered chromogen. The crimsons give a bluer colour with alkali due to the far slighter yellowing of the unaltered chromogen. The action of iron salts on the crimson pigment results in a brown, on the magenta in a green

colouration. It is probable that magenta and crimson are different pigments arising from the action of the same oxydase upon different chromogens.

Hence we see that in *Antirrhinum* three non-anthocyanic varieties exist. Ivory is a 'white' carrying the chromogen giving rise to magenta; yellow is a 'white' carrying the chromogen giving rise to crimson in oxidation and the true albino contains no chromogen but probably only the oxydase.

There is some resemblance between *Antirrhinum* and *Primula* in that one can detect in the latter two kinds of whites carrying chromogens giving rise to magenta and crimson respectively but the albino containing no chromogen has so far not been identified in *Primula* and may not exist.

There is moreover a further resemblance as regards the blueing factor; in both *Antirrhinum* and *Primula* the crimsons and magentas are of the purplish-red form of anthocyanin. When the blueing factor is absent, varieties arise containing red anthocyanin*. In *Antirrhinum* these are the 'rose dorée' (from magenta) and 'bronze' (from crimson). Similarly in *Primula*, loss of the blueing factor gives rise to 'pink' (from magenta) and 'orange king' (from crimson).

It is conceivable that the green-stemmed variety of *Primula* may represent, in particular so to speak, a class of plants in general which do not contain the requisite oxydase and hence can never produce red pigment. *Galanthus nivalis, Narcissus poeticus, Cucurbita Pepo, Helianthus annuus* and numerous others would be included in this class.

The red-stemmed white, on the contrary, may be the representative of a class of plants which are anthocyanic but in the flowers of which an inhibitor is present. Such plants have white or nearly white flowers, though the stems and foliage may contain more or less anthocyanin. Some marked examples of this class would be *Oxalis acetosella, Geranium Robertianum var. album*, white cultivated *Cyclamen persicum, Crataegus oxyacantha, Rosa arvensis, Nymphaea alba, Angelica Sylvestris*, and many other *Umbelliferae.*

Loss of the inhibiting factor from this latter class would give rise to a fully coloured variety from the white-flowered type, the exact converse of the origin of a true albino from a coloured type. Such cases are rather rare. The most striking examples are the origin under cultivation of the deep red and magenta and of the pink varieties from the original white-flowered *Cyclamen persicum* which has red stems and leaves. The origin of the tinted *Primula acaulis* from the wild type which has anthocyanin in its petioles and leaf-stalks only.

* Wheldale [20].

If the cultivated red varieties of *Helianthemum vulgare* are derived from the yellow type, this would constitute a further example. De Vries* quotes additional cases, *i.e.* red varieties of *Achillea Millefolium*, *Begonia semperflorens*, *Crataegus oxyacantha*, *Robinia Pseudacacia*, and the red-leaved Beech and Hazel.

In conclusion I should like to take this opportunity of expressing my thanks to Professor Bateson for his valuable advice and help in this research. I am also indebted to Mr H. O. Jones for suggestions in connection with the chemical portion of the subject.

Summary.

1. Chromogens, pale or deep yellow in colour (in the latter case the so-called soluble yellow or xantheïc pigments), and of the nature of the flavone and xanthone classes of natural colouring matters, are widely distributed in plants and are commonly found in connection with the pink, purplish-red and purple series of 'anthocyanic' pigments.

2. These chromogens exist in the plant probably in combination with various sugars, *i.e.*, as glucosides. Most of them may be regarded as compounds of phloroglucin with protocatechuic acid, hydroxybenzoic acid or sometimes resorcylic acid. Occasionally pyrocatechol or some other phenol takes the place of phloroglucin.

3. Overton's work on 'anthocyanin' from the point of view of nutrition, also points to the conclusion that this pigment is a glucoside compound of a tannic acid.

4. On the evidence of the recent work of Palladin in connection with plant respiration, chromogens of an aromatic nature are widely distributed and are able to produce red and purple pigments when acted upon by the peroxydase of Chodat and Bach in the presence of available oxygen. Hence it is highly probable that the bodies mentioned above may constitute these chromogens.

5. Evidence from cross-breeding in *Antirrhinum* goes to prove that for the production of the 'anthocyanin' of the type, two bodies are essential, *i.e.*, an aromatic chromogen of the flavone series and a reddening factor, in all probability an oxidising ferment.

* *Species and Varieties, their Origin by Mutation.*

6. Hence we may state that successive oxidation stages of an aromatic chromogen probably give rise to the colour series, pink, purplish-red, and purple of the more commonly occurring form of 'anthocyanin.'

7. An albino results either from the loss of the fundamental chromogen or from the loss of the reddening enzyme from the plant. The blueing enzymes depend for their manifestation on the presence of both the first two factors.

8. In *Antirrhinum* and *Phlox* the albino carrying *C*, the chromogen, is ivory-white in colour and is distinguishable from the dead-white albino carrying *R*, the enzyme, not only chemically but also in appearance. In *Lathyrus* and *Matthiola* no difference in appearance can be detected between the albino carrying *C* and the albino carrying *R*; in the two latter genera a thorough chemical examination of the albinos has yet to be made.

9. The less commonly occurring scarlet 'anthocyanin' usually gives no iron reaction; hence the difference in colour is probably connected with a difference in constitution.

10. The later oxidation stages producing purple 'anthocyanin' are frequently accompanied by a change from an iron-greening to an iron-blueing reaction. It is doubtful whether this is due to total oxidation and disappearance of the iron-greening chromogen or to a specific reaction of the 'anthocyanin' itself with iron salts.

BIBLIOGRAPHY.

1. Curtel, G. "Recherches physiologiques sur la fleur." *Ann. des Sci. naturelles*, Série viii., T. vi.
2. Dennert, E. "Anatomie und Chemie des Blumenblatts." *Bot. Centralb.*, 1889.
3. Gautier, A. "Sur la coloration rouge éventuelle de certaines feuilles et sur la couleur des feuilles d'automne." *Compt. Rend. Acad. des Sciences.* Paris. T. cxliii., 1906.
4. Ichimura, T. "On the Formation of Anthocyanin in the Petaloid Calyx of the Red Japanese Hortense." *Jour. Coll. of Sc. Imp. Univ. Tokyo*, Vol. xviii., 1903.
5. Mirande, M. "Sur un cas de formation d'anthocyanine sous l'influence d'une morsure d'insecte." *Compt. Rend. Acad. des Sciences.* Paris. T. cxliii., 1906.

   "Sur l'origine de l'anthocyanine déduite de l'observation de quelques insectes parasites des feuilles." *Compt. Rend. Acad. des Sciences.* Paris. T. cxlv., 1907.

6. Newbigin, M. I. "An Attempt to classify common Plant Pigments with some Observations on the Meaning of Colour in Plants." *Trans. and Proc. Bot. Soc.* Edinburgh, 1896.

7. Wiesner, J. "Einige Beobachtungen über Gerb- und Farb-stoffe der Blumenblätter." *Bot. Zeit.*, 1862. 20.

8. Wigand, A. "Einige Sätze über die physiologische Bedeutung des Gerbstoffes und der Pflanzenfarbe." *Bot. Zeit.*, 1862.
"Die rothe und blaue Färbung von Laub und Frucht." *Bot. Hefte. Forschungen a. d. bot. Garten zu Marburg*, 1887.

9. Zopf, W. "Über die Gerbstoff- und Anthokyan-behälter der Fumariaceen und einiger anderen Pflanzen." *Bibliotheca Botanica*, 1886. H. II.

10. Overton, E. "Beobachtungen und Versuche über das Auftreten von rothem Zellsaft bei Pflanzen." *Prings. Jahr. f. wiss. Bot.*, 1899.

11. Gautier, A. "Sur l'origine des matières colorantes de la vigne; sur les acides ampélochroïques et la coloration automnale des végétaux." *Compt. Rend. Acad. des Sciences.* Paris. T. CXIV., 1892.

12. Pfeffer. "Physiology of Plants."

13. Heise, R. "Zur Kenntniss des Heidelbeerfarbstoffes." *Arbeit a. d. Kaiserl. Gesundheitsamte*, 1894.

14. Waage, T. "Über das Vorkommen und die Rolle des Phloroglucins in der Pflanze." *Ber. d. D. bot. Gesell.*, 1890.

15. Nickel, E. "Zur Physiologie des Gerbstoffes und der Trioxybenzole." *Bot. Centralb.*, 1891.

16. Brissemoret, A. "Sur une réaction colorée des tannoïdes." *Bull. Soc. Chim. France*, 1907.

17. Gardiner, W. "On the general Occurrence of Tannins in the vegetable Cell." *Proc. Cam. Phil. Soc.*, 1883.

18. Grafe, V. "Studien über das Anthokyan." I. *Mitt. Sitzungsbericht d. k. Akad. d. wiss. Wien. Mat. nat. Kl.* Bd. CXV. Abt. I., 1906.

19. Wheldale, M. "On the Inheritance of Flower-Colour in *Antirrhinum majus*." *Proc. Roy. Soc.*, Vol. LXXIX., 1907.

20. Wheldale, M. "The Colours and Pigments of Flowers with special reference to Genetics." *Proc. Roy. Soc.*, 1909.

21. Perkin, A. G. and Cope, F. "The Constituents of *Artocarpus integrifolia*." Part I. *Jour. Chem. Soc. Trans.*, Vol. LXVII., 1895.

22. Perkin, A. G. and Geldard, J. "A Constituent of Persian Berries." *Jour. Chem. Soc. Trans.*, Vol. LXVII., 1895.

23. Perkin, A. G. and Pate, L. "Acid Compounds of some natural yellow Colouring Matters." Part I. *Jour. Chem. Soc. Trans.*, Vol. LXVII., 1895.

24. Perkin, A. G. "Luteolin." Part I. *Jour. Chem. Soc.*, Vol. lxix., 1896.

"Luteolin." Part II. *Jour. Chem. Soc.*, Vol. lxix., 1896.

"Acid Compounds of natural Colouring Matters." Part II. *Jour. Chem. Soc.*, Vol. lxix., 1896.

25. Perkin, A. G. and Bablich, H. "Morin." Part I. *Jour. Chem. Soc. Trans.*, Vol. lxix., 1896.

26. Perkin, A. G. and Gunnell, O. "The Colouring Matter of *Querbracho Colorado*." *Jour. Chem. Soc. Trans.*, Vol. lxix., 1896.

27. Perkin, A. G. and Hummel, J. J. "The Colouring Principle contained in the bark of *Myrica nagi*." *Jour. Chem. Soc. Trans.*, Vol. lxix., 1896.

"Occurrence of Quercetin in the outer Skins of the Bulb of the Onion." *Jour. Chem. Soc. Trans.*, Vol. lxix., 1896.

"The Colouring Matters occurring in British Plants." Part I. *Jour. Chem. Soc. Trans.*, Vol. lxix., 1896.

28. Perkin, A. G. and Young, G. "Colouring Matter of Sicilian Sumach, *Rhus Coriaria*." *Jour. Chem. Soc. Trans.*, Vol. lxix., 1896.

29. Perkin, A. G. "Apün and Apigenin." *Jour. Chem. Soc. Trans.*, Vol. lxxi., 1897.

"The yellow Colouring Principles of various Tannin Matters." *Jour. Chem. Soc. Trans.*, Vol. lxxi., 1897.

"Yellow Colouring Matters obtained from *Rhus rhodanthema*, *Berberis Œtnensis* and *Rumex obtusifolius*." *Jour. Chem. Soc. Trans.*, Vol. lxxi., 1897.

30. Perkin, A. G. and Martin, H. W. "Rhamnazin." *Jour. Chem. Soc. Trans.*, Vol. lxxi., 1897.

31. Perkin, A. G. "Azobenzene Derivatives of some natural Colouring Matters, Apigenin, Chrysin, Morin, Euxanthone and Gentisin." *Jour. Chem. Soc. Trans.*, Vol. lxxiii., 1898. "The yellow Colouring Matters contained in various Tannin Matters. *Rhus cotinus* and *R. rhodanthema*." *Jour. Chem. Soc. Trans.*, Vol. lxxiii., 1898.

32. Perkin, A. G. and Pilgrim, J. A. "The Colouring Matters of the Indian Dye-stuff Asbarg, *Delphinium Zalil*." *Jour. Chem. Soc. Trans.*, Vol. lxxiii., 1898.

33. Perkin, A. G. and Wood, P. J. "Yellow Colouring Principles contained in various Tannin Matters. *Pistacia lentiscus*, *P. terebinthus*, *Tamaris africana*, *T. gallica*, *Ailanthus glandulosa*, *Ficus carica*." *Jour. Chem. Soc. Trans.*, Vol. lxxiii., 1898.

34. Perkin, A. G. and Newbury, F. G. "Colouring Matters contained in Dyer's Broom, *Genista tinctoria* and Heather, *Calluna vulgaris*." *Jour. Chem. Soc. Trans.*, Vol. lxxv., 1899.

35. Perkin, A. G. "Apün and Apigenin." Part II. *Jour. Chem. Soc. Trans.*, Vol. lxxvii., 1900.

"Yellow Colouring Principles contained in various Tannin Matters. *Arctostaphylos Uva ursi, Haematoxylon Campeachianum, Rhus Metopium, Myrica gale, Coriaria myrtifolia*, and *Robinia pseudacacia*." *Jour. Chem. Soc. Trans.*, Vol. lxxvii., 1900.

36. Perkin, A. G. and Horsefall, L. H. "Genistein." Part II. *Jour. Chem. Soc. Trans.*, Vol. lxxvii., 1900. "Luteolin." Part III. *Jour. Chem. Soc. Trans.*, Vol. lxxvii., 1900.

37. Perkin, A. G. and Wilkinson, E. J. "The Colouring Matter of the Flowers of *Delphinium Consolida*." *Jour. Chem. Soc. Trans.*, Vol. lxxxi., 1902.

38. Perkin, A. G. "Myricetin." Part II. *Jour. Chem. Soc. Trans.*, Vol. lxxxi., 1902. "Robinin, Viola Quercitrin, Myrticolorin and Osyritrin." *Jour. Chem. Soc. Trans.*, Vol. lxxxi., 1902.

"Notes on Luteolin and Apigenin." *Jour. Chem. Soc. Trans.*, Vol. lxxxi., 1902.

39. Perkin, A. G. and Allison, J. R. "Rhamnazin and Rhamnetin." *Jour. Chem. Soc. Trans.*, Vol. lxxxi., 1902.

40. Perkin, A. G. and Phipps, S. "Some Natural Colouring Matters." *Jour. Chem. Soc. Trans.*, Vol. lxxxv., 1904.

41. Bidgood, J. "Floral Colours and Pigments." *Jour. Roy. Hort. Soc.*, Dec. 1905.

42. Naylor and Chappel. "Note on the Colouring Matter of *Rosa gallica*." *Pharm. Jour.* 1904.

43. Palladin, W. "Die Verbreitung der Atmungschromogene bei den Pflanzen." *Ber. d. D. Bot. Gesell.*, July 1908.

"Über die Bildung der Atmungschromogene in den Pflanzen." *Ber. d. D. Bot. Gesell.*, July 1908.

44. Bertrand, G. "Sur les rapports qui existent entre la constitution chimique des composés organiques et leur oxydabilité sous l'influence de la laccase." *Comp. Rend. Acad. des Sciences.* Paris, 1896.

"Recherches sur la laccase, nouveau ferment soluble à propriétés oxydantes." *Annales de Chimie et de Physique.* 7 Série. 1897.

45. Chodat, R. "Les ferments oxydants." *Jour. Suisse de Chimie et Pharmacie*, 1905.

"Nouvelles recherches sur les ferments oxydants." *Arch. des sciences physiques et naturelles*, 1907.

"Zur specifischen Wirkung der Peroxydasen—eine Prioritätsfrage." *Ber. d. D. Chem. Gesell.*, July, 1906.

46. Chodat, R. und Bach, A. "Untersuchungen über die Rolle der Peroxyde in der Chemie der lebenden Zelle." *Ber. d. D. Chem. Gesell.*, 1903, 1904.

"Recherches sur le rôle des peroxydes dans l'économie de la cellule vivante." *Arch. des sciences physiques et naturelles*, 1902.

47. De Stoecklin, E. "Contribution à l'étude de la peroxydase." *Université de Genève—Institut de Botanique*, 1907.

ADDITIONAL BIBLIOGRAPHY, IN CONNECTION WITH PLANT PIGMENTS.

Adrian et Trillat. "Sur la matière colorante de la Digitale." *Compt. rend. Acad. des Sciences.* Paris. T. cxxix., 1899.

Askenasy, E. "Über den Einfluss des Lichtes auf die Farbe der Blüthen." *Bot. Zeit.* 1876.

Bauer, K. "Untersuchungen über Gerbstofführende Pflanzen." *Österreichische Bot. Zeitschrift.* xl. Wien, 1890.

Bohm. "Physiologische Untersuchungen über blaue Passiflorabeeren." *Sitzungsber. der Kaiserl. Akad. der Wissenschaft.* Wien, 1857.

Claudel, L. "Sur les matières colorantes du spermoderme dans les Angiospermes." *Compt. rend. Acad. des Sciences.* Paris. T. cix., 1889.

Filhol, E. "Observations sur les matières colorantes des fleurs." *Compt. rend. Acad. des Sciences.* Paris. T. xxxix., 1854.

"Note sur quelques matières colorantes végétales." *Compt. rend. Acad. des Sciences.* Paris. T. l., 1860.

"Nouvelles recherches sur les matières colorantes végétales." *Compt. rend. Acad. des Sciences.* Paris. T. l., 1860.

Frémy, E. und Cloëz. "Über die Farbstoffe der Blumen." *Jour. für praktische Chemie*, 1854.

Fritsch, P. "Über farbige körnige Stoffe des Zellinhalts." *Prings. Jahr.*, 1883.

Gautier, A. "Sur les matières colorantes des vins." *Compt. rend. Acad. des Sciences.* Paris. T. lxxxvi., 1878.

Gertz, O. "Ett nytt fall af Kristalliseradt Anthocyan." *Botaniska Notiser.* H. vi., 1906.

"Studier öfver Anthocyan." *Akademisk Afhandling.* Lund, 1906.

Gillot, X. "Observations sur la coloration rosée ou érythrisme des fleurs normaliment blanches." *Bull. de la Soc. bot. de France.* T. xl. 1893.

Griffiths, A. B. "Die Pigmente des Geraniums und anderer Pflanzen." *Ber. d. D. Chem. Ges.*, xxxvi., 1903.

Griffon, E. "L'assimilation chlorophyllienne et la coloration des plantes." *Ann. des Sciences naturelles. Bot.* Sér. viii., T. x., 1899.

Hansen, A. "Die Farbstoffe der Blüthen und Früchte." Würzburg, 1884.

Hildebrand, F. "Anatomische Untersuchungen über die Farben der Blüthen." *Prings. Jahr.*, 1863.

"Die Farben der Blüthen." Leipzig, 1879.

"Über einige merkwürdige Färbungen von Pflanzentheilen." *Ber. d. D. bot. Gesell.*, 1883.

Hilger, A. "Zur chemischen Kenntniss der Blumenfarbstoffe." *Bot. Centralb.*, 1894.

Karzel, R. "Beiträge zur Kenntniss des Anthokyans in Blüthen." *Österreich. bot. Zeitschrift*, 1906.

Keeble, F. W. "The Red Pigment of Flowering Plants." *Sci. Prog.* Vol. i. New Series.

Kny, L. "Über Laubfärbungen." *Naturwissenschaftlichen Wochenschrift.* Vierter Band, 1889. Berlin.

"Zur physiologischen Bedeutung des Anthocyans." *Estratto dagli Atti del Congresso Botanico Internazionale.* 1892.

Kohl, G. "Untersuchungen über das Carotin und seine physiologische Bedeutung in der Pflanze." 1904.

Kraemer, H. "The Origin and Nature of Color in Plants." *Proc. Amer. Phil. Soc.*, lxiii., 1904.

Kraus, C. "Über die Ursache der Färbung der Epidermis vegetativer Organe der Pflanzen." *Flora.* 1873.

Kraus, G. "Die Entstehung der Farbstoffkörper in den Beeren von *Solanum pseudocapsicum.*" *Prings. Jahr.*, viii.

Kroemer, K. "Über das angebliche Vorkommen von violetten Chromatophoren." *Bot. Centralb.*, 1900.

Kutscher, E. "Über die Verwendung der Gerbsäure im Stoffwechsel der Pflanze." *Flora*, 1883.

Lepel, F. von. "Pflanzenfarbstoffe als Reagentien auf Magnesium Salze." *Ber. d. D. Chem. Gesell.*, 1880.

Linsbauer, L. "Einige Bemerkungen über Anthokyanbildung." *Österreich. Bot. Zeitschrift*, 1901.

Marshall-Ward and Dale, E. "On *Craterostigma pumilum.*" *Trans. Linn. Soc.*, Vol. v.

Mer, E. "Recherches sur les causes des colorations diverses qui apparaissent dans les feuilles en automne et en hiver." *Bull. de la Soc. bot. de France*, 1877.

Meilke, G. "Über die Stellung der Gerbsäuren im Stoffwechsel der Pflanzen." *Bot. Centralb.*, 1894.

Miyoshi, M. "Über die künstliche Aenderung der Blüthenfarben." *Bot. Centralb.*, Bd. lxxxiii., 1900.

Mobius, M. "Über den Glanz der gelben *Ranunculus*-blüthen." *Bot. Centralb.*, 1885.

"Die Färben in der Pflanzenwelt." Berlin, 1900.

"Das Anthophaeïn, der braune Blüthenfarbstoff." *Ber. d. D. Bot. Gesell.*, 1900.

Mohl, H. V. "Über die blaue Färbung der Früchte von *Viburnum Tinus*." *Bot. Zeit.*, 1870.

Molisch, H. "Über den Farbenwechsel anthokyanhältiger Blätter bei rasch eintretendem Tode." *Bot. Zeit.*, 1889.

"Blattgrün und Blumenblau." *Vorträge des Vereines zur Verbreitung naturwissensch. Kenntnisse in Wien*, xxx. Jahrg. 1890, H. 3.

"Die Krystallisation und der Nachweis des Xanthophylls (Carotins) im Blatte." *Ber. d. D. Bot. Gesell.*, Bd. xiv., 1896.

"Der Einfluss des Bodens auf die Blüthenfarbe der Hortensien." *Bot. Zeit.*, 1897.

"Über ein neues, einen carminrothen Farbstoff erzeugendes Chromogen bei *Schenckia blumenaviana*." *Ber. d. D. Bot. Gesell.*, 1901.

"Über amorphes und kristallisiertes Anthokyan." *Bot. Zeit.*, 1905.

Montanari, C. "Materia colorante rossa del pomodoro." *Staz. sperim. agrarie. Modena*, Vol. xxvii., 1904.

Morot, F. S. "Recherches sur la coloration des végétaux." *Ann. des Sciences naturelles.* Série iii., T. xiii., 1849.

Müller, N. J. C. "Spectralanalyse der Blüthenfarben." *Prings. Jahr.*, 1889.

Newbigin, M. I. "Colour in Nature." 1898.

Pantanelli, E. "Studii sul' albinismo nel Regno Vegetale." *Malpighia.* Vols. xv., xvi., xvii., xviii. and xix.

Pick, H. "Über die Bedeutung des rothen Farbstoffes bei den Phanerogamen und die Beziehungen desselben zur Stärkewanderung." *Bot. Centralb.*, 1883.

Pim, G. "Cell-sap Crystals." *Journ. of Bot.*, Vol. xxii., 1884.

Portheim, L. von und Scholl, E. "Untersuchungen über die Bildung und den Chemismus von Anthokyanen." *Ber. d. D. Bot. Gesell.*, July, 1908.

Reinitzer, F. "Bemerkungen zur Physiologie des Gerbstoffes." *Ber. d. D. Bot. Gesell.*, 1889.

Rosanoff, S. "Zur Morphologie der Pflanzenfarbstoffe." *Bot. Zeit.*, 1870.

Schimper, A. F. W. "Über die Entwickelung der Chlorophyllkörner und Farbkörper." *Bot. Zeit.*, 1883.

"Untersuchungen über die Chlorophyllkörper und die ihnen homologen Gebilde." *Prings. Jahr.*, 1885.

Schnetzler, J. B. "Über den rothen Farbstoff der Blätter von *Ampelopsis hederacea.*" *Bot. Centralb.*, 1880.

"Über Veränderungen des rothen Farbstoffes von *Paeonia officinalis* unter dem Einfluss chemischer Reagentien." *Bot. Centralb.*, 1880.

Schrötter-Kristelli, H. R. "Über ein neues Vorkommen von Carotin in der Pflanze." *Bot. Centralb.*, 1895.

Sorby, H. C. "On a definite Method of qualitative Analysis of animal and vegetable Colouring-matters by means of the Spectrum Microscope." *Proc. Roy. Soc.*, 1867.

"On comparative vegetable Chromatology." *Proc. Roy. Soc.*, 1873.

Stahl, E. "Über bunte Laubblätter." *Annales du Jardin Bot. de Buitenzorg*, Vol. xiii., 1896.

Tammes, T. "Über die Verbreitung des Carotins im Pflanzenreiche." *Flora*, 1900.

Tischler, G. "Über die Beziehungen der Anthocyanbildung zur Winterhärte der Pflanzen." *Beihefte zum Bot. Centralb.*, Bd. xviii., 1905.

Toni, J. B. de. "Observations sur l'anthocyane d'*Ajuga* et de *Strobilanthes.*" *Compt. rend. Ass. française Avanc. Sc.*, Paris, 1907.

Treviranus, L. C. "Über den Wechsel des Grünen und Rothen in den Lebenssäften belebter Körper." *Bot. Zeit.*, 1860.

Weigert, L. "Beiträge zur Chemie der rothen Pflanzenfarbstoffe." *Jahresber. der K. K. önolog. und pomolog. Lehranstalt zu Klosterneuberg bei Wien*, 1894—95.

Weiss, A. "Untersuchungen über die Entwickelungsgeschichte des Farbstoffes in Pflanzenzellen." *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Math.-natur. Classe.* Bd. liv. I. Abtheilung, Wien, 1866.

Wiesner, J. "Untersuchungen über die Farbstoffe einiger für chlorophyllfrei gehaltenen Phanerogamen." *Prings. Jahr.*, 1872.

The nature of the ionisation produced in a gas by γ rays.
By R. D. Kleeman, B.A., D.Sc., Emmanuel College, Cambridge.
(Communicated by Professor Sir J. J. Thomson, F.R.S.)

[*Read* 8 March 1909.]

The ionisation in a chamber placed in the path of γ rays may consist of three parts. One part may consist of ions ejected by the γ rays from the gas molecules with a velocity which is so small that they are unable to produce any further ions themselves. One of the other parts must consist of ions made by the cathode rays from the walls of the chamber, and the third part of ions made by the cathode rays of high velocity ejected from the gas molecules. The first of these parts is proportional to the mass of the gas and therefore proportional to the pressure, and this is also true for the second part since the ionisation of a gas by β rays has been shown to be proportional to the pressure. Since the number of electrons ejected from the gas is proportional to the pressure, and the number of ions each produces proportional to the pressure, the third part is proportional to the square of the pressure. The ionisation in the chamber may therefore be expressed in the form $\{(A + B)\, p + Cp^2\}$, where p denotes the pressure.

Laby and Kaye* have shown that the ionisation in air and carbon dioxide is proportional to the pressure over a wide range of pressures. The term Cp^2 is therefore small within this range. Experiments on the variation of the ionisation with pressure will not however give any information as to the relative values of A and B, since both the ionisation produced by the secondary cathode radiation from the walls of the chamber and that due to the ejection of slow-moving ions from the molecules of the gas by the γ rays vary as the pressure. Attempts have been made to obtain an estimate of the amount of ionisation in a chamber which is not due to the radiation from the walls. All these estimates depend on some calculations based on certain assumptions, generally involving the absorption, ionisation, etc., of the cathode rays produced by the γ rays. Now the writer has shown in a paper in the course of publication† that the γ rays give rise to secondary cathode rays of very different velocities, some of these rays have a range in air at atmospheric pressure of the order of 2 cm. while some are as penetrating as the penetrating β rays of radium. It cannot be assumed therefore that the

* *Phil. Mag.* p. 879, Dec. 1908.
† Read before the Royal Society.

absorption of the rays as a whole is the same as the β rays of uranium, or the penetrating β rays of radium. These calculations are therefore not satisfactory, they may easily be very considerably out. In fact McLennan* deduced that the ionisation in a chamber not due to the secondary radiation from the walls is equal to about one half of the total ionisation in the chamber, while Wilson† makes it equal to one sixth of the total ionisation. It does not therefore appear quite certain that the total ionisation is not due entirely to the secondary radiation from the walls. The writer therefore made some direct experiments to test this point.

Several slightly different arrangements were used, each of which involved the deflection of the secondary radiation from the apparatus by means of a magnetic field. A diagram of the apparatus first used is shown in figure 1. *A* is an ionisation

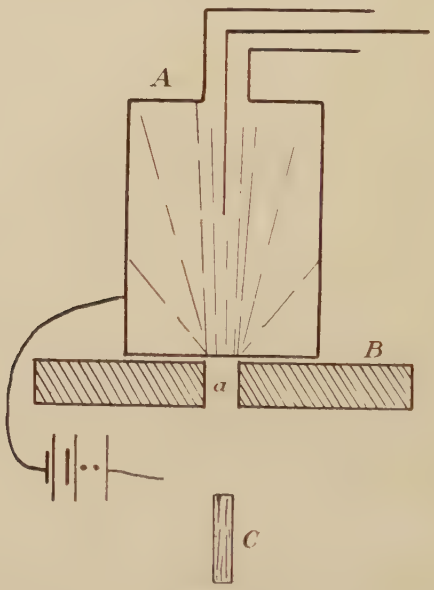


Fig. 1.

chamber on a lead block *B*, 5 cm. thick, the chamber being placed symmetrically with respect to an aperture *a* in the block. 30 mgrm. of radium were placed at *C* underneath the aperture. A magnet whose poles measured 5·5 cm. by 5·5 cm., was placed so that the lines of magnetic force traversed the chamber. Since the electrons produced by γ rays are ejected in the direction of

* *Phil. Mag.* Dec. 1907.
† *Phil. Mag.* p. 216, Jan. 1909.

propagation of the rays, there was a diffuse pencil of β rays projected into the chamber from its lower side. If the ionisation is produced by the β rays from the walls of the chamber, the ionisation ought to be principally due to this beam, and should therefore decrease considerably when a strong magnetic field is applied. It was found, however, that a magnetic field produces little change in the amount of ionisation. Thus in a particular case the ionisation in a sheet lead chamber 5·5 cm. high, 5·5 cm. broad, and 7·5 cm. long, was 1895 in arbitrary units. When a field of over 2000 units was applied, which was sufficient to bend the rays having the same velocity as the penetrating β rays from radium into a circle of radius less than 8 mm., the ionisation current decreased to 1645, or about 12 °/₀. When a chamber 7 cm. high, 4 cm. long, and 3 cm. broad was used, the current changed from 1560 to 1475, when the magnetic field was applied, a decrease of about 6 °/₀.

If all the ionisation in the gas was produced by the β rays from the walls of the chambers, the ionisation should obviously have decreased to a greater extent in each case, although a part of the path of each ray when deflected was still contained by the chamber, and consequently produced ionisation.

Since part of the leak in this experiment was always due to the β rays from the walls of the ionisation chamber, it was thought desirable to carry out the experiment in a somewhat different form. Figure 2 gives a diagram of the apparatus used.

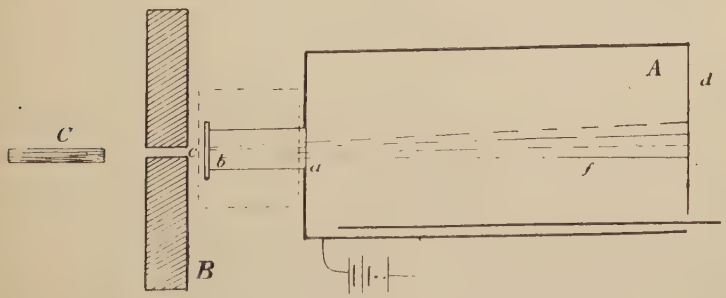


Fig. 2.

A is a cylindrical ionisation chamber 19 cm. long, and 8 cm. in diameter, to which the tube *ab* 6·5 cm. long and 3·5 cm. in diameter was co-axially attached. The chamber was placed so that the tube *ab* was between the poles of an electro-magnet. The end *a* of the tube was closed in one set of experiments by a sheet of thin aluminium leaf equivalent in mass to a layer of air ·7 cm. thick, the end *b* of the tube being closed with a metal plate *c*.

The end d of the chamber A was closed with a thin sheet of zinc. B is a lead block 3 cm. thick through which a circular hole e was drilled about 1 cm. in diameter. The axis of this hole and the glass tube C containing 30 mgrm. of radium (which was surrounded by sheet lead 2 mm. thick), were placed co-axially with the tube ab and the ionisation chamber A. The ionisation in the chamber was produced principally in the cone f by the γ rays from the radium and the secondary β rays from the plate c, the secondary β rays being initially projected in the direction of propagation of the γ rays*. The secondary β rays produced in a plate are to some extent scattered by the plate, but the larger part of the radiation proceeds approximately in the direction of propagation of the rays. Thus the writer has shown in the paper above mentioned that the pencil of secondary β rays emerging from a plate placed at one end of an aperture in a thick lead block while the radium is placed at the other end, is almost exactly of the same form as that obtained by shooting the β rays from the radium through the aperture. The larger part of the radiation from the plate c therefore entered the chamber.

The measurements were carried out by first placing a lead plug into the hole e of the lead block B and measuring the leak in the chamber. This gives the leak in the chamber due to imperfect screening of the lead block B. The leak was then measured with the plug removed. The difference between this leak and the former gave the ionisation in the cone f due directly to the γ rays and the secondary rays from the plate c. A magnetic field of sufficient strength to bend the β rays from c so that they did not enter the chamber A, was then applied, and the leak again measured. The difference between this leak and the first gave the leak in the cone approximately due to the direct action of the γ rays.

Whether the strength of the magnetic field was sufficient to bend the β rays from the lead plate c so that they did not enter the chamber A was tested as follows. The plate c and the lead sheeting surrounding the radium was removed so that a pencil of β rays from the radium now penetrated into the chamber. When the current used in these experiments was switched on to the magnet the ionisation was decreased to about 20 $^{\circ}/_{\circ}$ of its original amount, showing that practically all the β rays were prevented from entering the chamber by the magnetic field.

Firstly some measurements were made with no aluminium leaf placed at a. It was found that using a lead plate at c 2 mm. thick, the ionisation in the cone f decreased 15 $^{\circ}/_{\circ}$, when the current was switched on to the magnet; when the plate was of aluminium 3 mm. thick, the decrease was 23 $^{\circ}/_{\circ}$.

* Bragg and Madsen, *Trans. Roy. Soc. of S. Australia*, Vol. xxxii. Jan. 1908.

An aluminium leaf was then placed at *a*, the ionisation in the tube *ab* being now excluded from the leak. The decrease obtained with a lead radiator when the magnetic field was applied was 23 °/₀, and 31 °/₀ with the aluminium radiator.

The secondary β radiation from the thin zinc wall *d* of the ionisation chamber produced by the cone of γ rays *f* was much smaller than that from the plate *c* because the zinc wall was much thinner than the plate, and also because the returned β radiation from a plate is much smaller* than the radiation which is propagated in the same direction as the γ rays, the difference being the greater the thinner the plate. The figures obtained

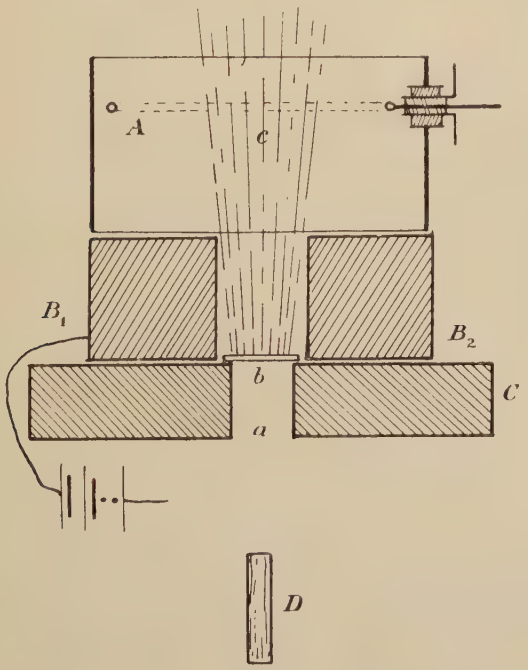


Fig. 3.

show therefore that more than 50 °/₀ of the ionisation in the cone *f* is due to the direct action of the γ rays on the air.

Some further measurements were carried out with a slight modification of the foregoing experiment. A diagram of the modified apparatus is shown in figure 3. *A* is an ionisation chamber 10·5 cm. long, 10·4 cm. broad, and 7 cm. deep, of which the upper and lower sides consisted of thin tightly stretched

* *Trans. Roy. Soc. of S. Australia*, 1908, Pt. 1.

tissue paper, equivalent in mass to a layer of air 1 cm. thick. The chamber was placed on the poles B_1 and B_2 of an electromagnet, which were resting on a lead block C 5 cm. thick. This lead block had an aperture a 3 cm. by 3·2 cm., which was placed in a symmetrical position with respect to the poles of the electromagnet and the ionisation chamber. D is the tube containing the radium placed at a distance of 10 cm. from the lead block. The electrode of the chamber consisted of a wire bent into the form of a square so that the principal stream of γ rays through the aperture did not impinge upon it. The ionisation in the chamber was, as before, principally due to the direct ionisation of the gas by the γ rays and the secondary β rays from the plate b placed over the aperture a. The plate b was of aluminium 4 mm. thick.

The readings were carried out in exactly the same way as in the foregoing experiment. The ionisation in the cone c decreased to about 55 °/₀ of its original value, when a magnetic field of sufficient strength to prevent the β rays from the plate b entering the chamber was applied. This remaining ionisation is almost entirely due to the action of the γ rays on the air in the cone c, the ionisation by the secondary radiation from the tissue paper being negligible since the paper is equivalent in mass to less than a third of the air in the chamber while the ionisation by the penetrating radiation from the air in a chamber of ordinary size is negligible.

The returned cathode radiation from the air outside the chamber may for our purpose be neglected in comparison with the radiation from the aluminium plate b. For if we assume that the air is equivalent to a carbon plate giving the maximum amount of radiation placed on top of the chamber, the returned radiation is about $\frac{1}{12}$ of the emerging radiation*. In the case of air it will be less than that, since it radiates to a larger extent sideways than the carbon plate. Now, at least one half of the radiation emerging from the aluminium plate b enters the chamber, and if we take the amount of emergence radiation of a plate of carbon and aluminium the same, which is approximately true, the returned radiation from the air is less than one-sixth of the radiation from the aluminium plate.

It appears therefore from these experiments that the ionisation in a chamber is due in part to the direct action of the γ rays on the gas it contains. And since Laby and Kaye have shown that the amount of ionisation produced by the secondary γ and β radiation from the gas is small, this ionisation consists of slow-moving β rays ejected by the γ rays, which have not sufficient velocity to produce any further ionisation themselves.

* Bragg and Madsen, *Trans. R. Soc. of South Australia*, Vol. xxxii. p. 4, 1908.

The ratio of this part of the ionisation in a chamber to that produced by the radiation from its walls, will depend on the ratio of the total inside surface of the walls of the chamber to its volume, and other conditions. The numbers obtained in the experiments just described suggest, however, that in most cases this ratio will probably be greater than one half, or the ionisation produced directly by the primary γ rays is in most cases greater than 50 °/₀ of the total ionisation.

The connection of this result with other quantities will now be considered.

Eve* has made a determination of the total number of ions produced per second by the γ rays from a gram of radium bromide in a volume of air which completely absorbs the rays. He measured the ionisation in a cylindrical chamber 51 cm. high and 23 cm. in diameter, the chamber being made of sheet aluminium ·4 mm. thick. Assuming the absorption of the γ rays by air to be the same as an equal mass of aluminium—whose coefficient of absorption is known—and knowing the volume of the chamber and its distance from the radium, etc., the number of ions produced per second by the γ rays of a gram of radium could be calculated. In this way Eve obtained $8{\cdot}9 \times 10^{14}$ for the number of ions produced.

The results obtained in this paper suggest that the ionisation in his chamber due to the δ rays produced directly by the γ rays, was roughly about half of the total ionisation; or, the number of δ rays produced per second directly by a gram of radium is equal to $4{\cdot}4 \times 10^{14}$. This number should be as near (if not nearer) to the true value of this quantity, as that given by Eve for the total number of ions produced by the γ rays of a gram of radium, in which case the penetrating β rays from the gas are supposed to spend all their energy in ionisation.

Crowther† has shown that the amount of ionisation produced by the secondary radiation from the molecules of a gas exposed to X rays is small in comparison with the total amount of ionisation produced, and that the ionisation in a gas is therefore principally due to the direct action of the X rays.

This can be shown to be true also in the case of β rays. The ionisation of the gas in a chamber exposed to β rays may be divided into two parts. One part consists of slow-moving ions or δ rays produced directly by the primary β rays and the secondary β rays from the walls of the chamber. The other part consists of the ionisation produced by the β rays of high velocity ejected from the molecules of the gas. The first part is proportional to the pressure, while the second part, from what has gone

* *Phil. Mag.* p. 192, Sep. 1906.
† *Proc. Camb. Phil. Soc.* p. 34, Vol. xv. Pt. 1, 1908.

before, is proportional to the square of the pressure. Now, Strutt* has shown that the ionisation in a gas by β rays is proportional to the pressure over a wide range of pressures. The second part of the ionisation is therefore small in comparison with the first. Thus the β rays also produce directly rays which do not themselves possess sufficient velocity to make any further ions.

Thus the ionisation of a gas by γ, β, or X rays, is a measure of the number of δ rays produced directly by the ionising agent.

Since these rays are not able to produce any further ions themselves they must have a velocity less than $2{\cdot}7 \times 10^8$ cm./sec., for at higher velocities, according to Townsend, ions are produced by collision.

The writer† has shown that the γ rays of radium produce cathode rays of very different velocities, the maximum velocity being equal to that of the penetrating β rays from radium. It appears therefore that the γ rays produce cathode rays ranging in velocity from $2{\cdot}7 \times 10^8$ to $2{\cdot}9 \times 10^{10}$ cm./sec.—the velocity of the penetrating β rays from radium. In the case of X rays cathode rays are produced probably ranging in velocity from $2{\cdot}7 \times 10^8$ to $8{\cdot}3 \times 10^9$ cm./sec., the latter velocity being that obtained by Innes for the penetrating cathode rays from a plate exposed to X rays.

Since the most penetrating cathode rays ejected by X rays have a range in air at atmospheric pressure equal to a fraction of a mm. only, the rays produced in the gas contained in an ionisation chamber are entirely absorbed by the gas. The ionisation in a gas by X rays is therefore a measure of the energy converted directly into cathode rays. Now, according to Crowther, the ionisation produced by the secondary cathode rays which are able to produce ions by collision is small in comparison with the ionisation produced directly by the X rays. It follows therefore that the energy expended in the production of penetrating cathode rays is small in comparison with that expended in the production of δ rays. And since the energy of one of the most penetrating cathode rays ejected by X rays is about 900 times that of a δ ray, the number of penetrating cathode rays ejected must be very small indeed in comparison with the number of δ rays ejected.

Whether this is also true for the cathode rays ejected by γ rays cannot yet be determined since there are not sufficient experimental data. The penetrating cathode rays ejected by γ rays are not absorbed in a distance of a fraction of a mm. in a gas like those ejected by X rays, but are able to cross an ionisation chamber without any appreciable absorption. And since the ionisation they produce in a chamber is small in comparison with the total ionisation, this quantity is, provided

* *Phil. Trans.*, A, vol. 196, p. 507, 1901.
† *loc. cit.*

the effect of the secondary radiation from the walls of the chamber is eliminated, a measure of the energy from the γ ray beam converted directly into δ rays. The energy converted into penetrating cathode rays may however be (in the case of γ rays) comparatively large, even if a comparatively small number of δ rays are produced, for the energy of one of the most penetrating rays is about 2000 times that of a δ ray.

The coefficient of absorption of a plate of material placed in the path of a beam of γ rays is a measure of the energy converted into energy of δ rays, penetrating cathode rays, and other forms of radiation. The ionisation values for different gases, since they are only a measure of the energy converted into energy of δ rays, cannot therefore be compared with the coefficient of absorption of materials without any knowledge of the comparative magnitude of the energies of these radiations. Some information on the subject could be obtained by comparing the coefficients of absorption of liquids with the ionisations of their vapours.

It is not improbable that the ejection of penetrating cathode rays by γ rays and X rays is accompanied by the production of secondary γ and X rays, and it would be interesting therefore, when sufficient data are available, to compare the relative secondary γ and X radiations from different materials with the relative penetrating cathode radiations produced, in order to see how much of the secondary γ and X radiation is accounted for in the above way.

Note on an abnormal pair of appendages in Lithobius. By L. Doncaster, M.A., Lecturer on Zoology, Birmingham University.

[*Received* 15 March 1909.]

In dissecting a specimen of the common English Centipede (*Lithobius forficatus*), a student in the Birmingham Zoological Laboratory found that it had an extra pair of appendages between the poison-claws and the second maxillae. I did not see the specimen before the appendages were removed, so cannot say exactly how they were attached. The additional pair is represented in fig. 1, from a camera-drawing, and fig. 2 represents the

Fig. 1.

Fig. 2.

poison-claws of the same specimen on the same scale. It will be seen that the extra appendages are more nearly like poison-claws than any other appendages of *Lithobius*, but differ in several important details. They are much smaller, are differently jointed, and instead of ending in one sharp tooth on which the poison-duct opens, they have at their extremity a triple tooth in which no duct is visible. It should be said that they had been soaked in potash before they were carefully examined so that a rudimentary poison gland and duct might have been present and destroyed. Another point of interest concerns the median part of the

appendage. The basal joints of the two sides are more completely fused than are the poison-claws, and in the position where the latter bear six small processes or teeth on each side, the extra appendages bear on each side two larger teeth, which have the appearance of being jointed on to the base of the appendage, and are not mere processes of it as they are in the normal pair.

The mandibles and both pairs of maxillae appeared to be perfectly normal, and behind the normal pair of poison-claws there were as usual fifteen pairs of walking legs.

On the migration of the thread-cells of Mœrisia. (Preliminary note.) By C. L. Boulenger, B.A., King's College.

[*Read* 22 February 1909.]

In a recent paper whilst describing the structure of the Egyptian medusa *Mœrisia lyonsi* I called attention to the fact that large nematocysts were constantly to be found among the endoderm cells of the manubrium. Careful examination of a larger series of sections has convinced me that these nematocysts are in process of migration through the tissues, a phenomenon similar to that recently described by Hadži in hydroids.

The main thread-cell batteries in *Mœrisia* are those around the oral opening and those on the four perradial tentacles.

The thread-cells of the oral battery develop in the more proximal parts of the manubrium and make their way through the endoderm and structureless lamella to the ectoderm of the mouth region, the movements being effected by the amœboid cnidoblasts of the thread-cells. Similarly, the nematocysts of the perradial tentacles are not formed *in situ* but probably develop in the large ocellar bulbs at the bases of the tentacles.

Action of Urethane on Esters of Organic Acids and Mustard Oils. By S. Ruhemann, M.A., Gonville and Caius College, and J. G. Priestley, M.A.

[*Read* 8 February 1909.]

The sodium-derivative of ethyl carbamate reacts with ethyl phenylpropiolate not by addition, but with formation of ethyl phenylpropiolylcarbamate. Similarly the esters of fatty saturated acids furnish acid derivatives of ethyl carbamate.

Phenyl mustard oil reacts with ethyl sodiocarbamate mainly according to the equation:

$$2C_6H_5NCS_5 + NaNH.CO_2Et = C_{15}H_{10}ON_3S_2Na + C_2H_6O,$$

and yields the anhydride of diphenylthiobiuretcarboxylic acid. Besides this compound, a small quantity of carboxyethylphenylthiocarbamide,

$$NH(CO_2Et)CS.NH.C_6H_5,$$

is formed. Analogous is the action of ethyl sodiocarbamate on other mustard oils.

Homologues of furfural. By Dr FENTON and F. ROBINSON, B.A.

[*Read* 8 February 1909.]

New syntheses have been effected by the application of the Friedel and Crafts reaction to the halogen derivatives of methylfurfural with various hydrocarbons and the results promise a wide field for further investigation. Improvements have been made in the mode of formation of the derivatives of methylfurfural and evidence is brought forward which appears to necessitate a modification in the generally accepted constitutional formula for the hydroxy-derivative.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Some fatigue effects of the cathode in a discharge tube. By R. WHIDDINGTON, B.A., Hutchinson Research Student of St John's College. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 17 May 1909.]

Skinner's* experiments on the vacuum tube discharge seem to suggest that charged hydrogen atoms emitted from the cathode play a considerable part in the carriage of the current.

Chrisler† has studied experimentally the effect of absorbed gases on the photoelectric activity of metals, coming to the conclusion that hydrogen again plays a large part in the photoelectric current.

The present note describes the results of some simple experiments designed to show what influence the gases occluded in the cathode have on the emitted cathode rays.

The method employed is qualitative and hardly capable of quantitative application. It consists of comparing the behaviour of a cathode, previously treated in various ways, with an exactly similar, and similarly situated untreated standard electrode, in the same tube. By this means the complicating influence of pressure changes was eliminated, for the change over from one cathode to the other was instantaneous leaving no time for a change in pressure.

If the looked-for effect has any existence it is reasonable to suppose that it might be greatest in the case of a charcoal cathode. For carbon can be made to absorb vast volumes of many gases.

* *Phys. Rev.* Vol. XXI. 1905.
† *Phys. Rev.* Vol. XXVII. 1908.

The first experiments accordingly were made with cocoanut charcoal electrodes.

Two circular carbon electrodes of equal thickness and about two centimetres in diameter were mounted symmetrically and parallel in a vertical straight tube only slightly larger, so as to face each other at a distance of about 30 centimetres. A subsidiary aluminium electrode was fused in, in a side tube midway between the two cathodes. Further, each carbon electrode was arranged so as to be at the geometrical centre of a concave aluminium electrode, which, when run as cathode, bombarded its attendant carbon electrode with an intensely heating beam of cathode rays.

The upper electrode was taken as the standard. The lower electrode was heated up by the convergent beam of rays from its concave cathode, the resulting spot of incandescence being moved about by means of a bar magnet. After four or five hours' continuous heating and pumping the carbon ceased to give off gas, indicated by the width of the Crookes' dark space remaining constant.

The spectroscope showed the emitted gas to be mainly oxides of carbon with a perceptible trace of hydrogen.

The tube now contained an upper electrode full of gas, and a lower one free of gas. The two electrodes were then connected direct to the secondary of an induction coil (giving a 6 in. spark). On making alternately the upper and lower carbon electrodes, cathode, no difference in the character of the discharge could be detected. The cathode ray emission as roughly indicated by the green glass phosphorescence was the same and the Crookes' dark space was in both cases of the same width.

Now, on running the lower carbon as cathode for 10 minutes, the subsidiary side electrode being anode, a further small quantity of gas was given off indicated by the shrinking in of the dark space. As before the gas appeared to be oxides of carbon and hydrogen.

A comparison of this now further depleted lower carbon with the standard showed a distinct alteration in properties. The green glass phosphorescence was very much less brilliant and the Crookes' dark space narrower and more sharply defined. After 20 minutes running as cathode, this fatigue effect, as it may be called, became more marked; but after half an hour the cathode seemed to get into a constant state. The state of affairs can be summed up in few words by saying that the upper standard cathode behaved as though the tube were hard, the lower one as though the tube were soft, yet the gas pressure was the same in either case.

The fact that the boundary of the Crookes' dark space

becomes more sharply defined with time of running, seems to suggest that the cathode rays emitted by the fresh cathode have various velocities, but that as the running is continued, the faster and more penetrating rays cease to be emitted in any quantity, until at last when the cathode is quite fatigued the rays are very homogeneous.

Having reduced the lower carbon to this fatigued state attempts were made to restore it to an equality with the standard. To this end it was caused to absorb hydrogen, air, carbon dioxide [and helium] by cooling in liquid air and admitting the gas.

No appreciable return to equality with the standard could be detected.

It should be mentioned that during the absorbing process the upper standard carbon electrode was kept hot to prevent any absorption taking place there. This meant that both oxides of carbon were being absorbed by the lower electrode during the admission of carbon dioxide, for carbon monoxide would be continuously produced by the union of the incandescent upper carbon with the surrounding carbon dioxide.

Platinum electrodes were next tried, the heating in this case being produced by the passage of an electric current. No certain results could be obtained owing to the erratic behaviour of the platinum after heating. The only point that could definitely be established was that, as in the case of the carbon, running as cathode could liberate gas which mere heating could not.

Aluminium electrodes were next tried but without any heating arrangement. The same fatigue effect though rather less marked than with carbon was observed. Again it was found impossible to restore the cathode to the original (standard) state by causing the metal to absorb various gases. The method of causing gas absorption in this case was to heat up the metal to about 400° C. in a glass tube containing the gas required and then allowing it to cool. In this way some little of the gas was always absorbed. A considerable volume of hydrogen could be introduced into the body of the metal in quite a different way. The cathode was made one of the electrodes in an electrolytic cell containing dilute sulphuric acid and the current passed so as to deposit hydrogen against the aluminium. Although this caused considerable absorption of hydrogen yet the fatigued metal showed no return to the standard state.

It is well known that the green phosphorescence excited on glass by cathode rays falls off in intensity as the bombardment is continued.

This effect was eliminated by using a horizontal discharge tube with the electrodes as before facing each other at either end, in the centre of which was suspended a Willemite screen carrying

a little magnet so that the screen could be rotated by an external magnetic control. The screen was midway between the two electrodes. When both electrodes were fresh the phosphorescence of the screen was independent of the cathode from which it received the exciting cathode rays. When, however, one of the electrodes was run as cathode for some time, the Willemite glowed much more vividly when it was responding to rays from the fresh cathode, the dark space being a centimetre or so in width and the distance of the screen about 5 cms.

This fact lends support to the view that at any rate some of the rays emitted from a fresh cathode are faster and therefore more penetrating than those emitted from a run cathode.

The surface of the aluminium used in the above (and future) experiments was always polished with the finest emery powder so as to be sure of always having a constant surface.

It was thought that the observed effects might be merely due to surface influences. This was proved to be not so, by filing off about ·2 mms. of the metallic surface and repolishing. The run cathode so treated appeared to be still as fatigued as ever.

A fatigued cathode also shows no very marked signs of recovery if left to itself. In one particular case a tube was left at atmospheric pressure for over three months and at the end of that time the fatigue effect was still as obvious as it had been immediately after the running period.

At first, running the fatigued cathode as anode did not appear to have any effect, but using a very small induction coil (giving 5 mm. spark) to produce the discharge, an effect was observed when the pressure was between certain limits. When the fatigued cathode which had been run as anode was made cathode again, there was a sudden burst, so to speak, of phosphorescence when the discharge first passed. This sudden brilliance however was only momentary, lasting but a small fraction of a second. This transient recovery did not seem to depend on the length of time the electrode was run as anode; making it anode momentarily produced as much effect as running for quite a long time. The effect was most marked in hydrogen but could be got in other gases as well.

Chrisler in the paper cited quotes figures showing that the photoelectric current for metals increases enormously if the electrode is run momentarily as anode in hydrogen. This falls into line with the cathode effect described above, but his further observation that running as anode in gases other than hydrogen actually diminished the photoelectric effect finds no counterpart in these experiments.

It was thought that the volume of metal forming the cathode might influence the time taken to produce the fatigue effect. Experiments on this point gave no definite results.

The direct results of these experiments are:

(i) That an electrode contains gas which cannot be driven off by an ordinary heating but which is easily evolved when the electrode is run as a cathode.

(ii) That an electrode after running as cathode for some time emits more slowly moving and more homogeneous cathode rays than it does when fresh.

(iii) That it is not possible to restore a "fatigued" cathode to its original state by causing it to absorb hydrogen, oxygen, nitrogen, carbon dioxide, carbon monoxide or helium.

It was thought that measurements of the cathode fall of potential might throw some light on (ii).

The cathode fall was measured only in the case of aluminium cathodes in air.

A platinum exploring point was used in conjunction with an Ayrton-Mather electrostatic voltmeter.

The instrument always took from half to three-quarters of a minute to reach its final reading owing both to the smallness of the collecting point and the large capacity of the electrometer quadrants. The arrangement could therefore not be used to measure rapid variations in the cathode fall.

In most cases the cathode fall started from a low value, rising rapidly in about three minutes to a maximum and then gradually falling to a final steady state. No value can be attached to the first rapid rise to a maximum as very often the time taken was comparable with the time of lag of the instrument. The value of the maximum recorded by the electrometer was very variable with different cathodes, even when they were cut out of the same sheet of metal. The limits of variation were between 280 and 400 volts. In only one case was the upper limit reached, usually 300 volts was about the value. The final value to which all cathodes converged was 265 volts and the time taken was of the magnitude of 10 minutes.

Thus the cathode fall for a new aluminium cathode is rather greater than for one which has been run some time, the difference usually being about 20 °/₀ but sometimes being as much as 60 °/₀.

This is what the qualitative experiments might have led one to expect; for a new cathode has a wider dark space than a run one.

The effect of absorbed gases was tried but with negative results.

Running the cathode as anode also produced no change that could be detected in the cathode fall. This again is what the previously described experiment might be taken as predicting; for the effect could only be a momentary one and could not be

detected owing to the great time lag of the instrument. No collecting point device could be expected to indicate changes in potential so rapid as this.

Some quite thin aluminium plate ·01 cm. thick was tried in the cathode fall tube. It was found that the cathode fall rose almost immediately to the final value 265 volts instead of starting from a high value and then slowly diminishing with time.

This observation suggested that the time taken for a cathode to give its final value for the cathode fall might depend on the volume of the cathode, the time being greater, the greater the mass of metal involved.

Such time effects were looked for but could not be quantitatively worked out owing to two main causes, firstly to the fact that different cathodes did not start from the same high value of the cathode fall although they all reached the same final value, and secondly to the fact that even two electrodes of the same thickness and size cut out of the same sheet of metal and starting from almost the same high value took very different times to reach the same final limit.

In general however it seemed certain that for heavy cathodes a considerable time was required to reach a steady state, while light cathodes reached it immediately, without any gradual fall.

This result led at once to the trial of very thin cathodes in the ordinary discharge tube. They did not show any fatigue effect, the dark space being sharply defined from the commencement.

It is possible that some gas is present in the new metal which is expelled by running as a cathode which is responsible for these fatigue effects. If the gas is one of those mentioned above it certainly cannot be made to enter the material of the cathode in the ordinary way. On the other hand it may be one of the rarer gases or even an unknown one.

It is possible that photographic methods using large masses of metal for the cathode may reveal this gas. But in these experiments the only gases evolved appeared to be oxides of carbon and hydrogen.

I have pleasure in thanking Prof. Sir J. J. Thomson for suggesting the work and for his kind interest while it was being carried out.

Note on the electrical behaviour of fluorescing iodine vapour. By R. WHIDDINGTON, B.A., Hutchinson Student of St John's College. (Communicated by Prof. Sir J. J. Thomson.)

[*Received* 4 August 1909.]

In view of the fact that no satisfactory theory of fluorescence exists, it was thought likely that a study of the electrical behaviour of iodine vapour when fluorescing might give valuable results.

The apparatus employed consisted of a wide glass tube into which a cylinder of copper gauze slipped tightly. The light could be concentrated within the tube through a hole cut in the gauze.

The electrodes were discs of copper placed parallel and at a distance of about three centimetres apart. One electrode was earthed and connected to the gauze sheath, the other being connected to a sensitive Wilson gold leaf electroscope.

In the first trials ebonite insulation was used, but as the iodine—which was liberally sprinkled within the vessel—was found to attack it, quartz was substituted in its stead.

The vessel was connected to a mercury pump and evacuated to the limit set by the vapour pressure of the iodine ($\frac{1}{10}$ mm. about).

When a beam of arc light was focussed between the electrodes, the fluorescence was plainly visible but no trace of ionization could be detected even when the applied electric field was almost enough to produce a discharge.

To be quite certain that the ionization chamber and electroscope were in good working order, a test was made with a weak sample of radium. Even at this low pressure there was a very rapid deflection of the gold leaf.

This experiment conclusively shows that ionization is not necessarily an accompaniment of fluorescence.

On the Reflection of Sound at a Paraboloid. By the Rev. H. J. SHARPE, M.A., late Fellow of S. John's College, Cambridge.

[*Received* 22 May 1909.]

38. In the year 1899 a Paper by the author with the above title was published in Vol. x. Part III. of the *Proceedings of the Cambridge Philosophical Society*. That Paper contained some minor and possibly some major Errata, which I should like some day to correct, but that is not my object at present. Rather it is to make an addition to my former Paper suggested by Arts. 10, 18 and 23, in which additional results are obtained, shewing a strange analogy with the Reflection of a point of Light at plane surfaces. Thinking it would be better to make this Paper intelligible *by itself*, I have ventured to repeat, as shortly as possible, some of the early part of the Analysis. Wishing also this Paper to be considered part of the first, I have made the numbering of the Articles, Equations, and figures run on continuously.

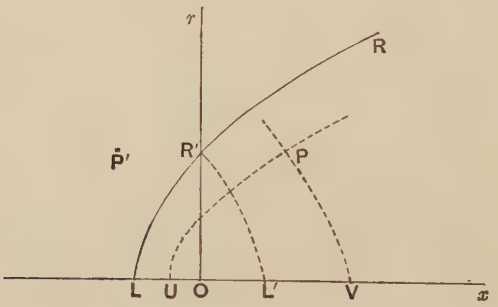


Fig. 8.

39. Suppose then (fig. 8) LR to be the material parabolic Reflector, O its focus and Ox the axis of x. The sound motion is supposed to be such that it is everywhere symmetrical round the axis, and in planes passing through it.

Let P be any point within the Paraboloid LR. Draw two confocal parabolas PU and PV passing through P with O as focus. Put $OU=\frac{u}{2}$, $OV=\frac{v}{2}$ and $OL=\frac{l}{2}$. It will be found that any sound motion within the Paraboloid must satisfy the following equation

$$\frac{4}{u+v}\left(v\frac{d^2F}{dv^2}+u\frac{d^2F}{du^2}+\frac{dF}{dv}+\frac{dF}{du}\right)=\frac{1}{a^2}\frac{d^2F}{dt^2}.$$

For sound motion F will be of the form

$$P \sin(2pat) + Q \cos(2pat),$$

where P and Q each satisfy the equation

$$\frac{1}{u+v}\left(v\frac{d^2P}{dv^2} + u\frac{d^2P}{du^2} + \frac{dP}{dv} + \frac{dP}{du}\right) + p^2P = 0.$$

As in the present Paper we shall only consider *stationary* vibration, either P or Q will always be zero. A particular solution of the last equation is $P = UV$, where U is a function of u only, and V of v only, and

$$v\frac{d^2V}{dv^2} + \frac{dV}{dv} + (p^2v - A)\,V = 0 \ldots\ldots\ldots\ldots(96),$$

$$u\frac{d^2U}{du^2} + \frac{dU}{du} + (p^2u + A)\,U = 0 \ldots\ldots\ldots\ldots(97),$$

where A is an arbitrary constant, pa/π is the frequency and a is the velocity of sound. It will be found that the condition of reflection at the paraboloid LR is that $\frac{dU}{du}$ should $= 0$ when $u = l$.

In all that follows it will be found to conduce much to brevity to use new independent variables, defined thus

$$pu \equiv u',\ pv \equiv v',\ pl \equiv l' \text{ and } A/p \equiv A';$$

(96) and (97) then become

$$v'\frac{d^2V}{dv'^2} + \frac{dV}{dv'} + (v' - A')\,V = 0 \ldots\ldots\ldots\ldots(98),$$

$$u'\frac{d^2U}{du'^2} + \frac{dU}{du'} + (u' + A')\,U = 0 \ldots\ldots\ldots\ldots(99),$$

from which p has disappeared and the condition of reflection becomes

$$\frac{dU}{du'} = 0 \text{ when } u' = l' \ldots\ldots\ldots\ldots(100).$$

We may now *drop the dashes*, if in all *final* results we remember to put

pu for u, pv for v, pl for l, and A/p for A.

By this we come back to the original notation.

40. We will first take a comparatively simple case. We will suppose $A = 0$. This case was partly treated in Arts. 10—17, but more will be added. In this case we get $V = J_0(v)$ and $U = J_0(u)$, and the condition of reflection is that $dU/du = 0$ when $u = l$.

It is known that the equations $J_0(u)=0$ and $\frac{d}{du}J_0(u)=0$ have an infinite number of real roots, so that l has an infinite number of values appropriate to the present solution. The same can be said of the *original pl* (Art. 39), and (as we suppose p constant) of the latus rectum of the Reflector. It will be interesting to find the various points on the axis where the air-velocity is a maximum, and especially the position and intensity of the *greatest* maximum. We shall suppose the vibration stationary, then it can be shewn (Art. 5) that the air-velocity in the direction OV (fig. 8) at any point V on the axis is $2\frac{dV}{dv}\times$ time factor, which $=2\frac{d}{dv}J_0(v)\times$ time-factor. We shall, however, for brevity always omit the time factor. The position on the axis to the right of O where the air-velocity is a maximum is determined by the equation $\frac{d^2}{dv^2}J_0(v)=0$, which is the same thing as $\frac{d}{dv}J_1(v)=0$.

Now the zeros and turning points of $J_1(x)$ are given by the following Table, where, for simplicity, we have retained only two places of decimals.

When $x=$ 1·84, 5·33, 8·54, 11·71, 14·86, 18·00, &c.

Then $\frac{d}{dx}J_0(x)=-J_1(x)=$

$$-\cdot58,\ +\cdot34,\ -\cdot27,\ +\cdot23,\ -\cdot20,\ +\cdot19,\ \&c.,$$

and the roots of $J_1(x)=0$ are given by

$$x=0,\quad 3\cdot83,\quad 7\cdot01,\quad 10\cdot17,\quad 13\cdot32,\quad \&c.$$

Looking at fig. 8, if P' be the optical image of P in OR', we see that the u and v of P will be equal to the v and u of P' respectively, and when $A=0$, it will be found (see Art. 5) that the sound motion to the left of OR' is the image of the sound motion to the right of OR'. It follows that if in fig. 8 we draw the parabola $L'R'$ the image of LR', there will be no air-velocity normal to $L'R'$; and L', *as well as* L, will be a point of rest.

We can now see something as to the nature of the air motion and sound intensity, at any rate at points along the axis of the reflector LR'. To the right of OR' (see fig. 9) along the axis Ox we have an infinite series of points v_1, v_2, v_3, v_4, etc., whose abscissae are $Ov_1=1\cdot84$, $Ov_2=5\cdot33$, etc. The ordinates of the wavy curve represent the magnitudes and directions of the corresponding maximum air-velocities.

For instance, at v_1 the magnitude is ·58 to the left,

" " v_2 " " ·34 " right,

" " v_3 " " ·27 " left, and so on;

r_1, r_2, r_3, r_4, etc., are intermediate points of rest.

It will be observed that as we pass from infinity along the axis from x towards O the maxima air-velocities gradually increase in intensity till at v_1 we have the greatest maximum.

On the left of OR' we have a state of air-velocity at u_1, u_2, u_3, etc., ρ_1, ρ_2, ρ_3, etc., exactly the image, in the line OR', of that which exists on the right of OR'. Of course on the right of O v_1, v_2, v_3, etc., are real foci of reflection. But on the left of O (as the figure is drawn) only u_1 and u_2 are real foci, all the rest u_3, u_4, etc., are virtual foci of reflection. And here it must be noted that the point r_2 in fig. 9 is supposed to be identical with the point L' in fig. 8. The points L and L' being points of no air-velocity must be made identical with some of the points r_1, r_2, etc., and ρ_1, ρ_2, etc.,

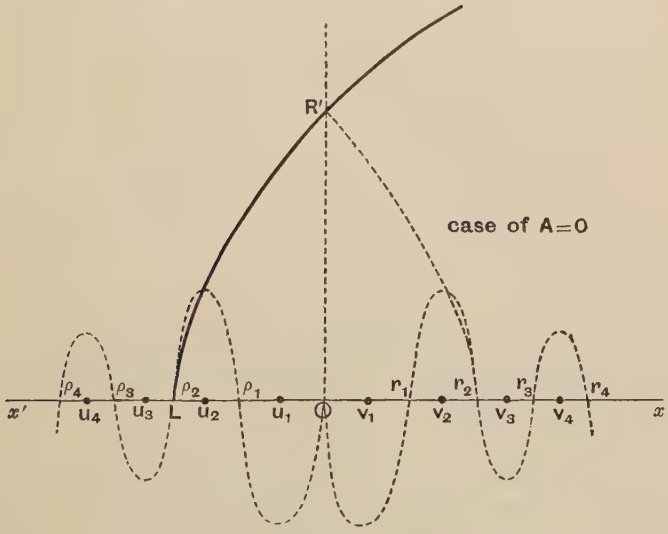


Fig. 9.

in fig. 9—*taken in pairs* as $r_1\rho_1$, $r_2\rho_2$, $r_3\rho_3$, etc. As we have chosen the size of the Reflector so that L' coincides with r_2 (fig. 9) we shall have l (or, Art. 39, going back to the *original* notation) $pl = 7{\cdot}01$. Suppose l to be 8 inches (the size of a portable Sound Reflector I actually made). Then $p = 10{\cdot}51$, a high note just within the range of a piano.

Suppose (fig. 9) we had made L' to coincide with r_1 (instead of r_2) we should have got $pl = 3{\cdot}83$, and then p would be $5{\cdot}74$. We thus see that as we experiment with higher and higher notes, more and more points of maximum velocity are crowded into the space LL' (see fig. 8) that is in the neighbourhood of the focus O. It must, however, be noticed that in this case where $A = 0$, which we have been considering, O is always a point of rest.

41. We will next suppose A finite. We have now to use the equations (98) and (99)—but we will drop the dashes and no mistake will arise if we are careful to remember the remark at the end of Art. 39.

In (98) and (99) A is one of the roots of the equation in A

$$dU/du = 0, \text{ when } u = l \quad \ldots\ldots\ldots\ldots\ldots\ldots(100).$$

Suppose, if possible, A_1 and A_2 to be two different roots of this last equation, and when $A = A_1$, A_2 respectively, let U become U_1, U_2 respectively. Then

$$u\frac{d^2U_1}{du^2} + \frac{dU_1}{du} + (u + A_1)\, U_1 = 0,$$

$$u\frac{d^2U_2}{du^2} + \frac{dU_2}{du} + (u + A_2)\, U_2 = 0.$$

It is easy to shew that

$$\frac{d}{du}\left[u\left(U_2\frac{dU_1}{du} - U_1\frac{dU_2}{du}\right)\right] + (A_1 - A_2)\, U_1U_2 = 0.$$

Integrate this with regard to u between the limits 0 and l, when we get

$$\int_0^l U_1U_2\,du = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(101).$$

This shews that the equation (100) considered as an equation in A has all its roots real, for otherwise, if A_1 and A_2 were two conjugate unreal roots, (101) could not be satisfied.

42. We will next suppose that in (99) u is much larger than A, then it is evident that, in the neighbourhood of points which satisfy this condition, U does not differ much from $J_0(u)$, and the curve whose ordinates give dU/du is a wavy or periodic curve which cuts the axis. This is corroborated by Art. 23, where it is shewn that if $u > A$ we have approximately

$$U = \frac{B}{u^{\frac{1}{2}}}\cos\left(u + \tfrac{1}{2}A\log u + \alpha\right) \quad \ldots\ldots\ldots\ldots(102),$$

where B and α are constant functions of A.

[N.B. In Art. 23 we have V, v and $-A$, but it will be found that exactly similar reasoning may be applied to the case U, u and $+A$.]

We will next suppose that v is much larger than A, then we can shew in like manner that, near points which satisfy this condition, the curve whose ordinates give dV/dv is a wavy curve

which cuts the axis, also when $v > A$ we have approximately from Art. 23

$$V = \frac{B'}{v^{\frac{1}{2}}} \cos\left(v - \tfrac{1}{2} A \log v + \alpha'\right) \quad \ldots\ldots\ldots\ldots(103),$$

where B' and α' are constant functions of A.

43. We will next suppose that in (99) u is much smaller than A. Then divide each side of (99) by A and put $Au \equiv z$. We then get approximately (see Art. 19 of this Paper), if z is large,

$$U = J_0(2z^{\frac{1}{2}}) = \frac{\cos\left\{2(Au)^{\frac{1}{2}} - \frac{1}{4}\pi\right\}}{(Au)^{\frac{1}{4}} \cdot \pi^{\frac{1}{2}}} \quad \ldots\ldots\ldots(104).$$

This shews that in the neighbourhood of points which satisfy the condition u/A small (A being large) the curve whose ordinates give the velocity dU/du is a wavy curve which cuts the axis.

We will next suppose that in (98) v is much smaller than A. Proceed as before. Divide by A and put $Av \equiv z$. We shall then get approximately (see Art. 20 of this Paper)

$$V = Z = 1 + z + \frac{z^2}{(2!)^2} + \frac{z^3}{(3!)^2} + \text{etc.} = J_0\{2(-z)^{\frac{1}{2}}\}.$$

It is evident from this, that in the neighbourhood of points satisfying the condition $v < A$ (A being large) no positive value of v can satisfy the condition of dV/dv being a maximum or minimum, and that therefore near such points the curve, whose ordinates give dV/dv, is either not a wavy curve at all, or if it *is* a wavy curve, that it *does not cut the axis.* This kind of curve is shewn by the wavy line in fig. 10. Or it is possible, but not likely, that between O and A the velocities might *uniformly* increase or uniformly diminish without there being maxima or minima. Perhaps we might be allowed to speak of this kind of curve as an exponential curve. All this is further corroborated by the fact that when z is large the approximate value of V takes an *exponential* form (see Art. 20)

$$V = \frac{\epsilon^{2(Av)^{\frac{1}{2}}}}{\pi^{\frac{1}{2}}(Av)^{\frac{1}{4}}} \quad \ldots\ldots\ldots\ldots\ldots(105).$$

44. We have now examined the shape of the air-velocity curve in the neighbourhood of four classes of points defined thus

$$u > A, \quad u < A, \quad v > A, \quad v < A.$$

It remains to examine the same near four other classes of points defined thus

u does not differ much from A in excess or defect,

v „ „ „ A „ „

We shall take the two last cases first. In (98) put $v = A\mu$ and suppose both v and A large, and μ nearly $= 1$.

And first we shall suppose $\mu < 1$, equation (98) then becomes

$$\mu \frac{d^2V}{d\mu^2} + \frac{dV}{d\mu} - A^2V(1-\mu) = 0 \ldots\ldots\ldots\ldots (106).$$

With the above suppositions, a solution (68) is given of this equation in Art. 26. [It must, however, be admitted that (68) fails if μ be too near 1. For this case and for the case of μ being actually $= 1$ another solution must be sought.] The solution (68), however, answers very well if say μ is near $\frac{1}{2}$. It will be further observed that the solution (68) is of an *exponential* character, so it seems to follow by reasoning similar to that used before that (for points for which $\mu < 1$, but v and A both large) the air-velocity curve is exponential. It is important to observe that this result agrees very well with the latter part of Art. 43.

45. Next, with the above suppositions, we shall suppose $\mu > 1$. In this case it is better to write the equation for V thus

$$\mu \frac{d^2V}{d\mu^2} + \frac{dV}{d\mu} + A^2V(\mu - 1) = 0 \ldots\ldots\ldots\ldots (107).$$

A solution (70) of this equation is given in Art. 27.

[As before, we must admit that this solution fails when μ is very near 1 or actually $= 1$, but it answers very well if μ is near or $= \frac{1}{2}$.] It will be further observed that the solution (70) is of a trigonometrical or periodic character, so it seems to follow as before that (for points for which $\mu > 1$, but v and A both large) the air-velocity curve is wavy and cuts the axis. Again it will be observed that this agrees very well with the latter part of Art. 42.

46. According to the beginning of Art. 44 it would now be our duty to examine the value of U when u does not differ much from A in excess or defect. In (99) put $u = A\mu$, and suppose u and A both large, then (99) becomes

$$\mu \frac{d^2U}{d\mu^2} + \frac{dU}{d\mu} + A^2U(1+\mu) = 0 \quad \ldots\ldots\ldots (108).$$

As μ is supposed nearly $= 1$ this equation does not differ much from

$$\mu \frac{d^2U}{d\mu^2} + \frac{dU}{d\mu} + 2A^2U = 0.$$

Put $2A^2\mu = \nu$. Then

$$\nu \frac{d^2U}{d\nu^2} + \frac{dU}{d\nu} + U = 0.$$

Then $$U = J_0(2\nu^{\frac{1}{2}}) = J_0(2^{\frac{3}{2}} A \mu^{\frac{1}{2}}).$$

As A is large and μ about 1, we get nearly

$$U = \frac{\cos(2^{\frac{3}{2}} A \mu^{\frac{1}{2}} - \frac{1}{4}\pi)}{(\pi A)^{\frac{1}{2}} 2^{\frac{1}{4}} \mu^{\frac{1}{4}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (109).$$

This shews that the air-velocity curve near the points defined as above is wavy and cuts the axis. It seems probable that this solution will take the same form whether μ differs from 1 in excess or defect.

47. I will now give a figure (fig. 10) which will illustrate at one view the chief results arrived at in Arts. 42 to 46. The figure is of course not drawn to scale. The Numbers 42 (1) etc. refer to Art. 42, First Part, etc. The little dots close together near a large point on the axis mean that the air-velocity curve near the point is wavy, like that in fig. 9, and cuts the axis.

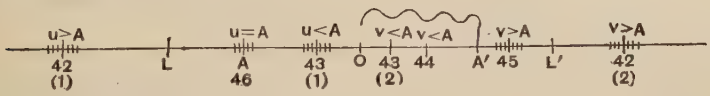


Fig. 10.

In this figure $LAOA'$ is the axis of the Reflector of which O is the focus and L the vertex. OA is measured to the left of $O = A$ and OA' to the right $= A$. One curious result is that at all points between O and A' on the right of O the velocity curve is "exponential."

Discussion of a difference equation relating to the tension of overhead wires supported by equidistant poles. By A. A. Robb, M.A., St John's College.

[*Received* 22 May 1909.]

Section I. *On the Form of the Solution.*

When overhead wires are suspended from poles for electrical purposes, certain precautions must be taken in adjusting the tensions, since otherwise the snapping of the wire may lead to the breakage of a large number of poles.

The mathematical problem thence arising has already been considered by several writers and among others by Messrs Hawthorne and Morton in the *Philosophical Magazine*, Vols. XI. and XII., 1906.

Their result is somewhat vitiated through taking the quantity denoted by "f" as a constant; whereas it generally varies through a wide range (Vol. XI., p. 634).

The attention of the writer was drawn to the problem by Mr W. H. Logeman who has himself investigated it by a graphical method which is however quite different from the following.

In practice the wire is stretched between "anchor poles" which are often long distances apart and may be regarded as rigid; while between these it is supported by ordinary poles which in normal circumstances are not subject to transverse stress. When a break occurs in the wire, the ordinary poles are subject to forces tending to deflect them and the deflecting force on a pole is equal to the difference between the horizontal components of the tensions in the two adjoining sections of wire.

If these forces lie within the limits of safety the deflections of the poles will be proportional to them and we may assume a constant of proportionality H, such that the horizontal displacement of the point of attachment of the wire $= H$ (deflecting force).

Let T_1, T_2, T_3, ... be the horizontal tensions in successive sections of the wire counting from the break and let L be the common distance between successive poles.

If λ_n be the horizontal displacement of the top of the nth pole counting from the break, we have

$$\lambda_n = H(T_n - T_{n-1}),$$

$$\lambda_{n+1} = H(T_{n+1} - T_n).$$

Thus
$$\lambda_{n+1} - \lambda_n = H(T_{n+1} + T_{n-1} - 2T_n).$$

The total distance apart of the extremities of the nth section of the wire will accordingly be

$$L + \lambda_{n+1} - \lambda_n = L + H(T_{n+1} + T_{n-1} - 2T_n).$$

We must now make use of a well-known property of the elastic catenary. (See, for instance, Routh's *Analytical Statics*, p. 373.) Let x be the horizontal distance of any point of this curve measured from its lowest point while s_1 is the unstretched length of the intervening portion. Let further T_h be the horizontal component of the tension; let w be the natural weight of the material per unit length and let E be the elastic modulus.

We have then

$$x = \frac{T_h}{E} s_1 + \frac{T_h}{w} \log \left\{ \frac{ws_1}{T_h} + \sqrt{1 + \frac{w^2 s_1^2}{T_h^2}} \right\}.$$

Treating our wire as perfectly flexible we may identify T_h with the horizontal tension in any section and $2x$ with the total distance apart of the extremities, while $2s_1$ will be the natural length of the portion of wire. We have accordingly

$$L + H(T_{n+1} + T_{n-1} - 2T_n) = \frac{2T_n}{E} s_1 + \frac{2T_n}{w} \log \left\{ \frac{ws_1}{T_n} + \sqrt{1 + \frac{w^2 s_1^2}{T_n^2}} \right\}.$$

If we expand this logarithm in powers of $\frac{ws_1}{T_n}$, neglecting those powers beyond the third (as is permissible if the sag of the wire be not too great), we get

$$L + H(T_{n+1} + T_{n-1} - 2T_n) = \frac{2s_1}{E} T_n + 2s_1 - \frac{w^2 s_1^3}{3T_n^2},$$

or

$$T_{n-1} + T_{n+1} = \frac{2s_1 - L}{H} + 2\left(1 + \frac{s_1}{EH}\right) T_n - \frac{s_1^3 w^2}{3HT_n^2}.$$

At a great distance from the break the tension in a section will be practically the same as it was before the break occurred and may accordingly be denoted by T_∞. Our problem is thus reduced to solving the above equation subject to the conditions

$$T_0 = 0,$$
$$T_\infty = T$$

say, where T is a finite quantity.

The equation may be written briefly

$$T_{n-1} + T_{n+1} = a + 2bT_n - \frac{c}{T_n^2}.$$

Putting $n = \infty$ we get

$$\frac{c}{T^2} - 2(b-1)T = a.$$

In order to solve the equation we shall assume that T_n may be expressed in the form of a series of a particular type and shall

afterwards justify the assumption by investigating the convergency of the series. Let us assume then, if possible, that

$$T_n = T\left(1 - a_1 \frac{C}{K^n} - a_2 \frac{C^2}{K^{2n}} - a_3 \frac{C^3}{K^{3n}} - \dots\right),$$

where $a_1, a_2, a_3, \dots$ C and K are quantities to be determined.

We have then

$$T_{n-1} = T\left(1 - a_1 K \frac{C}{K^n} - a_2 K^2 \frac{C^2}{K^{2n}} - a_3 K^3 \frac{C^3}{K^{3n}} - -\right),$$

$$T_{n+1} = T\left(1 - \frac{a_1}{K} \frac{C}{K^n} - \frac{a_2}{K^2} \frac{C^2}{K^{2n}} - \frac{a_3}{K^3} \frac{C^3}{K^{3n}} - -\right),$$

$$-2bT_n = -2bT\left(1 - a_1 \frac{C}{K^n} - a_2 \frac{C^2}{K^{2n}} - a_3 \frac{C^3}{K^{3n}} - -\right),$$

$$-a = 2(b-1)T - \frac{c}{T^2},$$

$$\frac{c}{T_n^2} = \frac{c}{T^2}\left\{1 + 2a_1 \frac{C}{K^n} + (3a_1^2 + 2a_2) \frac{C^2}{K^{2n}} + (4a_1^3 + 6a_1a_2 + 2a_3) \frac{C^3}{K^{3n}} + \dots\right\},$$

thus

$$T_{n-1} + T_{n+1} - 2bT_n - a + \frac{c}{T_n^2}$$

$$= -T\left[\left\{K + \frac{1}{K} - 2b - \frac{2c}{T^3}\right\} a_1 \frac{C}{K^n} + \left\{\left(K^2 + \frac{1}{K^2} - 2b - \frac{2c}{T^3}\right) a_2 - \frac{3ca_1^2}{T^3}\right\} \frac{C^2}{K^{2n}} + \left\{\left(K^3 + \frac{1}{K^3} - 2b - \frac{2c}{T^3}\right) a_3 - \frac{c}{T^3}(4a_1^3 + 6a_1a_2)\right\} \frac{C^3}{K^{3n}} + +\right].$$

Thus we get

$$K + \frac{1}{K} - 2\left(b + \frac{c}{T^3}\right) = 0,$$

an equation which determines K.

The calculation of the successive coefficients (as well as the value of K itself) may be considerably facilitated by the help of a table of hyperbolic functions*. In order to do this let us write

$$\cosh \omega = b + \frac{c}{T^3} = 1 + \frac{s_1}{3EHT^3}(3T^3 + s_1^2 w^2 E)$$

and

$$\theta = \frac{c}{2T^3} = \frac{s_1^3 w^2}{6HT^3}.$$

* Good tables of these functions by C. Burrau have recently been published; Berlin, 1907. Georg Reimer.

Then $$\frac{1}{2}\left(K+\frac{1}{K}\right)=\cosh\,\omega,$$

and since K must be greater than unity if T_∞ is to be finite, we have

$$K=e^{\omega}.$$

Further $$\frac{1}{2}\left(K^m+\frac{1}{K^m}\right)=\cosh\,m\omega,$$

so that the value of these expressions may be at once obtained by the use of the tables and considerable calculation thereby avoided.

We then have

$$a_2=\frac{\theta}{\cosh\,2\omega-\cosh\,\omega}\,3a_1^2,$$

$$a_3=\frac{\theta}{\cosh\,3\omega-\cosh\,\omega}\,(4a_1^3+6a_1a_2),$$

.....

.....

Since $\cosh\,m\omega-\cosh\,\omega$ is always positive, these coefficients will all be positive if a_1 is so. The value of a_1 is arbitrary, but the presence of the constant C enables us to make it unity without loss of generality. We have therefore

$$a_1=1,$$

$$a_2=\frac{3\theta}{\cosh\,2\omega-\cosh\,\omega},$$

$$a_3=\frac{\theta}{\cosh\,3\omega-\cosh\,\omega}\left\{4+\frac{18\theta}{\cosh\,2\omega-\cosh\,\omega}\right\},$$

.....

.....

Thus

$$T_n=T\left\{1-\frac{C}{K^n}-\frac{3\theta}{\cosh\,2\omega-\cosh\,\omega}\frac{C^2}{K^{2n}}\right.$$
$$\left.-\frac{\theta}{\cosh\,3\omega-\cosh\,\omega}\left(4+\frac{18\theta}{\cosh\,2\omega-\cosh\,\omega}\right)\frac{C^3}{K^{3n}}--\right\}.$$

Now since $T_0=0$ we get

$$1-C-\frac{3\theta}{\cosh\,2\omega-\cosh\,\omega}C^2$$
$$-\frac{\theta}{\cosh\,3\omega-\cosh\,\omega}\left(4+\frac{18\theta}{\cosh\,2\omega-\cosh\,\omega}\right)C^3---=0,$$

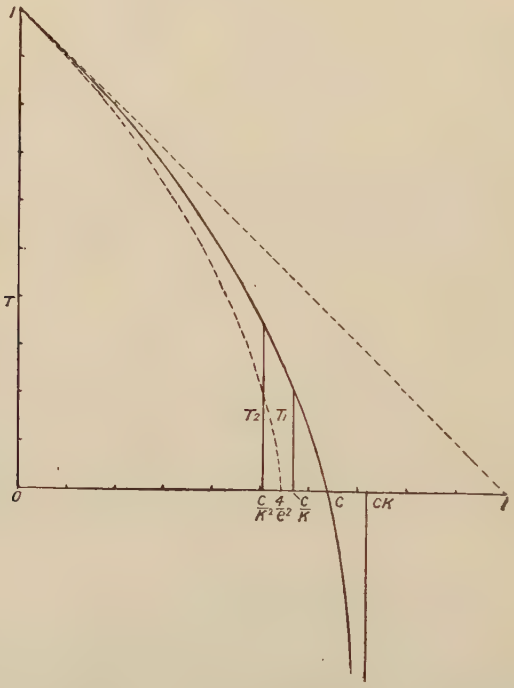
an equation which determines C.

This equation may be best solved by tracing the curve

$$y = 1 - x - \frac{3\theta}{\cosh 2\omega - \cosh \omega} x^2 - \frac{\theta}{\cosh 3\omega - \cosh \omega}\left(4 + \frac{18\theta}{\cosh 2\omega - \cosh \omega}\right) x^3 - - \ldots$$

and noting where it cuts the axis of x.

This series changes sign if we substitute first $x = 0$ and then $x = 1$, so that there must be a root between 0 and 1 if it be convergent. Further, since the terms involving x are all of one sign, there cannot be more than one positive root which must be the quantity we require.



Having in this way found the value of C, we may find the tension in the nth section by substituting $\frac{C}{K^n}$ for x.

The corresponding ordinate will then bear to unity the same ratio that T_n bears to T.

It may be readily shown that the curve meets the axis of y, so as to form an angle of 45°, and has an asymptote parallel to that axis and at a distance from it equal to CK.

If T be represented by unity we shall have

T_1 represented by the ordinate at $x = \frac{C}{K}$,

T_2 „ „ „ „ $x = \frac{C}{K^2}$,

.....

.....

the general form of the curve being that shown in the figure.

It may be proved that

$$\frac{4}{\gamma}\left\{\frac{\sqrt{\gamma}}{\sqrt{1-\gamma}+1}\right\}^{\frac{2}{\sqrt{1-\gamma}}} < C < 1,$$

where

$$\gamma = \frac{s_1^2 w^2 E}{3T^3 + s_1^2 w^2 E}.$$

As a limiting form of the above inequality we have

$$\frac{4}{e^2} < C < 1,$$

or

$$\cdot 54134\ldots < C < 1.$$

For the safety of the system of poles it is requisite that the maximum deflecting force should lie within a certain limit. This maximum deflecting force is clearly equal to T_1.

Section II. *Investigation of Convergency.*

In order to complete our investigation we must examine into the question of the convergency of the series which we have obtained. In order to do so we write

$$\gamma = \frac{2\theta}{\cosh \omega - 1}$$

$$= \frac{s_1^2 w^2 E}{3T^3 + s_1^2 w^2 E}.$$

The equations connecting the successive coefficients then become

$$a_2 = \frac{\cosh \omega - 1}{\cosh 2\omega - \cosh \omega} \frac{\gamma}{2} 3a_1^2,$$

$$a_3 = \frac{\cosh \omega - 1}{\cosh 3\omega - \cosh \omega} \frac{\gamma}{2} (4a_1^3 + 6a_1 a_2),$$

.....

.....

Consider now the expression

$$\frac{\cosh \omega - 1}{\cosh m\omega - \cosh \omega}.$$

We may expand numerator and denominator in powers of ω and the series are always convergent.

Thus

$$\frac{\cosh \omega - 1}{\cosh m\omega - \cosh \omega}$$

$$= \frac{\frac{\omega^2}{2!} + \frac{\omega^4}{4!} + \frac{\omega^6}{6!} + + \ldots}{(m^2-1)\frac{\omega^2}{2!} + (m^4-1)\frac{\omega^4}{4!} + (m^6-1)\frac{\omega^6}{6!} + +}$$

$$= \frac{1}{m^2-1} \frac{\frac{1}{2!} + \frac{\omega^2}{4!} + \frac{\omega^4}{6!} + +}{\frac{1}{2} + (m^2+1)\frac{\omega^2}{4!} + (m^4+m^2+1)\frac{\omega^4}{6!} + +}.$$

Thus $\dfrac{\cosh \omega - 1}{\cosh m\omega - \cosh \omega}$ has the limit $\dfrac{1}{m^2-1}$ for $\omega = 0$ and for $\omega > 0$ it has always a smaller value.

Let γ now be kept constant in our series and let a_1', a_2', a_3', ... be the limiting values of the coefficients as ω approaches zero.

We have

$$a_1' = a_1 = 1,$$

$$a_2' = \frac{1}{2^2-1}\frac{\gamma}{2} 3a_1'^2,$$

$$a_3' = \frac{1}{3^2-1}\frac{\gamma}{2}(4a_1'^3 + 6a_1'a_2').$$

It is thus clear that the coefficients a_2, a_3, a_4, ... are respectively less than a_2', a_3', a_4' ....

Now let

$$v = \sqrt{1-2x}$$

$$= 1 - x - \frac{1}{2!}x^2 - \frac{1.3}{3!}x^3 - \frac{1.3.5}{4!}x^4 - \ldots,$$

the series being convergent when

$$|x| < \tfrac{1}{2}.$$

We shall write this

$$v = 1 - b_1 x - b_2 x^2 - b_3 x^3 - b_4 x^4 - \ldots.$$

We have further

$$\frac{1}{v^2} = \frac{1}{1-2x}$$
$$= 1 + 2x + 2^2x^2 + 2^3x^3 + 2^4x^4 + \dots,$$

which is also convergent when

$$|x| < \tfrac{1}{2}.$$

But
$$b_m = \frac{1.3.5 \dots (2m-3)}{m!}$$

and so we may write

$$\frac{1}{v^2} = 1 + 2b_1 x + \frac{2.4}{1} b_2 x^2 + \frac{2.4.6}{1.3} b_3 x^3 + \dots.$$

We have, however, on the other hand

$$\frac{1}{v^2} = 1 + 2b_1 x + (3b_1^2 + 2b_2) x^2 + (4b_1^3 + 6b_1b_2 + 2b_3) x^3 + \dots.$$

Equating coefficients we get

$$b_2 = \frac{1}{2\left(\frac{4}{1} - 1\right)} 3b_1^2,$$

$$b_3 = \frac{1}{2\left(\frac{4.6}{1.3} - 1\right)} (4b_1^3 + 6b_1b_2),$$

.....

.....

while
$$b_1 = 1.$$

We shall next show that if $m > 2$

$$\frac{4.6.8 \dots 2m}{1.3.5 \dots (2m-3)} < m^2.$$

We have in the first place

$$\frac{2.4.6}{1.3.5} < \frac{7}{2},$$

also
$$\frac{8}{7} < \frac{9}{7},$$

$$\frac{10}{9} < \frac{11}{9},$$

.....

.....

$$\frac{2m}{2m-1} < \frac{2m+1}{2m-1}.$$

Multiplying corresponding sides we get

$$\frac{2.4.6.8\ldots 2m}{1.3.5.7\ldots(2m-1)} < \frac{2m+1}{2}$$

if $m > 2$. Or

$$\frac{4.6.8\ldots 2m}{1.3.5\ldots(2m-3)} < m^2 - \frac{1}{4}.$$

Thus finally

$$\frac{4.6.8\ldots 2m}{1.3.5\ldots(2m-3)} < m^2.$$

For $m = 2$ this inequality becomes an equality.

It follows that $b_2, b_3, b_4, b_5, \ldots$

are respectively greater than

$$a_2', a_3', a_4', a_5', \ldots$$

where $a_1' = 1,$

$$a_2' = \frac{1}{2(2^2-1)}\gamma\, 3a_1'^2,$$

$$a_3' = \frac{1}{2(3^2-1)}\gamma\,(4a_1'^3 + 6a_1'a_2'),$$

.....

.....

and γ is less than unity.

A fortiori

$$b_2, b_3, b_4, \ldots$$

are respectively greater than

$$a_2, a_3, a_4, \ldots$$

and therefore for values of

$$|x| < \tfrac{1}{2}$$

the series

$$y = 1 - x - a_2x^2 - a_3x^3 - a_4x^4 - \ldots$$

converges more rapidly than the series

$$v = 1 - x - \frac{1}{2!}x^2 - \frac{1.3}{3!}x^3 - \frac{1.3.5}{4!}x^4 - \ldots.$$

If we include terms up to that involving x^m the remainder for this latter series is known to be numerically less than

$$\frac{1.3.5\ldots(2m-1)}{m+1!}\,\frac{x^{m+1}}{1-2x},$$

which is accordingly a superior limit to the error involved if we neglect the powers of x above the mth in the expansion of y, provided

$$x < \tfrac{1}{2}.$$

We have in fact for such values of x

$$\sqrt{1-2x} < y < (1-x).$$

If now we take a value of x less than $\frac{1}{2}$ and form the series

$$T(1 - x - a_2x^2 - a_3x^3 - \ldots)$$

it will represent some value of the function T_n where, however, n is unknown and is not necessarily an integer.

We can similarly form the series

$$T\left(1 - \frac{x}{K} - a_2\frac{x^2}{K^2} - a_3\frac{x^3}{K^3} - \ldots\right)$$

which will also converge and will represent T_{n+1}.

Now T_n is positive and the series which represents it is absolutely convergent and therefore its square forms a convergent series with the same radius of convergency. Now it is known (see Bromwich's *Theory of Infinite Series*, p. 216) that the circle of convergence of the reciprocal of a power-series is either the same as that of the original series, or else reaches up to the zero of the given series which is nearest to the origin. Thus $\frac{1}{T_n^2}$ forms a convergent series, since the absolute value of x does not extend as far as a zero of the function T_n.

We thus see that

$$a + 2bT_n - \frac{c}{T_n^2} - T_{n+1}$$

forms a convergent series in x.

But this will represent T_{n-1} which is accordingly given by our series and must be absolutely convergent so far at least.

If now it should so happen that T_{n-1} is positive, we may repeat this process and $\frac{1}{T_{n-1}^2}$ will give a convergent series in x and therefore T_{n-2} is given by a convergent series.

If, however, T_{n-1} is negative we have passed over a root of the equation

$$1 - x - a_2x^2 - a_3x^3 - \ldots = 0,$$

and consequently $\frac{1}{T_{n-1}^2}$ will no longer converge and the series will not give T_{n-2}.

Now the series is continuous within the limits of convergency and therefore there will be a definite value of x which satisfies this equation and is the quantity denoted by C.

Thus KC is the radius of the circle of convergence.

Since $\sqrt{1-2x} < y < (1-x)$ for values of x less than $\frac{1}{2}$ it follows that $\frac{1}{2} < C < 1$.

We have, however, mentioned that

$$\frac{4}{\gamma}\left\{\frac{\sqrt{\gamma}}{\sqrt{1-\gamma}+1}\right\}^{\frac{2}{\sqrt{1-\gamma}}} < C < 1.$$

This result is obtained by considering the series

$$w = 1 - x - a_2'x^2 - a_3'x^3 - \ldots$$

which is the limit of the series

$$y = 1 - x - a_2x^2 - a_3x^3 - \ldots.$$

The series for w can readily be shown to satisfy the differential equation

$$2x^2\frac{d^2w}{dx^2} + 2x\frac{dw}{dx} = (3\gamma - 2) + 2(1-\gamma)w - \frac{\gamma}{w^2}.$$

A solution of this equation may be obtained in finite form for the case where

$$w = 1$$

and

$$\frac{dw}{dx} = -1, \quad \text{when } x = 0.$$

This solution takes the form

$$\frac{\{\sqrt{1-\gamma}\,w^{\frac{1}{2}} + \sqrt{(1-\gamma)w+\gamma}\}^{\frac{2}{\sqrt{1-\gamma}}}(1-w)}{\{w^{\frac{1}{2}} + \sqrt{(1-\gamma)w+\gamma}\}^2} = \frac{\{\sqrt{1-\gamma}+1\}^{\frac{2}{\sqrt{1-\gamma}}}}{4}x,$$

which gives

$$w = 0$$

when

$$x = \frac{4}{\gamma}\left\{\frac{\sqrt{\gamma}}{\sqrt{1-\gamma}+1}\right\}^{\frac{2}{\sqrt{1-\gamma}}},$$

and it is clear that

$$w < y < 1$$

for values of x less than the above*.

It follows that

$$\frac{4}{\gamma}\left\{\frac{\sqrt{\gamma}}{\sqrt{1-\gamma}+1}\right\}^{\frac{2}{\sqrt{1-\gamma}}} < C < 1.$$

* The expansion of w in powers of x must converge up to this point, since there is no singularity of the function w anywhere nearer to the origin.

The complicated form of the solution of the differential equation renders it less convenient than the simple function

$$v = \sqrt{1 - 2x}$$

for estimating the degree of convergence, although the former is valid over a greater range and gives a closer approximation.

Concluding Remarks.

In the foregoing investigation we have treated the problem as a statical one in which an equilibrium condition has been attained.

The extent to which this supposition is permissible will depend upon the circumstances of the break, and the rate at which energy is dissipated in the system. If the wire is suddenly cut the nearest pole will initially be subject to a force equal to T, but this force will quickly diminish before it has produced much effect in bending the pole.

In any case the dynamical problem appears too complicated to lead to trustworthy results even if the purely mathematical difficulties should be overcome.

Another important point is the actual number of poles which must be passed over, counting from the break, before the statical tension T_n may be practically regarded as equal to T_∞ or T.

It is clear that this depends almost entirely on the value of K, and n will be very large if K is nearly equal to unity.

We may estimate the size of n by considering that $\frac{C}{K_n}$ must be negligible in comparison with unity. If quantities less than ϵ be regarded as negligible then we must have

$$\frac{C}{K^n} < \epsilon.$$

That is

$$n > \frac{\log C - \log \epsilon}{\log K}.$$

It may perhaps be worthy of note that a superior limit to the value of T_1 is given by

$$T_1 < T\left\{1 - \frac{\bar{x}}{K} - a_2\left(\frac{\bar{x}}{K}\right)^2 - a_3\left(\frac{\bar{x}}{K}\right)^3 - \cdots\right\}$$

where

$$\bar{x} = \frac{4}{\gamma}\left\{\frac{\sqrt{\gamma}}{\sqrt{1-\gamma}+1}\right\}^{\frac{2}{\sqrt{1-\gamma}}}.$$

On a property of summable functions. By A. C. Dixon, Sc.D., Trinity College.

[*Received* 22 March 1909.]

[*Read* 3 May 1909.]

1. It is a known theorem*, due to de la Vallée-Poussin, that if $f(x)$ is a limited integrable real function of x in the interval $(-\pi, \pi)$ and if

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt,$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,dt,$$

then $\frac{1}{2}a_0^2 + \Sigma(a_n^2 + b_n^2)$ is a convergent series, whose sum is

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \{f(t)\}^2\,dt.$$

It is also known that if $a_0', a_1', \ldots, b_1' \ldots$ are the Fourier constants of a second such function $\phi(x)$, formed from it in the same manner as $a_0, a_1, \ldots, b_1, \ldots$ from $f(x)$, then

$$\tfrac{1}{2}a_0a_0' + \Sigma(a_na_n' + b_nb_n')$$

is a convergent series, whose sum is

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,\phi(t)\,dt.$$

If we suppose $f(x)$ to be periodic, with period 2π, and take $\phi(x) = f(x+y)$, we have

$$a_n' = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t+y)\cos nt\,dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos n(t-y)\,dt$$

$$= a_n\cos ny + b_n\sin ny,$$

and $b_n' = b_n\cos ny - a_n\sin ny$ similarly;

also $a_0' = a_0$.

Thus $a_na_n' + b_nb_n' = (a_n^2 + b_n^2)\cos ny,$

and $$\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)f(t+y)\,dt = \tfrac{1}{2}a_0^2 + \sum_1^\infty (a_n^2 + b_n^2)\cos ny.$$

* For proof and references see Hobson, *Functions of a Real Variable*, pp. 715–7, 723–5; Bôcher, *Annals of Mathematics*, ser. 2, vol. VII. p. 107.

This series is uniformly convergent for all real values of y, and hence its sum is a continuous function of y; putting x for y we have that

$$\int_{-\pi}^{\pi} f(t) f(t+x)\, dt$$

is a continuous function of x, if $f(x)$ is any limited integrable function, with the period 2π.

The first object of the present note is to give a proof of this theorem, independent of the Fourier theory.

A special case is that

$$\operatorname*{Lt}_{x \to 0} \int_{-\pi}^{\pi} f(t) f(t+x)\, dt = \int_{-\pi}^{\pi} \{f(t)\}^2\, dt = \int_{-\pi}^{\pi} \{f(t+x)\}^2\, dt,$$

from which it follows that

$$\operatorname*{Lt}_{x \to 0} \int_{-\pi}^{\pi} \{f(t+x) - f(t)\}^2\, dt = 0.$$

This special case will be proved first, and the more general theorem then derived from it. The method of proof is applied to unlimited, as well as to limited, functions, and thus it is possible to prove de la Vallée-Poussin's theorem for all cases in which it has a meaning.

2. Let $f(x)$ be a limited summable function in an interval (a', b') which includes a, b as internal points, and let U, L be its upper and lower boundaries in (a, b). Divide the interval (L, U) into $n-1$ equal parts, at $a_2, a_3 \ldots a_{n-1}$, and let $a_1 = L$, $a_n = U$.

Let the set of values of x in (a, b) for which $a_{r-1} < f(x) \leqslant a_r$ be called e_r $(r = 1,\ 2 \ldots n)$.

Enclose e_r in a set of intervals Δ_r, not overlapping, and the complementary set $C(e_r)$ in intervals Γ_r, not overlapping, so that Δ_r, Γ_r have a common part $< \alpha$. This can be done, for $f(x)$ is summable.

Let the intervals of Δ_r, in descending order of length, be δ_{r1}, $\delta_{r2} \ldots$ and take an integer p, such that

$$\sum_{m=1}^{p} \delta_{r,m} > \Delta_r - \epsilon \quad (r = 1,\ 2 \ldots n).$$

Let $\epsilon_{r,m}$ denote the part of $\delta_{r,m}$ which is also in Γ_r.

The similarly formed system of sets and intervals in $(a+\theta,\ b+\theta)$, shifted back a distance θ, will serve for the function $f(x+\theta)$; let them be distinguished in this new position by dashes, so that $\delta'_{r,m}$ is $\delta_{r,m}$ in the new position. Of course $\theta < b' - b$.

Then for any x which is common to $\delta_{r,m}$ and $\delta'_{r,m}$ and which is not in Γ_r or Γ'_r,

$$|f(x+\theta)-f(x)|<(U-L)/(n-1).$$

Now the part common to $\delta_{r,m}$ and $\delta'_{r,m}$ is $\delta_{r,m}-\theta$, and of this at most $2\epsilon_{r,m}$ lies in Γ_r or Γ'_r.

Hence $|f(x+\theta)-f(x)|<(U-L)/(n-1)$

over intervals, E, which are together

$$\geqslant \sum_{r=1}^{n} \sum_{m=1}^{p} (\delta_{r,m}-\theta-2\epsilon_{r,m}).$$

Also $$\sum_{r} \sum_{m=1}^{p} \delta_{r,m} > \Sigma\Delta_r - n\epsilon > (b-a)-n\epsilon,$$

$$\sum_{m=1}^{p} \epsilon_{r,m} < \alpha, \quad \sum_{r} \sum_{m=1}^{p} \epsilon_{r,m} < n\alpha.$$

The sum of the intervals E is therefore within

$$n(\epsilon+2\alpha)+np\theta$$

of the length of the whole interval (a, b).

Again, $|f(x+\theta)-f(x)|$ can nowhere exceed $U-L$.

Hence $\int_a^b \{f(x+\theta)-f(x)\}^2\,dx$ does not exceed

$$\left(\frac{U-L}{n-1}\right)^2 (b-a)+(U-L)^2\{n(\epsilon+2\alpha)+np\theta\},$$

the first term arising from E, and the second from the rest* of (a, b).

Here we may give n any fixed value, and put $\alpha=\epsilon=\frac{1}{n^2}$, so fixing p; then if $\theta<\frac{1}{n^2 p}$, the whole will be $<\frac{1}{n}$ of a quantity independent of n, and will tend to zero as n is increased.

Hence $\operatorname{Lt}_{\theta\to 0} \int_a^b \{f(x+\theta)-f(x)\}^2\,dx=0$, when $f(x)$ is any limited summable function.

The same method shews that

$$\int_a^b |f(x+\theta)-f(x)|^g\,dx$$

* Here, as in § 4 below, such end-points of the intervals constituting $\epsilon_{r,m}$ as lie within $\delta_{r,m}$ form an enumerable set, whose measure is zero, so that it does not matter whether they are included, excluded, or counted twice over.

tends to zero with θ, if g is any positive number, and that the same is true of

$$\int_a^b F\,|f(x+\theta)-f(x)|\,dx,$$

where Fx is any function that is finite when x is finite and tends to zero with x.

3. To prove that

$$\int_a^b f(t)\,f(t+x)\,dt,$$

say $\phi(x)$, is continuous, if $(a,\ b)$, $(a+x,\ b+x)$ are both included in $(a',\ b')$, we have

$$\phi(x)-\phi(y)=\int_a^b f(t)\,\{f(t+x)-f(t+y)\}\,dt,$$

and therefore

$$\{\phi(x)-\phi(y)\}^2\leqslant\int_a^b\{f(t)\}^2\,dt\int_a^b\{f(t+x)-f(t+y)\}^2\,dt,$$

in which the first factor is finite, and the second tends to zero with $x-y$ by what has been proved.

Hence $\phi(x)-\phi(y)$ tends to zero with $x-y$, and $\phi(x)$ is a continuous function of x.

If we now put $a=-\pi$, $b=\pi$ and suppose $f(x)$ to have the period 2π, it is easy to find the Fourier expansion of $\phi(x)$.

First, $\phi(x)$ is an even function of x.

Secondly*,

$$\begin{aligned}\int_{-\pi}^{\pi}\phi(x)\cos nx\,dx&=\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}f(t)\,f(t+x)\cos nx\,dt\,dx\\&=\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}f(t)\,f(y)\cos n(y-t)\,dy\,dt\\&=\int_{-\pi}^{\pi}f(t)\cos nt\,dt\times\int_{-\pi}^{\pi}f(y)\cos ny\,dy\\&\quad+\int_{-\pi}^{\pi}f(t)\sin nt\,dt\times\int_{-\pi}^{\pi}f(y)\sin ny\,dy.\end{aligned}$$

Hence the Fourier constants of $\phi(x)$ are

$$\begin{array}{llll}\pi a_0^2, & \pi(a_1^2+b_1^2), & \pi(a_2^2+b_2^2), & \ldots\\ & 0, & 0, & \ldots\end{array}$$

* In calculating $\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}f(t+x)f(t)\cos nx\,dt\,dx$, we first change the order of integration, and this is justified for $f(x)$ and $f(x+y)$ are both summable in any rectangle in the xy plane: the set for which $k>f(x)>l$ consists of lines parallel to the y axis and the set for which $k>f(x+y)>l$ of lines which make equal intercepts on the axes.

if those of $f(x)$ are

$$a_0, \quad a_1, \quad a_2, \ldots,$$
$$b_1, \quad b_2, \ldots.$$

Since $\frac{1}{2}a_0^2 + \sum_1^n (a_n^2 + b_n^2) < \frac{1}{\pi}\int_{-\pi}^{\pi} \{f(x)\}^2\, dx$

for all values of n, the Fourier series is absolutely and uniformly convergent, and since $\phi(x)$ is continuous, the sum of the series is equal to $\phi(x)$, that is,

$$\phi(x) = \tfrac{1}{2}\pi a_0^2 + \pi \sum_1^\infty (a_n^2 + b_n^2) \cos nx.$$

(Hobson, *F. R. V.*, p. 713.)

4. Suppose now that $f(x)$ is not a limited function, but is still summable in an interval (a', b'), which includes a, b as internal points. It will be proved that

$$\int_a^b \{f(x+\theta) - f(x)\}^2\, dx$$

tends to zero with θ, if $\int_a^b \{f(x)\}^2\, dx$ exists.

Take a finite quantity h, which will afterwards be made to diminish without limit. Let $e_r\,(r = 0, \pm 1, \pm 2 \ldots)$ denote the set of points in (a, b) where $(r-1)h < f(x) \leqslant rh$.

Take a whole number n, so that $\int \{f(x)\}^2\, dx$, taken over the set complementary to $e_{1-n} + \ldots + e_0 + e_1 + \ldots + e_n$, is $< \gamma$.

Enclose e_r in a set of intervals Δ_r, not overlapping, and $C(e_r)$ in intervals Γ_r, not overlapping, so that Δ_r, Γ_r have a common part not exceeding $\alpha/(2r+1)^2$. The sum of these common parts for all values of r is then $< 3\alpha$. Then we shall have

$$\Delta_0 + \Delta_1 + \ldots + \Delta_n + \Delta_{-1} + \ldots + \Delta_{-n} > (b-a) - \beta, \text{ where } \beta = \gamma/n^2h^2.$$

Let the intervals of Δ_r, in descending order of length, be δ_{r1}, δ_{r2} ... and take a whole number p, such that

$$\sum_{m=1}^{p} \delta_{r,m} > \Delta_r - \epsilon \quad (r = 0, \pm 1, \ldots \pm n).$$

Let $\epsilon_{r,m}$ be the part of $\delta_{r,m}$ which is also in Γ_r, and let the dash again indicate a displacement θ to the left.

Then for any x which is common to $\delta_{r,m}$ and $\delta'_{r,m}$ and is not in Γ_r or Γ'_r,

$$|f(x+\theta) - f(x)| < h,$$

and as before $|f(x+\theta)-f(x)|<h$ over intervals E which are together

$$\geqslant \sum_{r=-n}^{n} \sum_{m=1}^{p} (\delta_{r,m} - \theta - 2\epsilon_{r,m}).$$

In this expression $\Sigma\Sigma\delta_{r,m} > \Sigma\Delta_r - (2n+1)\,\epsilon$

$$> (b-a) - \beta - (2n+1)\,\epsilon,$$

$$\Sigma\Sigma\epsilon_{r,m} < 3\alpha,$$

and the intervals E fall short by less than

$$\beta + (2n+1)\,\epsilon + 6\alpha + p\,(2n+1)\,\theta$$

of the whole interval (a, b).

The value of

$$\int_E \{f(x+\theta)-f(x)\}^2\,dx < (b-a)\,h^2.$$

For the rest of (a, b) we have

$$\int\{f(x+\theta)-f(x)\}^2\,dx \leqslant 2\int\{f(x)\}^2\,dx + 2\int\{f(x+\theta)\}^2\,dx.$$

To the first term of this, the set where $|f(x)|>nh$ contributes a quantity $<2\gamma$, and the rest a quantity

$$< 2n^2h^2\,\{\beta + (2n+1)\,\epsilon + 6\alpha + p\,(2n+1)\,\theta\};$$

treating the other term similarly, and putting γ for $n^2h^2\beta$, we have

$$\int_a^b \{f(x+\theta)-f(x)\}^2\,dx < (b-a)\,h^2 + 8\gamma + 4n^2h^2\,\{(2n+1)\,\epsilon + 6\alpha + p\,(2n+1)\,\theta\}.$$

Since now $\int_a^b \{f(x)\}^2\,dx$

exists we may take $\gamma = h^2$, thus fixing n.

We can also take $\alpha = \dfrac{1}{n^2}$, $\epsilon = \dfrac{1}{n^3}$,

the latter condition fixing p. Then if $\theta < \dfrac{1}{n^3 p}$, the whole is less than a certain constant multiple of h^2, and can be made as small as we please by diminishing h.

Thus $\underset{\theta\to 0}{\mathrm{Lt}} \int_a^b \{f(x+\theta)-f(x)\}^2\,dx = 0$, if $\int_a^b \{f(x)\}^2\,dx$

exists, $f(x)$ being a function which is summable in an interval (a', b') which includes (a, b).

The same method enables us to prove that

$$\operatorname*{Lt}_{\theta \to 0} \int_a^b |f(x+\theta) - f(x)|\, dx = 0 \text{ if } \int_a^b |f(x)|\, dx$$

exists, by means of the inequality

$$|f(x+\theta) - f(x)| < |f(x)| + |f(x+\theta)|.$$

5. The deduction of the continuity of

$$\int_a^b f(t+x) f(t)\, dt,$$

and that of the Fourier expansion for it, still hold good, and in particular it follows that *the series* $\frac{1}{2}a_0 + \Sigma(a_n^2 + b_n^2)$ *converges to the sum*

$$\frac{1}{\pi} \int_{-\pi}^{\pi} \{f(x)\}^2\, dx$$

even when $f(x)$ *is unlimited, if this last integral exists and is finite.*

6. It follows from the results of § 2 that a necessary condition for a limited function $f(x)$ to be summable is that the superior* integral of $|f(x+\theta) - f(x)|$ tend to zero with θ. The question is at once suggested whether this condition is sufficient, and if not whether a satisfactory definition can be given for the integral of a function which satisfies this condition, but is not assumed to be summable.

* In Lebesgue's sense, see Hobson, *F. R. V.* p. 577.

On certain phenomena of the kathode region. By J. A. ORANGE, B.A., Major Scholar of Trinity College. [Communicated by Professor Sir J. J. Thomson.]

[*Read* 17 May 1909.]

(PLATES IV—IX.)

In the discharge through rarefied gases, the appearances surrounding the kathode seem to be quite independent of the form and situation of the anode, and also of the shape of the containing vessel, provided that the latter does not approach too near to the kathode. For instance, if one arranges any kind of kathode, of linear dimensions up to about 4 cms., in the middle of a glass bulb of about 10 cms. diameter, the various luminous effects associated with the kathode and known as glows and rays (together with the secondary effect of phosphorescence on the walls of the vessel) appear to have no relation to the position of the side tube which contains the anode, but are determined solely by the characteristics of the kathode, the intensity of the discharge, and the pressure of the gas through which it passes.

Throughout the whole range of what are termed low pressures (pressures less than 1 mm. of mercury, say) the surface of demarcation between the Crookes' dark space and the negative glow is one of the most conspicuous features of the region and, moreover, its form is of considerable theoretical interest. The rays which have been observed in the neighbourhood of the kathode fall into two classes, the first, which is now fairly well defined, consisting of *the* kathode rays, rays that are twisted readily by the application of a weak magnetic field, while the second consists of a number of rather indefinite radiations to which Goldstein* has applied the name of 'The Canal Ray Group.'

Both classes of rays are in a sense more difficult of treatment than the glows and dark spaces, for there is no one characteristic by which a radiation may be apprehended, throughout the whole range of pressures at which it undoubtedly exists. For example, kathode rays have a maximum of luminosity at a certain pressure. If the exhaustion is carried further the rays gradually become less luminous, but at the same time another property is coming into evidence, namely, the power of producing phosphorescence on the glass, until finally we have quite invisible rays manifested solely by the vivid phosphorescence at their termination. The canal ray group shows luminosity within a still more restricted range of pressure and gives rise to much inferior phosphorescent effects. By piecing together the rather fragmentary evidences of the

* E. Goldstein, *Verhandl. d. D. Physik. Gesellsch.* (iv), p. 228, 1902; *Phil. Mag.* March 1908, p. 372.

various rays it is possible, nevertheless, to map out their courses with considerable precision.

Much of the work on the distribution of the kathode rays has been performed by Goldstein. His paper, "Ueber den Einfluss der Kathodenform auf die Vertheilung des Phosphorescenzlichtes Geissler-'scher Röhren"* deals very thoroughly with the phosphorescent patterns produced on the glass walls of discharge-tubes by kathode rays arising from concave kathodes of various geometrical outlines.

Somewhat related to the same subject was the work of Campbell Swinton† on the kathode rays from concave carbon kathodes.

A much more recent paper by Goldstein‡ deals with the phenomena of the double or sandwich kathode, while supplementary observations have been made by Kunz§.

The subject has also been treated in a rather different way by Prof. J. J. Thomson‖.

The general method in most of this previous work seems to have been to obtain phosphorescent effects and to establish the paths of the rays from these. The experiments described below were intended to demonstrate the paths as far as possible directly by virtue of the luminosity of the rays themselves.

The primary object of these experiments was to test the view put forward by Prof. J. J. Thomson of the beams of rays obtained with a double kathode composed of equilateral triangles. He observed that well-marked pencils of kathode rays arose from the middle points of the sides of the triangles, while minor ones proceeded from the corners. His explanation¶ was that the rays will occur principally in the regions where the lines of force are straight or nearly straight, since in such circumstances any positive ions produced by the kathode rays will strike the kathode very near the source of those rays, which would not be the case in regions of strongly curved lines of force.

In my first experiment I employed a double triangular kathode of extremely unsymmetrical outline, the lengths of the sides of the triangles being 36, 24 and 16 mm., i.e. $a : b = b : c = 3 : 2$. It was thought that with this arrangement the straight lines of force would occur, not at the middle points of the sides, but at other points which could be determined roughly. Since, however, the field is due not only to the charged kathode but also to a large extent to the distribution of charge throughout the region,

* E. Goldstein, *Wied. Ann.* xv. 1882, p. 254.
† A. A. Campbell Swinton, *Proc. Roy. Soc.* Vol. LXI. (1897), No. 370, p. 79.
‡ E. Goldstein, *Phil. Mag.* March 1908, p. 372.
§ J. Kunz, *Phil. Mag.* July 1908, p. 161.
‖ J. J. Thomson, *Phil. Mag.* Oct. 1908, p. 657.
¶ *Ibid.* p. 666.

the question is more complicated than at first sight appears. For other reasons the experiment failed with regard to the primary object, as will appear in the following account.

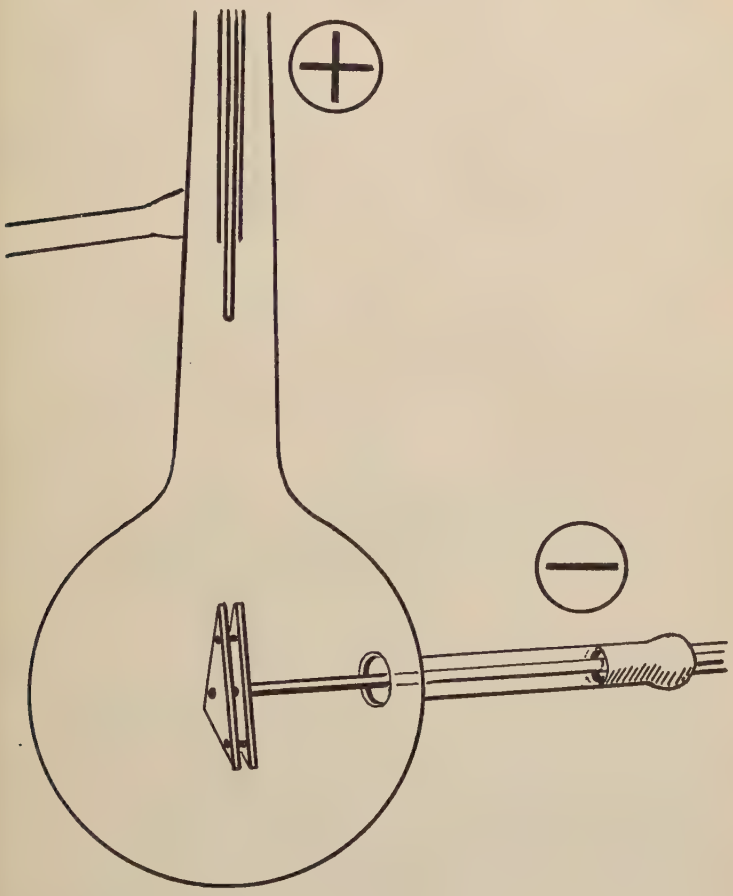


Fig. 1.

Experiment I.

The discharge vessel employed was of the form shown in Fig. 1. It was in continuous connection with a Toepler pump to which the usual phosphorus pentoxide bulb was attached. The gas used was generally air* but for a special purpose hydrogen was introduced

* At times apparatus was used in which many joints, made air-tight by the use of sealing wax and varnish, occurred, and in consequence the gas was largely carbon monoxide.

at a later stage; this was attained by having a small bulb containing dry sodium formate in connection with the apparatus and heating to 250° C. when the hydrogen was required.

The kathode is shown separately in Fig. 2; it was made of aluminium and carried on a lead of the same material protected by a glass tube.

An induction-coil with the common interrupter maintained the discharge and in the usual way a condenser was connected to the interrupter, but it was found necessary (with the higher gas pressures at least) to insert a 'point and plane' spark gap in the secondary circuit, to secure freedom from reversals.

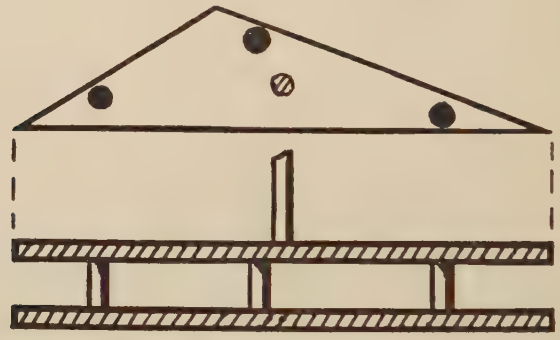


Fig. 2.

At first it was thought that sufficiently accurate records of the appearance of the discharge could be obtained by sketching, but owing to the glare this was painful to the eyes and did not give very satisfactory results. Recourse was then had to photography and as a rule little difficulty was encountered. The camera employed could be extended considerably and consequently could be set up very near to the discharge-tube, so that photographs were obtained of nearly natural size. Using Imperial Ortho plates, exposures of from 2 to 5 minutes were given, the aperture being $\frac{F}{32}$.

Fig. 1, Pl. IV*, shows the peculiar appearance of the kathode region when the Crookes' dark space envelopes only the corners of the triangle. [Cp. Kunz, *loc. cit.* Fig. 16.]

* It should be noted that certain features of these photographs are quite accidental, being reflections from the glass discharge vessel, *e.g.* the eye-shaped patch of light opposite the shortest side of the triangle, and two rectilinear markings opposite the side of intermediate length.

Further, the positive column, which extends a variable distance down into the bulb, should be clearly distinguished.

This would indicate that at this pressure the discharge is taking place almost entirely from the corners of the kathode, the probable explanation being that the electrostatic field at this stage is somewhat similar to the simple field due to a charged triangular conductor. If this is so, those portions of the sides of the kathode which are free from dark space will correspond very well with the regions of weakest electric intensity. But these regions of low intensity are the places where the lines of force are straightest. Hence, when the mean free path in the gas is sufficiently great for the untrammelled formation of kathode rays, the discharge will tend to occur to a greater extent than previously from those parts (according to Prof. Thomson's view given above), and the Crookes' dark space will surround the whole triangle, although it will be somewhat narrower opposite these places. [e.g. ctr. Figs. 1 and 2, Pl. IV.]

In the next photograph, Fig. 2, Pl. IV, the pressure is a little lower and the kathode rays have become visible. They appear to come off from practically the whole lengths of the sides of the triangle, and show maxima of luminosity at certain places. It seems only too probable, however, that the three connecting pins, the ends of which appear in the photograph, play a large part in determining the positions of these maxima. Notice in this and the following photograph, Fig. 1, Pl. V, the dimpled boundary of the Crookes' dark space opposite each of the three sides of the triangle, as referred to above.

The next photograph, Fig. 2, Pl. V, has a very vague appearance because at this stage the kathode rays, although now past their maximum of brightness, are still fairly active photographically, by virtue of their bluish colour, whereas the canal rays, in spite of their prominence visually, are inadequately recorded by the camera owing to their reddish colour. We thus have a medley of two kinds of rays which it would be difficult and unprofitable to unravel at this stage. The beginning of the green phosphorescence due to kathode rays is visible in this photograph in three patches corresponding to the three sides of the triangle, but the next view, Fig. 1, Pl. VI, associated with an increased exhaustion, shows the phosphorescence very well. The blaze of light which masks the kathode in this case, represents the phosphorescence on the glass wall nearest to the camera. The actual space within the vessel is almost free from luminosity, but it is still possible to make out the boundary of the Crookes' dark space and a few traces of the rays.

The last photograph of this set, Fig. 2, Pl. VI, is certainly the most interesting owing to the distinct appearance of the 'canal rays' [so-called by Goldstein*]. It corresponds very nearly to

* E. Goldstein, *Phil. Mag.* March 1908, p. 378.

Fig. 2, Pl. V, but hydrogen was used instead of air in order to obtain canal rays of a more reddish colour than those in air; further, a yellow glass screen, which cut out the blue end of the spectrum, was used in conjunction with orthochromatic plates. It should be remembered that the kathode rays are still just visible at this stage, as shown in Fig. 2, Pl. V, but are suppressed in the present example owing to the photographic method used. [An exposure of 40 minutes was necessary and this gave rise to

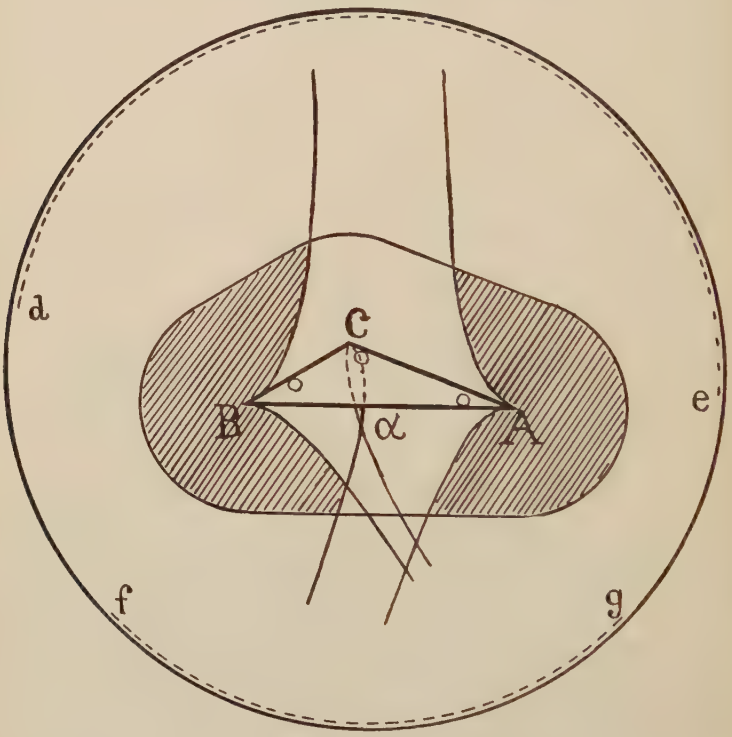


Fig. 3.

difficulties at first owing to the strong tendency of the vacuum to 'harden' with continued running of the discharge. By judiciously fixing the initial pressure of the gas, however, it is possible to obtain steady conditions if the discharge is given a preliminary run.]

The configuration of the canal rays can be made clearer by means of a diagram, Fig. 3*. As the pressure is reduced still

* It may be stated here that subsequent experiments have shown that the three connecting pins do not affect the boundaries of these beams.

further, the rays extend through a smaller distance from the kathode; the boundaries of the beams also change gradually, so that at some distance from the kathode the beams become more constricted.

In previous work, kathodes of symmetrical or nearly symmetrical outline have been used, and consequently what one might call the roots of the beams of canal rays have been hidden by the kathode. The shape of these has however been inferred from markings which are found on the inner surfaces of the kathode, after the discharge has been running for some time*. [*Vide* Fig. 4, taken from Goldstein's paper.] The case described above indicates that the boundaries of the beams are similar in character whether they fall within the outline of the kathode or not.

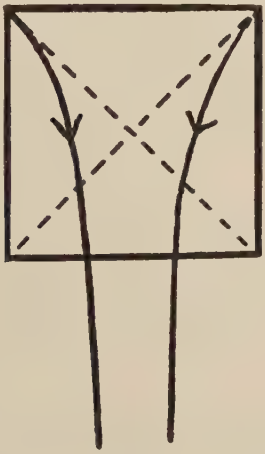


Fig. 4.

But, arguing from the cases of more regular kathodes, Goldstein, *loc. cit.*, and Kunz† have advanced an explanation of the curved boundaries which only holds when the curvature occurs within the kathodic interspace.

Goldstein says, "From the concave boundary of the traces and of the pencils themselves, it may be inferred that the rays proceeding from any side are subject to an attraction due to the neighbouring sides. We may, for example, imagine that in the case of the square the rays are originally convergent along straight lines, somewhat after the fashion indicated by the dotted lines in Fig. 7 [Fig. 4 of the present paper], and that it is in consequence of the attraction exerted on them by the adjacent sides that they assume the actually observed form (shown by the full lines)."

* E. Goldstein, *loc. cit.* p. 377.

† J. Kunz, *Phil. Mag.* July 1908, p. 180.

It would seem now that no such simple explanation based on the geometry of the kathode will suffice. Each beam is symmetrical about the central normal to the corresponding side of the triangle, although the two corners which bound that side differ considerably. Thus we cannot invoke the aid of the electrostatic field; this is shown also by the way in which the two narrower beams cross each other, at α of Fig. 3, as if operated quite independently. The latter fact is also opposed to the supposition that

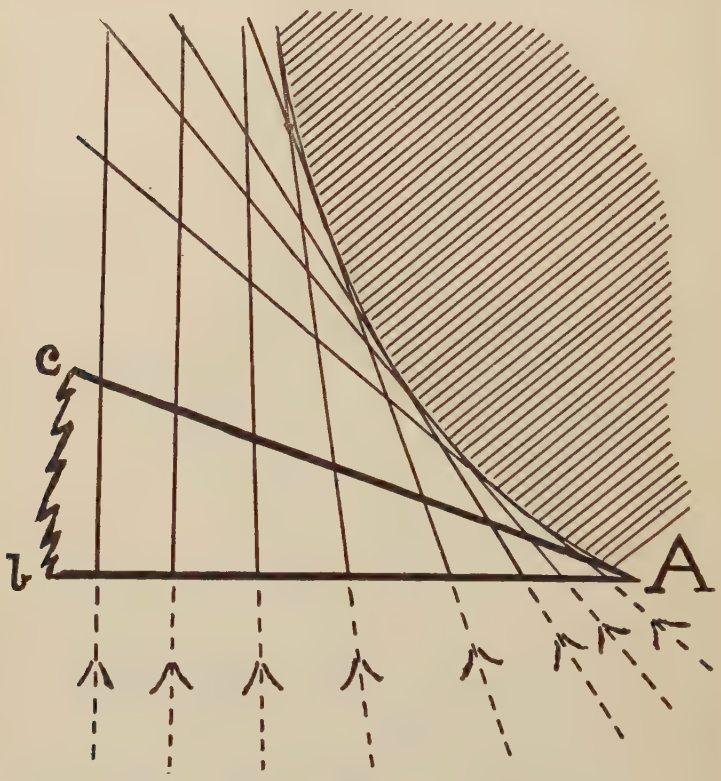


Fig. 5.

the constituent particles of the beams are strongly mutually repulsive, a supposition that might have been advanced to account for the curved boundaries.

The author puts forward the suggestion that the latter are similar to caustic curves, that is, are the envelopes of straight rays, as shown diagrammatically in Fig. 5.

We thus avoid the necessity for supposing the beams to be acted on by peculiar systems of forces, systems which would seem

almost impossible. If, then, we consider the beams to be constituted of straight rays instead of curved ones, the problems which face us are two much simpler ones.

(i) To account for the directions of the constituent straight rays, and

(ii) To account for the occurrence of the luminosity of the rays after, and only after, they have passed through the kathodic interspace.

With regard to the first question, let us consider the canal beam which arises from the longest side of the triangle and runs towards the left of the picture (Fig. 2, Pl. VI).

Adopting Prof. Thomson's explanation of the mutual interdependence of kathode and canal rays, we suppose this beam to have its origin in the region to the right of the kathode. It would be interesting then to know where the kathode rays which, starting from the longest side of the triangle, run to the right, accomplish the ionization which gives rise to the canal rays.

Reference to Fig. 1, Pl. VI, tells us roughly where the kathode rays in question excite phosphorescence on the glass vessel. This has been indicated by the dotted line *fg* in the diagram, Fig. 3. There is thus at least a rough correspondence between the region where ions are produced by the kathode rays and the backward prolongation of the visible canal rays. The same argument seems to apply to the other two canal ray beams. (It is evident however, that one limit of the phosphorescence, at *e*, Fig. 3, is rendered uncertain by the occurrence of the anode side tube at that part of the bulb.)

The second question, that of the luminosity of the canal rays after passing the kathode, remains to be considered. If the canal rays be regarded as positive ions which move up to the kathode with continually increasing velocity and, after passing the kathode, lose their velocity by the reverse process, it is indeed difficult to account for their luminosity occurring only *after* passing the kathode. It can be shown quite readily, however, that the positive ions are much more likely to be neutralized when moving away from the kathode than when approaching it. For in the former case they encounter negative ions moving in nearly the same direction, while in the latter they encounter them travelling in nearly the opposite one. Thus if recombination is associated with luminosity, as is probable, we can see why the canal rays are visible only after passing the kathode. We may consider the recombination to occur entirely within a short distance of the kathode, in which case the visible length of the rays can be explained by supposing the ions to vibrate for about 5×10^{-8} sec., or we may consider that recombination occurs throughout the whole visible length of the beam.

The former view seems better as yet, because

(1) the luminous beams do not seem to be under the influence of the electrostatic field, and

(2) Kunz* states that "Prof. J. J. Thomson, who applied a very strong (magnetic) field to these rays, could find so far no deflexion at all." (The rays referred to were obtained with an equilateral triangular double kathode.)

It was next proposed to devise apparatus in which the two kathode plates were not connected by pins, but carried on separate leads, and since we did not know definitely by what amount the plates should be separated, to secure the best results, it was decided that this should be susceptible of adjustment while the discharge was running.

Experiment II.

The apparatus is represented in Fig. 6. One anode A and one kathode B are supported from above while the other kathode C and the anode D, which are mechanically connected by ebonite, E, can be moved up and down by means of a barometer column with moveable reservoir. (The float H, which is used in this process, makes contact between the kathode lead and the mercury of the column. The anode lead is attached to the fine spiral of wire G, so that it is readily moveable.)

The kathode plates were similar to those used in the first experiment but were bevelled as shown in the figure, with the object of simplifying the section of the kathodes and hence of the electrostatic field. The two plates, or rather their connections, were joined externally by a wire. The two anodes were used in determining whether the kathode region were quite independent of the situation of the anode. On observing that this was so, the lower anode was disconnected and the upper one alone was employed throughout.

Although it was possible to obtain views of the kathode region similar to those of the first experiment, photography in this direction, i.e. vertically downwards, was out of the question owing to the inconvenience of the arrangements. It could be seen, however, that the canal rays formed beams very similar to those obtained with the original double kathode.

The canal rays are most conspicuous when the separation of the plates is about 3 mm., but they are observed quite easily with separations of from 1 to 15 mm. With the larger degrees of separation the beams are more convergent than those described in Expt. I.

* J. Kunz, *loc. cit.* p. 177.

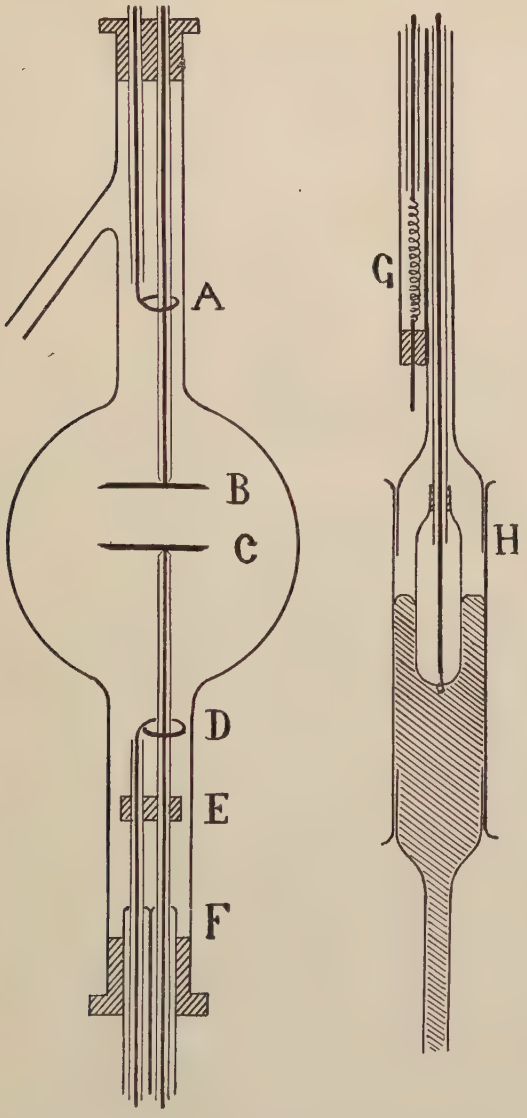


Fig. 6.

The apparatus is shown in two parts, the upper one lying to the left.

The inconvenience of the apparatus in the respect mentioned above was amply compensated for by the ease with which the region could be viewed in other directions (that is to say, in horizontal directions). Of the appearance in one of these directions, viz. that showing the kathodes lengthwise, there is little to say. Fig. 6, Plate IX, shows the one stage of any interest; the form of the Crookes' dark spaces in the kathodic interspace is peculiar, but it seems to be quite in accordance with the views previously stated.

Examination of the region in the remaining principal direction revealed much more interesting features (Pls. VII and VIII), and what follows refers entirely to appearances viewed in this way. (i.e. by looking horizontally so that the line of sight is parallel to the longest sides of the kathodes, these longest sides lying to the left, while the obtuse angles of course fall on the right.)

The photographs cited are selected from a large number of records taken. In Figs. 2 and 7, Pl. VIII, the positive column intrudes but does not seriously mar the view; it would appear to have no direct connection with the phenomena discussed in this paper. The Crookes' dark spaces are in all cases clearly defined, and it is noticeable that while they are still unfused, as in Figs. 1, 2 and 6, Pl. VII, the thickness of the dark space is greater opposite the remote faces of the kathodes than it is over the interfaces.

The striking feature, however, in most of this series, is the distribution of kathode rays in the form of a very sharply-defined cross of St Andrew's. The pencils are especially well defined on the left side, that is in connection with the longest sides of the triangles. On the right some slight confusion occurs owing to the way in which this part of the triangle is viewed. The system of rays really forms two sheets, somewhat like slightly truncated pyramids, the base of the lower one being visible at the lowest pressures as a phosphorescent trace which encircles the bulb like a line of latitude (Figs. 2 and 7, Pl. VIII). The pencils are remarkably restricted transversely at the lower pressures, e.g. Fig. 7, and, moreover, they pursue a slightly curved path. In all cases where the two Crookes' dark spaces either adjoin each other or are fused together, kathode rays are also emitted abundantly along the median plane of the interkathodic space, e.g. Fig. 1, Pl. VIII.

I think we may correlate the oblique rays with a phenomenon described by Kunz*. Kunz used as kathode a hollow cylinder and observed on the containing vessel phosphorescent rings opposite the mouths of the cylinder. If the rays described above correspond to those producing Kunz's rings, then the latter's explanation of the phenomenon must be wrong, as a reference to his diagram† will show.

* J. Kunz, *loc. cit.* pp. 164 *et seq.* † *Ibid.* p. 170.

The way in which the pencils change with the gas pressure and the separation of the plates is easily summarized qualitatively:

The inclination of the rays to the plane of either kathode

(1) increases with the distance separating the plates, and

(2) increases with the pressure.

There is a transition at a certain stage of exhaustion; at the higher pressures the rays being sharply bounded on one side only, while at lower pressures they form true sheets which are well-defined on both sides.

[Figs. 1 and 6, Pl. VII, Fig. 3, Pl. VIII are typical of *higher* pressures,
„ 2 and 7, „ „ „ „ „ rather lower „
„ 3 „ is „ „ *transitional* „
„ 4 „ „ „ „ still lower „
„ 5 „ Fig. 6, Pl. VIII are „ „ *low* „]

Figs. 4 and 5, Pl. VIII are prints from a single negative. The first shows details of the rays near the kathodes very well while the second shows up the general appearance. At this stage the two "oblique" beams on the left as well as the median beam are practically horizontal.

In seeking an explanation of the pencils, let us first disregard the *origin* of the rays and consider only their directions and curvature. If we except the comparatively high pressures—at which the rays are ill-defined—it may be observed that the asymptotes of the rays always pass approximately through the central point of the kathode interspace. This would imply that the electrostatic field is determined largely by the existence of free charge in the region containing the kathodes, as distinct from the charge on the kathodes themselves, and, in fact, that in the cases considered there is a region of negative charge corresponding roughly with the brightly luminous part of the interspace.

It has been shown by various workers that the electric intensity in the ordinary Crookes' dark space is considerable at all points but increases rapidly as the kathode is approached; Schuster's* experiments indicate that there is a distribution of free positive electricity in the dark space, such that the volume density decreases in geometrical progression as the distance from the kathode increases in arithmetical progression†.

In such circumstances, the kathode rays in their progress away from the kathode will acquire a high velocity in the initial stages of their motion, and subsequently will be affected but little by the weaker electrostatic fields which they traverse.

Consider the rays from the opposed faces of the two kathodes in our experiment. The rays from each must in due course enter

* Schuster, *Proc. Roy. Soc.* XLVII. p. 526, 1890.

† J. J. Thomson, *Conduction through Gases*, 2nd ed. p. 541.

the dark space of the óther. The particles will lose kinetic energy as they approach the second kathode, most of the loss occurring very near the latter owing to the character of the field, and they will tend to drift a little in the direction parallel to the kathodes. They will not quite reach the second kathode since they must have lost some energy by ionization *en route*; they may subsequently travel back towards the kathode of origin and continue to swing across the interkathodic space, but always trending laterally outwards. Finally, they will have travelled to the right or left considerably and when their velocity is a miuimum, i.e. when near one of the kathodes, they will be borne out of the interspace in a manner determined by the electrostatic field.

As was stated above, the visible paths of the pencils indicate the presence of free negative charge in the middle of the interspace, a view which can be supported as follows:

For two reasons there is much ionization (and hence luminosity) in the interspace.

(1) The kathode rays are not as swift here as elsewhere (since they start in the weaker field evidenced by the thickness of the dark space), and

(2) the kathode rays, and also some proportion of the negative ions to which they give rise, oscillate about the median plane, so that at any instant there is an exceptionally large amount of ionization proceeding.

The concentration of negative ions must be high in the neighbourhood of the median plane and many of them doubtless travel out along that plane and constitute the horizontal sheet of rays seen in the photographs.

In summary then:

We regard the electrostatic field as being due mainly to three things.

(1) The actual charge on the kathodes; the effect of this cannot be great, since the covered supports of the kathodes produce little or no visible effect.

(2) The distribution of positive charge so that it is especially abundant very near to the surfaces of the kathodes, as assumed from Schuster's experiments. (*loc. cit.*)

(3) An accumulation of negative charge about the median plane of the interspace. This must be postulated, I think, to account for the considerable divergence of the oblique ray sheets and their peculiar curvature.

The oblique sheets are regarded as the kathode rays arising primarily from the kathodic interfaces and losing energy by ionization of the gas as they travel backwards and forwards between the kathodes. As long as the particles, in their surgings,

arrive within close range of the kathodes, they will reverse their paths in such a powerful field that they will gain sufficient impetus to travel in a fairly direct line towards the other kathode. When they have lost so much energy that they fail to reach the high intensity zone at the end of one of their oscillations, they will be dominated much more by the influence of the postulated region of negative charge at the centre of the system, and will be repelled out to form the oblique sheets.

The difference between the sheets at the higher and lower pressures can be explained simply thus: At the stage shown in Fig. 1, Pl. VII, the kathode rays produced at the interfaces will lose much energy in crossing over (because of the higher gas pressure chiefly).

Consequently the rays when they start out laterally will be at various distances from the kathode; that is, they will have lost very different amounts of energy, according to the number of times they have crossed over. There will, however, be a minimum of energy lost corresponding to the case of a single traverse. The sharply defined boundary of the oblique ray sheets can be taken as the expression of this minimum. The other boundary of the rays is at this stage scarcely marked at all, owing to the indefinite range of possible energy losses of the particles in their preliminary movements. Probably, also, the spreading of the rays is conditioned partly by the great range of intensity of field as we cross the narrow Crookes' dark space.

At lower pressures (e.g. Fig. 5, Pl. VII), the particles will lose little energy in their surgings, and so when finally ejected they form a confined sheet. The large dark spaces characteristic of these lower pressures are associated with more uniform fields; this also will be favourable to the production of confined sheets of rays.

To test these views of the phenomena a few experiments were made with the two kathodes at different potentials. This was attained by having the lower plate connected as before to the coil terminal, while the upper plate was connected to it through a considerable electrolytic resistance. [The photographs obtained are unfortunately slightly out of focus (except Figs. 1 and 3, Pl. IX), but they are clear enough to show the points referred to.] The series, Figs. 1 to 5, Pl. IX, shows various combinations of potentials and pressures. In Fig. 1 the upper plate was quite disconnected externally. In the other cases it was connected through various amounts of resistance.

The first thing noticeable is the inequality of the two Crookes' dark spaces, the upper one varying continuously with the potential and apparently vanishing when the plate is disconnected. This

seems to confirm the observations of Schuster* on the relation between the thickness of the dark space and the strength of the current.

As regards the sheets of rays, the appearances in these experiments agree fairly well with the explanations advanced above. In Fig. 1, Pl. IX, the kathode rays from the lower plate strike the upper one violently. Some of the rays graze the edge of the plate and produce the sharp, nearly vertical shadow which may be noticed. The upper plate emits secondary kathode rays, which are not sufficiently powerful to travel far towards the lower plate, but are very soon thrown out sideways as three highly luminous, but ill-defined sheets, corresponding to the three sides of the triangle.

In Figs. 2 and 4, Pl. IX, the form of the lower ray sheet is clear and it will be noticed that in these cases it is much further away from the kathode than in any of the examples on Pls. VII and VIII. This is probably due to the fact that rays from the upper (low potential) plate will not have sufficient energy to approach near to the lower kathode and hence when they finally start moving sideways they are at a considerable distance from it. No such feature is associated with the upper plate. The rays starting from near the latter form two distinct beams (two on each side that is) in most of these examples. (e.g. Figs. 2 and 4, Pl. IX.)

These two beams might be explained as being due to

(i) the primary rays from the upper plate which have travelled back after visiting the neighbourhood of the lower one, and

(ii) the secondary rays from the upper plate due to the impact of the primary rays from the lower one.

The change which has occurred in the position and direction of the "median" beams, owing to the alteration of the potential of the upper plate, is quite what one would anticipate. (Fig. 2, Pl. IX.)

In conclusion, I wish to thank Prof. Sir J. J. Thomson for his valuable suggestions and kindly interest in these investigations.

My gratitude is due also to Mr E. Everett for his ready assistance at various times.

* Schuster, *loc. cit.* p. 556.

Fig. 2

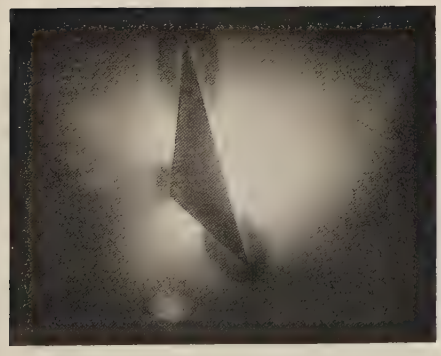


Fig. 1

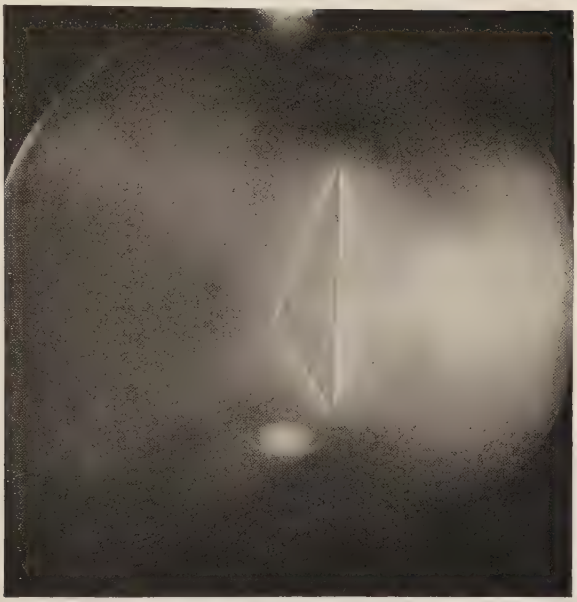


Fig. 2

Fig. 1

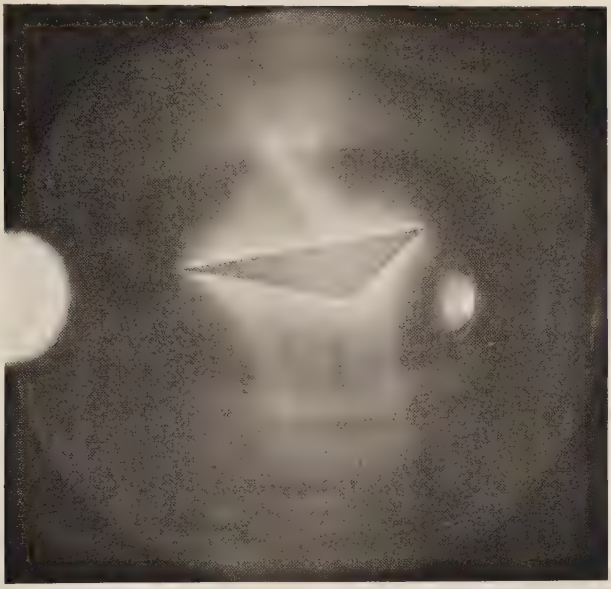


Fig. 2

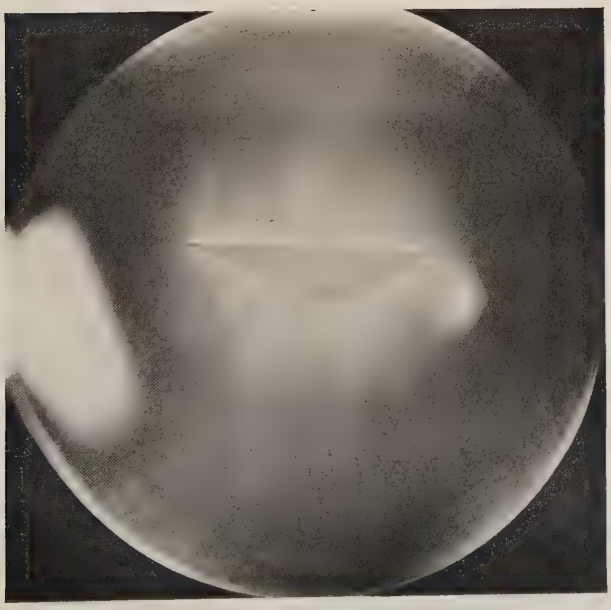


Fig. 1

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 1

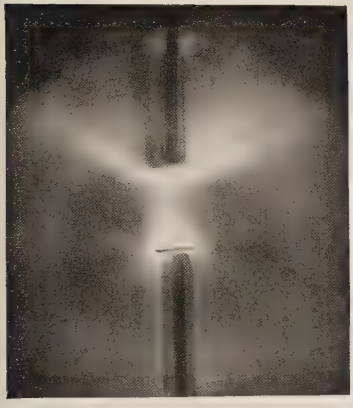


Fig. 2

Fig. 3

Fig. 4

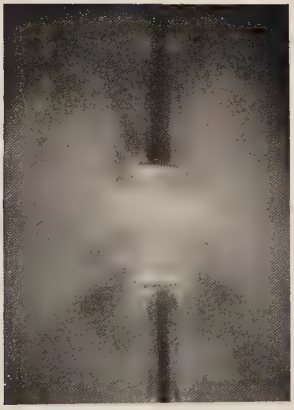
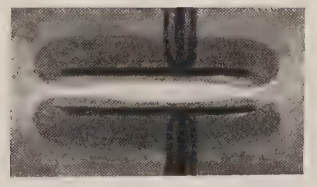


Fig. 5

Fig. 6



ERRATUM

P. 233, line 10 from top, *for* species *read* examples.

Note on two new Leeches from Ceylon. By W. A. Harding, M.A., Peterhouse.

[*Read* 3 May 1909.]

The leeches here described were collected in Ceylon by Miss Muriel Robertson, who very kindly sent them to me for examination, together with information respecting their habits and the hosts upon which they were found.

The material placed at my disposal was not in a condition very favourable for the examination of external features. It comprised examples of two species hitherto unrecorded, of which a brief description is given below.

1. *Ozobranchus shipleyi*, nov. sp.

Eleven species, fixed in corrosive sublimate and preserved in alcohol, were examined. The largest specimen was 5 mm. in length; this species therefore is one of the smallest of known leeches.

Body depressed, divided into two distinct regions and generally resembling *Branchellion* in form. Complete somite formed of three rings. Abdominal region of the body provided with eleven pairs of digitate branchiae, the posterior smaller and less branched than the anterior pairs.

Anterior sucker not distinct from the body. Acetabulum cupuliform, shallow, centrally-attached and very large, its diameter being equal to nearly one-third of the total length of the animal.

Eyes two (?).

Habitat:—Fresh-water; parasitic on the Ceylon terrapin, *Nicoria trijuga*.

I have associated with this species the name of Mr A. E. Shipley, to whom I am indebted for much kind help and advice.

2. *Glossiphonia ceylanica*, nov. sp.

The species is founded upon one specimen preserved in alcohol. This had assumed a brown colour, somewhat lighter upon the ventral surface. Irregular, black, median and marginal patches and traces of four longitudinal brown stripes were seen on the dorsal surface.

Head distinct from the much depressed and slender body. Six eyes arranged in two sub-parallel rows, the first pair approximate.

Counting the first oculiferous ring as the first ring, the eyes occur on the first, second and fourth rings; the male genital opening lies between the 26th and 27th rings and the female opening between the 29th and 30th rings (?); the anus lies between the 68th and the 69th (and last) ring; and rings 66, 67 and 68 are partly double.

Length, in alcohol, about $7\frac{3}{4}$ mm.

Habitat:—Ceylon, in fresh-water; parasitic on the soft-tortoise, *Emyda vittata*.

Note on the abnormal pair of appendages in Lithobius.

By F. G. Sinclair, M.A., Trinity College.

[*Received* 3 June 1909.]

The presence of an abnormal extra pair of appendages is of interest to me, as I have examined a great number of abnormalities in the Myriapoda, but I am inclined to give a different interpretation to the facts from that given in the note referred to by Doncaster*, in which the appendage is taken to be a reduplication of the poison claw.

The abnormalities which are very frequent in Myriapods fall, in all cases that are familiar to me, into two classes. A reduplication in the transverse axis of the body or appendage, or a reduplication in the longitudinal axis of the body or appendage. The former is the more frequent—an example being a reduplication of part of the antenna or leg forming a bifurcation.

The description of the small processes described as teeth, jointed to the base of the appendage and the drawing of them, remind me strongly of two small processes which are commonly present on the end of the larval appendages in Myriapods. They are moveable by muscular fibres, and, as the animal grows up, one of them usually degenerates and disappears while the other changes to a claw or spike.

In the reduplication of a part in the longitudinal axis I should think it more likely that the abnormal part would be posterior to that of which it was a copy, following the order in which the normal segments are formed.

It results from these considerations that I take the abnormal appendage described in the note to be a reduplication, not of the poison claw, but of the second maxilla. The basal joint with the two small processes I believe to be an imperfectly formed transverse reduplication of the other branch, so that if the development had gone a step further there would have been two similar branches on either side.

According to my view the undivided plate at the base would be—not the coalesced basal joints of the poison claws, but a metamorphosed sternal plate.

Of course if a poison gland and duct had been found present it would have upset this view, but that does not seem to have been the case. It is worth remembering that in the Myriapoda portions of proliferous tissue from which complete segments are formed normally are present till comparatively late in life. The complete number of segments in Diplopoda is not present for a long period.

* *Proc. Camb. Phil. Soc.* xv. p. 178.

On a specimen of the cone Calamostachys binneyana (*Carr.*). By H. Hamshaw Thomas, B.A., Downing College. [Communicated by Mr E. A. Newell Arber.]

[*Read* 3 May 1909.]

Forty years have now elapsed since the first description of the cone *Calamostachys binneyana* was published*, but hitherto no petrified specimen has been found showing its connection with vegetative organs of any kind.

In December last, I received a specimen of a cone with four whorls of Calamite leaves attached to the base, derived from the Halifax Hard Bed at Huddersfield. The cone was at least 14 mm. long and about 5 mm. broad. It was cylindrical in shape and eight whorls of sporangiophores are preserved.

The structure of the cone agrees closely with that of specimens previously described†, and the axis and sporangiophores do not present any unusual features. Several of the bracts are very well preserved and show some new points of interest. As usual, they are coherent at the base to a disc, and, on becoming free, turn sharply upwards. Each is provided with a vascular bundle, the centre of which is occupied by a group of very small xylem tracheides surrounded on all sides by small thin-walled cells, probably phloem. The bundle may possibly therefore be concentric in structure and not collateral as formerly supposed, though it is very difficult to arrive at any certain conclusion. The free portion of the bract bears a striking resemblance to a leaf. It consists largely of sclerenchymatous fibres on the adaxial side, while a layer of rudimentary pallisade tissue forms the other side. The small bundle occurs in the centre of the bract, and on the inner side of the pallisade tissue, a zone of cells with dense black contents is found. There is a thin epidermis on the outside.

At the base of the cone, there is a whorl of bracts resembling the leaves still more closely. Immediately below this we have a whorl of appendages differing completely from the leaves, bracts, and sporangiophores, and seen in longitudinal section as a broad

* Carruthers, W. "On the fruit spike of Calamites." *Journal of Botany*, vol. v. 1867.

† Binney, E. W. "Observations on the Structure of Fossil Plants." *Mem. Palæontographical Soc.* p. 24. 1868. Williamson, W. C. "On the organisation of the Fossil Plants of the Coal measures," Parts iv. x. xi. xv. *Phil. Trans.* 1873, 1880, 1881, 1889. Hick, T. "On *Calamostachys binneyana*." *Proc. Yorks. Geol. and Polyt. Soc.* vol. xii. pt. iv. 1893. Williamson, W. C. and Scott, D. H. "Further Observations on the organisation," etc. *Phil. Trans.* 1894.

ring of parenchymatous tissue. This may be compared with the annulus of the cones of modern Equisetums. It contains however some rudiments of vascular tissue. The discovery of this organ further illustrates the similarity between the Palæozoic Calamites and modern Equisetaceæ.

Below the "annulus" there are four whorls of foliage leaves. These are small linear, rather falcate structures, about 3 mm. long, less than 1 mm. broad, and free at the base. They are almost identical in their size, form, and arrangement, with those known from impressions as *Calamocladus grandis* (Sternberg). The structure of the leaves differs somewhat from those which have been already described*. The upper portion consists of a large strand of sclerenchymatous fibres running the whole length of the leaf but smaller than that occurring in the bracts. A small bundle occupies the centre of the leaf, and below, there is a band of cells with dense black contents. The latter is crescentic in transverse section and separates the pallisade tissue from the bundle and fibres; its function is not yet certain. The pallisade tissue is greater in amount than in the bracts, and consists of radially elongated cells with thin walls and numerous intercellular spaces. On its outside, there is a narrow epidermis. Stomata are seen in some places, and were probably slightly sunk below the surface of the leaf.

The internodes of the stems between the leaf whorls seem to have been covered with small hairs, which also occur on the lower part of the cone. They are preserved as small, dense, black bodies, and may perhaps have been glandular.

The form of the cone and leaves shows that it is probably identical with the impressions named by Weiss† *Paracalamostachys williamsoni*, and also with Zeiller's *Calamostachys grandis*‡. It seems possible that the name *Calamostachys binneyana* should be applied as indicating a type of structure rather than a species, and hence the name *Calamostachys grandis* is perhaps the best designation for the cone described here.

The discovery of this specimen is of interest for several reasons. It furnishes for the first time direct evidence that this well-known cone was borne on a Calamite plant. The question of its affinities—at one time debated with keenness—was practically settled some years ago, but the stem and leaves of the plant have

* Hick, T. "On the structure of the leaves of Calamites." *Mem. and Proc. Manchester Lit. and Phil. Soc.* vol. IX. p. 179. 1895.

† Weiss, C. E. "Steinkohlen-Calamarien II." *Abhandlungen zur geologischen Specialkarte von Preussen*, Band v. p. 193. 1884.

‡ Zeiller, R. *Flore fossile du bassin houiller de Valenciennes*, p. 376. Pl. 59, Figs. 4—7. Paris, 1886.

hitherto remained unknown. The leaves have now been discovered, and there is little doubt that, before long, a petrified stem bearing leaves of this kind will be found. The "annulus" and the hairs on the stem are here described for the first time.

The great similarity of structure between the foliage leaves and the bracts of the cone is also a point of some interest, as indicating the foliar nature of the latter. This, together with the probable occurrence of an annulus in Calamostachys, is of some importance in the discussion of the morphology of the cones of the Equisetales.

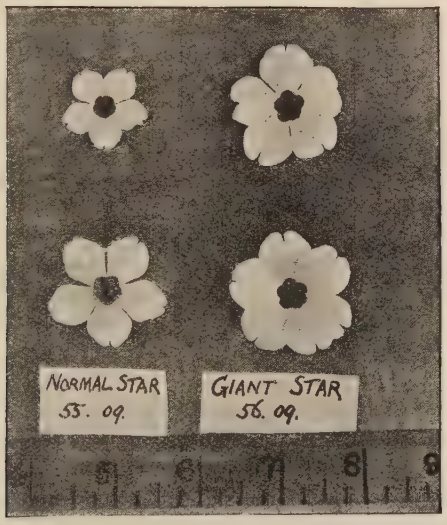
My best thanks are due to Mr Arber for his kind advice and assistance in this work.

Note on the Histology of the Giant and Ordinary Forms of Primula sinensis. By R. P. Gregory, M.A., St John's College.

[*Read* 17 May 1909.]

(Plate X.)

During the last few years the Giant form of *Primula sinensis* has become well established in cultivation, and is now known in many of the numerous horticultural varieties of this genus. Nothing definite is known as to the origin of the form, nor can we at present say anything as to the behaviour of the Giant and Ordinary characters in cross-breeding, although experiments are now in progress in this connexion.



Flowers of the two plants described. In each case the upper flower is newly opened, the lower is older. Scale divided in inches. From a photograph.

Plants of the Giant form are of a somewhat coarser general habit than those of the usual form. As compared with the latter, they are characterized by the stoutness of their stems and petioles, and by their larger leaves and flowers. In Giants of the "Stellata" type, the petal-lobes are broad, and, unlike those of the ordinary form, overlap one another to a greater or less degree (see text figure). My experience of the seeds of the Giant form is limited to those obtained this year from a few plants; they were, on the average, larger than those of the ordinary type. I am indebted

to Mr Leonard Sutton for further information on this point; he writes: "With regard to the size of the seeds, we do not find much difference in the Stellata forms, but in the Florist's type, seed of the Giant varieties, though variable from year to year, is always larger and flatter than that of the smaller flowered sorts."

At Professor Bateson's suggestion I have made observations with a view to comparing the nuclei of the two forms. The plants chosen for the purpose were (1) a Giant White Star with dark red stems, and (2) a White Star of the ordinary habit, but closely resembling the Giant in all other characters. For both of these plants I am indebted to Messrs Sutton of Reading.

The primary object of the investigation was to discover whether there might be a difference in the number of the chromosomes in the two forms, an idea suggested by the results of Miss Lutz* and Gates† in *Oenothera gigas*. Upon this point the answer is definitely in the negative; the number of the chromosomes is the same in both forms of *Primula sinensis*, viz. 12 (reduced) and 24 (somatic). The reduced number of chromosomes is shown with quite diagrammatic clearness in sections transverse to the spindle at metaphase or anaphase of either of the two meiotic divisions (figs. 1 and 2). In the somatic divisions the chromosomes are more closely crowded and their precise number is less easy of determination; but several counts have given numbers ranging from 22 to 24, and there can be no doubt, I think, that the number may be correctly stated at 24. I have no evidence which suggests that there is any tendency to variation in the number of chromosomes which occur in the somatic mitoses found in various regions of the floral organs.

But although the two forms are alike in the number and form of the chromosomes, they give very distinctly the impression that a difference exists between them in the size of the chromosomes (at any rate as they appear at metaphase of the heterotype division), in the size of the resting nuclei, and correspondingly in the size of entire cells.

In testing this point the comparison between the two forms was made by means of figures drawn to exactly the same scale with the help of a camera lucida, and by measuring the resting nuclei with an ocular micrometer, the measurements being taken along, and at right angles to, the long axis of such nuclei as were not circular in outline. The drawings and measurements were made upon material which had been carefully fixed, and showed little or no signs of shrinkage during the process. The material was cut in paraffin into sections sufficiently thick to contain entire nuclei which had been untouched by the knife of the microtome. In a

* "A Preliminary Note on the Chromosomes of *Oenothera Lamarckiana* and one of its Mutants, *O. gigas*." *Science*, N. S. 26, p. 151, 1907.

† "The Chromosomes of *Oenothera*." *Ibid.* N. S. 27, p. 193, 1908.

few test cases the vertical measure of the nucleus was estimated by focussing on to the upper and lower walls of the nucleus by means of the micrometer fine adjustment, taking the mean of three readings. In each of these cases the measure so obtained approximated very closely to those taken in the horizontal plane.

It is of course desirable that the tissues selected for comparison should be composed of cells which approach, as nearly as may be, to uniformity of shape and size. Such tissues are to be found in various epithelial layers, notably in the very regular layer of cells which surrounds the embryo sac in the mature unfertilized ovule. This layer is the "couche de revêtement" of Warming* and Vesque†, and the "tapetum" of Billings‡, all of whom have recorded its existence in species of *Primula*. Other layers whose nuclei have been compared are the epidermis of the young stigma as seen in approximately median longitudinal section, and the epithelia of the young developing ovules. Equally favourable for purposes of comparison are the nuclei of the pollen mother cells, since the individual cells at the same stage of development are remarkably uniform in size, and the stages are marked by easily recognizable characters of the nucleus, so that a just comparison can be made. I hoped to test the matter further by measurements of the cells of the young embryos as they developed within the seed, but owing to the resistance offered by the thickening integuments to the penetration of the fixing fluids I was unable to obtain sufficiently good material. The apex of the young radicle may perhaps allow a further test, when it becomes available later in the season.

In dealing with the size of the chromosomes of the two forms, I have had to be content with a comparison of the drawings made with a camera lucida, since the chromosomes are too small to be measured with any degree of accuracy by means of an ocular micrometer. Figs. 1 and 2 show the groups of chromosomes of the Ordinary and Giant forms, respectively, as they appear at metaphase of the heterotype division. The figures represent the view transverse to the spindle; the outlines of the chromosomes were carefully drawn, and I think the difference between the two is not exaggerated. Spindles seen in longitudinal view show that the two forms had reached as nearly as possible identical stages; a similar slight difference in the size of the chromosomes of the two forms is apparent also in these longitudinal views. In the somatic mitoses which I have seen the chromosomes were too crowded to permit of fair comparison.

* "De l'Ovule." *Ann. des Sci. Nat.*, Ser. VI. Tome 5, p. 235, 1878.

† "Sur le Développement du Sac embryonnaire." *Ibid.*, Ser. VI. Tome 8, p. 360, 1879.

‡ "Beiträge z. Kenntniss d. Samenentwickelung." *Flora*, 88, p. 277, 1901.

I give in the table (p. 243) some measurements taken from edge to edge of the group of chromosomes in the equatorial plane of the heterotype division, but any difference in size thus indicated is perhaps not very trustworthy, since it is slight and its measurement depends entirely on reading by the eye to fractions of the micrometer scale.

But in the nuclei of the pollen mother cells and in the resting nuclei of the various epithelial layers the differences indicated by a comparison of the camera drawings are clearly borne out by a comparison of the measurements (see table), all of which show the same result, namely, that the nuclei of the Giant plant are, on the average, larger than those of ordinary form, in the proportion of from 4 to 10% of each diameter. In the case of the developing pollen mother cells great care was taken in choosing exactly comparable stages, though, as a matter of fact, there appears to be only slight change in size when the cells have reached the stages under consideration. Figs. 3 and 4 represent the stage whose measures are recorded under the head "Prophase, with looped thread"; figs. 5 and 6 represent those recorded under the head "Prophase, diakinesis." The nuclei selected for measurement were chosen at random, with the provision that any nuclei that showed any signs of shrinkage or distortion during fixation, or were cut by the knife, were excluded. In making the measurements the divisions of the micrometer scale were read to fifths by the eye; to this extent therefore the measurements are only approximate, but the preponderance of the measurements obtained for the Giant over those obtained for the ordinary form, in the average of a number of measurements, although not great, is, I think, too consistent and regular to be accounted for by experimental error.

Perhaps the clearest evidence is to be obtained from an examination of the columnar cells of the ovule, to which reference has already been made. Both in the measurements and in the drawings these show a very consistent difference in size. Fig. 7 represents nuclei from this layer, the upper row being those of the Giant, the lower row those of the ordinary form. It will be noticed that the third and fifth are the largest nuclei in the lower row; they are almost exactly equal in size to the first and fourth of the upper row, which are, to say the least, distinctly not the largest of that row. In the ordinary form some care was exercised in the selection of the nuclei to be drawn, lest any difference might be unintentionally exaggerated. The nuclei shown in the figure were afterwards measured; the average of the measures of the six nuclei of the ordinary form was found to be $3{\cdot}7 \times 3{\cdot}4$ scale divisions, while the average in the case of the six nuclei of the Giant was $3{\cdot}9 \times 3{\cdot}7$. If these figures are compared with those given in the table, it will be seen that the difference is, if anything, diminished rather than exaggerated in the figure.

The measures were recorded in terms of the divisions of a Zeiss Ocular Micrometer. Used with a Zeiss 2 mm. Apochromatic Objective, each scale division is approximately $= 1{\cdot}74\,\mu$. The divisions were read approximately to fifths by the eye.

			No. of nuclei measured	Average Size		Largest (Scale Divisions)	Smallest (Scale Divisions)	Remarks
				Scale Divs.	$\times 1{\cdot}74 = \mu$ (ap px.)			
Pollen mother cells	Prophase, with looped thread (figs. 3 & 4)	Giant	10	5·3 × 4·8	9·2 × 8·4	5·4 × 5·2	5·0 × 4·6	
		Ordinary	10	5·0 × 4·6	8·7 × 8·0	5·2 × 5·0	4·6 × 4·6	
	Prophase, diakinesis (figs. 5 & 6)	Giant	15	5·5 × 5·1	9·6 × 8·9	5·8 × 5·2	5·4 × 5·0 (two)	
		Ordinary	16	4·9 × 4·5	8·5 × 7·9	5·2 × 4·8	4·4 × 4·4	
	Metaphase Plates — Long. Sect.	Giant	6	3·1	5·4			
		Ordinary	6	3·0	5·2			
	Metaphase Plates — Trans. Sect.	Giant	6	3·1 × 3·0	5·5 × 5·2			
		Ordinary	6	3·0 × 3·0	5·3 × 5·2			
Epidermis, in Median Long. Section of Young Stigma		Giant	10*	3·3 × 2·8	5·7 × 4·8	3·8 × 3·0 (two)	2·8 × 2·6	* The nuclei of successive cells were measured, omitting any that were dividing or were cut by the knife
		Ordinary	10*	3·0 × 2·7	5·3 × 4·7	3·4 × 3·0	2·8 × 2·6	
Epithelial cells, in Trans. Sect. of developing ovules		Giant	25	3·8 × 3·5	6·6 × 6·1	4·0 × 4·0 (three)	3·4 × 3·2 (two)	† The largest showed signs of the onset of mitosis and was much larger than any others in that ovule
		Ordinary	25	3·4 × 3·2	5·9 × 5·6	3·8 × 3·8 †	3·0 × 3·0 (two)	
Columnar cells surrounding mature unfertilized embryo-sac		Giant	20	4·0 × 3·7	6·9 × 6·4	4·4 × 4·0 (two)	3·6 × 3·4	
		Ordinary	20	3·6 × 3·3	6·3 × 5·8	4·0 × 3·8 (two)	3·2 × 3·2	

As a further test drawings were made of the six largest nuclei to be found in this layer in one section passing through the ovule in an approximately median plane. The result is shown in fig. 8, in which the left hand row represents nuclei of the ordinary form, the right hand row nuclei of the Giant. The difference here is obvious. Lest chance should have favoured the Giant, further drawings were made of the ordinary form, choosing the largest nuclei which could be found in the serial sections of some four or five ovules which were mounted on the same slide. The six largest of these are reproduced in fig. 9; it will be noticed that they are rather smaller than the six nuclei drawn from one section of one ovule of the Giant form.

Evidence has been given above which I think clearly suggests the existence of a difference in size between the nuclei of the Giant and ordinary forms. It remains to compare the size of entire cells. It is very desirable that in instituting this comparison measurements should be made of the living pollen mother cells and living pollen grains, but at the time when material of this kind was available the trend which this work has since taken was unforeseen. Microtome sections of growing tissues are, at the best, unfavourable for making measurements of entire cells, and in default of measurements of isolated cells such as might be provided by the pollen, I have had recourse to hand sections of the living tissues of the stems and petioles. Transverse sections were cut from the living material and mounted in water, so that no disturbance due to shrinkage was possible. Care was taken to avoid any disturbance due to the drag of the razor in cutting. Measurements of the diameters taken when the sections were cut show that the stems and petioles of the Giant are thicker than those of the ordinary form, the proportional difference being about 4 to 12 °/₀ of the diameters, or of the same order of difference as that found in the size of the nuclei. The size of the individual cells encountered in such sections is naturally very divergent, but by drawing the outlines of the cells to exactly the same scale the difference in their size in the two forms is strikingly shown. A number of drawings were made in this way from sections of various comparable regions of the stems and petioles. Two pairs of these drawings are shown in figs. 10 and 11, and 12 and 13. The figures represent sections, chosen at random from a number floating in a watch glass, of the main flowering stems of the two forms, in each case taken half way between the base of the stem and the lowest umbel of flowers; figs. 10 and 11 show the region of the pith, and figs. 12 and 13 the cortical region, in the ordinary and Giant respectively. It will be noticed—what has been found to be the case in all the sections made—that the cells of the Giant

are, cell for cell, larger than those of the ordinary form, and that to a degree sufficient in itself, I think, to account for the greater thickness of the Giant. Just the same resemblance in structure and difference in size is found in other regions of the stem and in the petioles of the leaves; the difference is sufficiently illustrated in the figures of the stem, and I have not thought it necessary to multiply the number of figures in order to illustrate the petioles.

In these observations there is, I think, strong ground for the suggestion that the difference between the Giant and ordinary forms may be referred to a difference in the size of the cells; that is, the character of Giantness manifests itself in the cells themselves and not merely in the plant as a whole. At present only a suggestion can be made, for we are as yet ignorant as to the extent to which individual variations among plants of the same type may affect these results; my observations have been limited, for want of others, to one pair of plants. I hope in the future to make a comparison between a number of Giant and ordinary plants, grown, as were the pair described here, under similar conditions.

EXPLANATION OF PLATE X.

The figures were all drawn with a Camera lucida. Figs. 1—6, 8, and 9 were drawn with a Zeiss 2 mm. Apochromatic Obj., 18 Ocular. For fig. 7 the same objective with Ocular 8 was used. Figs. 10—13 were drawn with a $\frac{2}{3}$ in. objective.

Fig. 1. Ordinary form; metaphase of the heterotype division in the pollen mother cell, the section being transverse to the spindle. × about 3200.

Fig. 2. Giant form; as fig. 1.

Fig. 3. Ordinary form; prophase of the heterotype division of the pollen mother cell. × 3200.

Fig. 4. Giant form; as fig. 3.

Fig. 5. Ordinary form; later prophase (diakinesis) of the heterotype division, the chromosomes in the form of gemini. × 3200.

Fig. 6. Giant form; as fig. 5.

Fig. 7. Nuclei from the layer of cells surrounding the embryo sac; the upper row from the Giant, the lower row from the Ordinary form. × 1420.

Fig. 8. Nuclei from the same layer; the left hand row from the Ordinary form, the right hand row from the Giant form. The nuclei shown were the largest to be found in one section of an ovule in each case. × 3200.

Fig. 9. Nuclei from the same layer. The nuclei shown were the six largest to be found in the serial sections of four or five ovules of the Ordinary form. × 3200.

Fig. 10. Pith cells and primary xylem from a T. S. of the flowering-stem of the Ordinary form. × 210.

Fig. 11. The same in the Giant form. × 210.

Fig. 12. Cortical cells from a T. S. of the flowering-stem of the Ordinary form. × 210.

Fig. 13. The same in the Giant form. × 210.

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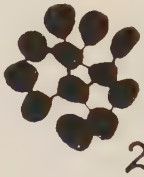

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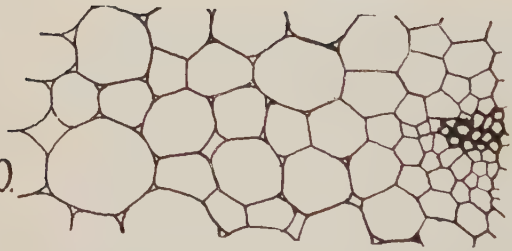

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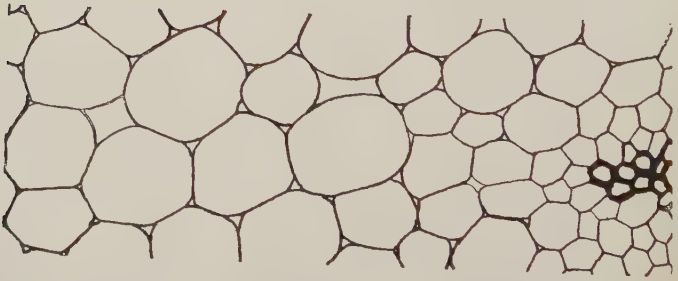

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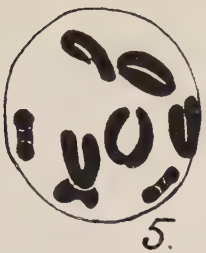

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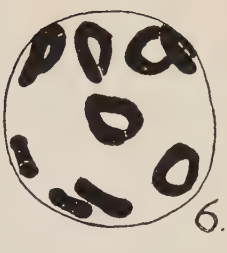

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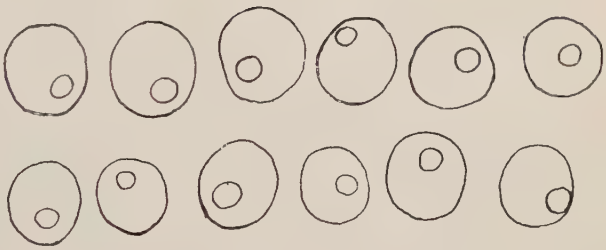

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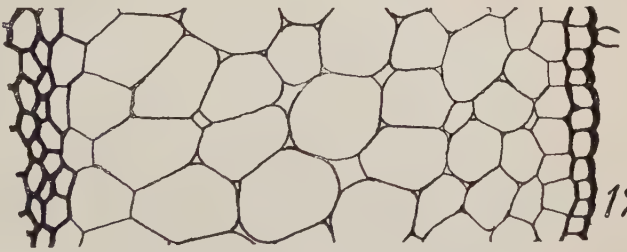

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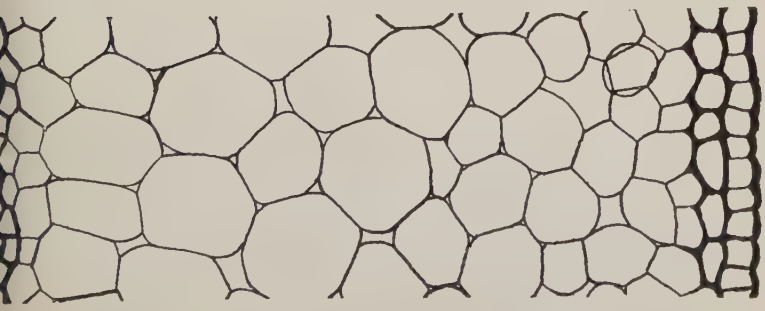


13

The influence of dilution on the colour and the absorption spectra of various permanganates. By J. E. Purvis, M.A., St John's College.

[*Read* 17 May 1909.]

[Plate XI.]

It has always been considered that permanganate solutions show similar absorption bands whatever be the nature of the element or group of elements acting as the positive ion. Ostwald's experiments * comparing the absorption spectra of the solutions of a number of permanganates, whereby there appeared to be no change either in the position or the width or the intensity of the bands, appeared to prove conclusively that such absorptions were independent of the nature of the positive ion. The only criticism which could be brought against the experiments was, and is, that sufficient dispersion of light had not been used. It is probable that any changes would be exceedingly small, and in order to distinguish small differences, if there are any, it would seem to be desirable to use a greater dispersion than was used by Ostwald.

It is also well known that dilute solutions of permanganates gradually change their colours when they are kept for some time, and it is usual to explain such changes by the action of light. The change is only noticed in very dilute solutions, and, ordinarily, a brown colour is produced. No observations are recorded with regard to changes in the absorption bands during the colour changes; nor are there any observations to indicate that the changes take place in the dark, or out of contact with the air. The following description gives some account of experiments conducted in these directions.

Experimental part.

The separate bands of solutions of permanganates can only be seen when the solutions are very dilute: and, after some preliminary trials, the following standard solutions of the permanganates of barium, zinc and potassium were made:

0·375 gram barium permanganate dissolved in 1000 c.c. distilled water,

0·262 gram zinc permanganate dissolved in 1000 c.c. distilled water,

0·316 gram potassium permanganate dissolved in 1000 c.c. distilled water.

Three separate solutions of each salt were investigated, and

* *Zeits. physik. Chem.* Vol. IX. (1892), p. 579.

each solution contained the same amount of permanganate. The solutions were placed in their respective cells or troughs, one of which was 5 mm. long, a second one was 154 mm. long, and a third one was 310 mm. long. The comparative solutions were, therefore, made by diluting the strongest one 30·8 and 62 times respectively.

The apparatus used in these experiments has been previously described* by the author in some experiments on the absorption spectra of solutions of various salts of didymium and erbium. The source of light was a Welsbach burner without the glass, whose rays passed through a small circular aperture and thence carried through the solutions by a quartz lens; and the emerging light was focussed on the slit of the spectroscope. The times of exposure varied from 1, 3 and 5 minutes; but the same length of time was allowed for each series of comparative observations. In order to make the comparison as strict as possible, blank tubes containing distilled water and of exactly the same lengths and diameters as those containing the solutions were placed in front of the slit when photographs were taken of the stronger solutions contained in the 5 mm. cell. By this arrangement, the light rays passed through the same thickness of water.

In the preliminary series of observations photographs were taken immediately after the solutions were made; and although there appeared to be no difference in the colour of the solutions, there were differences in the width of the bands as well as in the amount of general absorption of light in the more refrangible parts of the spectrum. After the solutions had stood for some hours changes in the colour of the dilute solutions were also observed. Fresh solutions of the permanganates of barium and potassium were then made, and they were allowed to stand two days when photographs of the absorption spectra and changes in the colour were noticed.

The following notes describe these observations; and in order to shorten the description it should be stated that the phrases "5 solution," "154 solution," and "310 solution" mean that there was the same amount of permanganate in the tubes whose lengths were 5 mm., 154 mm., and 310 mm. respectively.

The bands observed were λλ 570, 546, 524, 504, 486†, and they are numbered in the following notes so that

$$\begin{aligned} \lambda\ 570 &= 1, \\ 546 &= 2, \\ 524 &= 3, \\ 504 &= 4, \\ 486 &= 5. \end{aligned}$$

* *Proc. Camb. Phil. Soc.* Vol. XII. pt. III. p. 206.

† Lecoq de Boisbandran, *Spectres lumineux*, p. 109.

The other bands were too faint to make any exact observations of them.

I.

Solutions of the permanganates of barium and potassium were made and allowed to stand for two days in the full glare of a well lighted room. They were then examined; and the 5 solutions showed the well known permanganate tint; whilst the 154 solutions were pink, and the 310 solutions were yellowish brown.

A. Barium permanganate } 154 solutions.
Potassium do. }
do. do. 5 solutions.

Band 1 was narrower in the 154 solutions than in the 5 solutions; band 2 was also narrower, and the differences were well marked; band 3 was also narrower but the difference was not so marked as in band 2, whilst bands 4 and 5 were a little wider in the 154 solutions than in the 5 solutions. Also the general absorption was much greater in the 154 solutions; and the position of the bands in the 5 solutions was more towards the red end of the spectrum than in the 154 solutions.

Barium permanganate } 310 solutions.
Potassium do. }
do. do. 5 solutions.

Bands 1, 2 and 3 were narrower in the 310 solutions, and these differences were more marked than in the corresponding bands of the 154 solutions; and the bands 4 and 5 were also much wider than the corresponding bands of the 5 and 154 solutions. Also the general absorption had greatly increased, and it was greater in the 310 solutions than in the 154 solutions. The position of the bands in the 5 solutions was also more towards the red end of the spectrum: the difference in this respect was more marked when compared with the 310 solutions than with the 154 solutions.

B. The solutions were allowed to stand two days longer and they were again examined. In the 154 solutions the general phenomena were very like those observed before, except that the general absorption had slightly increased. In the 310 solutions the general absorption had also slightly increased, and the differences in the width of the bands 2 and 3 were much more marked.

C. The solutions then stood for three weeks and they were again examined. The 154 solution of barium permanganate was a reddish brown colour; the bands 1, 2 and 3 were much narrower than the corresponding bands in the 5 solution, and the bands

4 and 5 were wider than those in the 5 solution. The general absorption was more marked in the 154 solution than in the 5 solution. The 154 solution of barium permanganate showed that the bands 1, 2 and 3 had also become narrower, and the bands 4 and 5 were wider; but the general absorption had not altered much from the previous observation. The 310 solution of barium permanganate was of a light brownish yellow colour and perfectly clear. The 5 solution showed the five bands well marked; and the 310 solution showed that the bands 1, 2 and 3 had almost disappeared, whilst the bands 4 and 5 had quite disappeared and just beyond these there was complete general absorption.

The colour of the 154 solution of potassium permanganate was brown, and the solution was quite clear. The bands 1, 2 and 3 were much narrower than the corresponding bands in the 5 solution, whilst the bands 4 and 5 were a shade wider, and just beyond band 5 there was complete general absorption; whereas there was little or no difference in the colour or the general absorption in the 5 solution. In the 310 solution of potassium permanganate the bands 1, 2 and 3 were only just visible; bands 4 and 5 had completely disappeared, and there was complete general absorption just beyond; whilst in the 5 solution the bands were just as before, and there was no apparent change in the general absorption.

The general results of the above observations indicate a development of considerable differences in the more diluted solutions both as regards colour and absorption bands. The usual explanation is that light vibrations are the cause of such changes. In order to test this, similar solutions of the permanganates were made in the dark, and they stood covered up in a dark cupboard so that no light could enter. The solutions were examined after two days; and the following notes describe the observations.

II.

Solutions of permanganates of barium and potassium made in the dark and allowed to stand for two days well covered up and placed in a dark cupboard and then examined.

The colours of the 154 solutions had changed from the well known permanganate tint to a pinkish colour; and the colours of the 310 solutions had become a rose colour with a tinge of brown. The strong 5 solution did not appear to have changed. The absorption spectra of these solutions were also examined.

D. Barium permanganate } 154 solutions.
Potassium do. }
do. do. 5 solutions.

The change in colour was accompanied by some changes in the appearance of the absorption bands. There appeared to be little or no change in the band 1; band 2 was not so wide in the dilute solutions; and similarly in band 3, although the difference was not so well marked as in band 2; whilst in bands 4 and 5 there was little or no difference in the solutions. Besides alterations in the bands, there was much more general absorption in the 154 solutions.

E. Barium permanganate, Potassium do. } 310 solutions.
do. do. 5 solutions.

Band 1 in the 310 solutions was a shade narrower than the corresponding band in the stronger solutions: the width of band 2 was much less in the 310 solutions (cf. the 154 solutions): band 3 had also decreased in width but the difference was not so marked as in band 2; band 4 was also very slightly decreased in width: whilst band 5 did not appear to have decreased, although there was a slight difference in the intensity.

Also, there was much more general absorption in the 310 solutions. Besides these changes, the position of the bands 1 and 2, and in a less degree, band 3, is not so far towards the red end as in the 5 solution, and the same remark applies to bands 4 and 5 although the difference is not so marked.

The above solutions stood two days longer in the dark, and they were again examined.

The colours of the 154 solutions were of a rose red tint—they were not so brown as the solutions made two days earlier and kept in the light; whilst the 310 solutions were yellow brown.

G. Barium permanganate, Potassium do. } 154 solutions.
do. do. 5 solutions.

There appeared to be little or no change in band 1 in all the solutions; band 2 was much narrower in the 154 solutions; band 3 in the 154 solutions was also narrower than in the 5 solution, but the difference was not so marked as in band 2; bands 4 and 5 did not show a marked difference, although they appeared to be a shade wider in the 154 solutions.

There was more general absorption in the 154 solutions, and it had increased a little since the observations of two days ago.

H. Barium permanganate, Potassium do. } 310 solutions.
do. do. 5 solutions.

Band 1 was not quite so wide in the 310 solutions as in the

5 solution; band 2 had decreased in width very considerably in the 310 solutions; band 3 was also not so wide in the 310 solutions, but the difference was not so striking as in band 2; and bands 4 and 5 have very slightly increased in width in the 310 solutions.

Besides these changes, there was more general absorption in the 310 solutions, but the increase since the observations of two days ago was not very great.

Also, the shift of the bands towards the red end as noticed in E was similarly marked.

J. The solutions stood for three weeks longer in the dark and they were again examined.

Barium	permanganate	154 solutions.
Potassium	do.	
do.	do.	5 solutions.

The 154 barium permanganate solution was of a purplish colour, and there was a slight opalescence; the 154 potassium permanganate solution was rose pink coloured and perfectly clear.

In the 154 barium permanganate solution the bands 1, 2 and 3 were again narrower than the corresponding bands in the 5 solution; whilst bands 4 and 5 were slightly wider, and general absorption was more pronounced in the 154 solutions than in the 5 solution. Similar remarks apply to the 154 potassium permanganate solution.

K.	Barium	permanganate	310 solutions.
	Potassium	do.	
	do.	do.	5 solutions.

These solutions were perfectly clear, and showed no opalescence. The colours of the 310 solutions were brownish yellow.

The 5 barium permanganate solution showed the bands well marked, and very little increase in the general absorption from the earlier observations—the bands 1, 2 and 3 were very faint and narrow in the 310 solutions, and just beyond this towards the ultra violet there was complete general absorption.

In the 310 potassium permanganate solution the bands 1, 2 and 3 were faint but they were better marked, and bands 4 and 5 were only just visible, but they were wider, than the corresponding bands in the 5 solution. Just beyond band 5 on the more refrangible side there was complete general absorption in the 310 solutions.

So that these experiments prove that the changes of the solutions made in the dark and kept from the influence of light were very similar to those which had been subjected to the action of light; although light appeared to accelerate the rapidity of the changes.

III.

In order to test the accelerative effect of light, solutions of permanganate of zinc were made; one series in the light, and allowed to stand in the full glare of the light of the laboratory, and another series of solutions made in the dark and allowed to stand well covered up with black cloth and placed in a cupboard to which no light could penetrate. The solutions standing in the light were examined by the eye from time to time. After six hours a very slight change in the usual permanganate tint of the 310 solution was noticeable; but none in the 154 solution. After 10 hours standing the 154 solution was still pink and the 310 solution was rose coloured. On standing 24 hours, the 310 solution was light yellow brown in colour, and the 154 solution was rose red. The solutions appeared to retain this colour during another 24 hours.

After two days standing, the solutions which had been made in the dark, and which had stood in the dark during this period of time, were then examined and compared with those kept in the light, and there appeared to be exactly the same changes in colour in them.

L.	Zinc permanganate	in dark	154 solutions.
	do. do.	in light	
	do. do.	in dark	5 solution.

Band 1 was not quite so wide in the 154 dark and light solutions as in the 5 solution; band 2 was also narrower in the 154 solutions; band 3 was also narrower but not so well marked; whilst bands 4 and 5 appeared to be slightly wider in the 154 dark and light solutions. The general absorption in the solutions kept in the dark and in the light was about the same in each, and greater than that of the 5 solution. Also there was a slight shift of the bands in the 5 solution towards the red end more than in the 154 solutions, but the difference was not well marked.

M.	Zinc permanganate	in dark	310 solutions.
	do. do.	in light	
	do. do.	in dark	5 solution.

Band 1 was again not so wide in the dark and light 310 solutions as the corresponding one of the 5 solution; bands 2 and 3 were also much narrower in the dark and light solutions; whilst bands 4 and 5 appeared to be slightly wider in the dark and light 310 solutions than in the dark 5 solution.

The general absorption of light in the solution kept in the light was a shade more marked than that kept in the dark;

and both general absorptions were greater here than those in the 154 solutions.

The shift towards the red end of the bands of the 5 solutions was much better marked when compared with those of the 310 solutions than those of the 154 solutions.

The solutions were allowed to stand as before in the dark and the light for two days longer and they were again examined.

N. Zinc permanganate in dark } 154 solutions.
do. do. in light }
do. do. in dark 5 solution.

There was a very doubtful increase in the general absorption of the 154 solutions as compared with the previous observations in L, otherwise the differences in the appearances of the bands were very like those in the previous observations. There was no difference in the width and appearances of the bands of the solutions kept either in the light or dark; the bands 2 and 3 were narrower in these solutions than in the 5 solution, whilst the bands 4 and 5 were very slightly wider.

O. Zinc permanganate in dark } 310 solutions.
do. do. in light }
do. do. in dark 5 solution.

There appeared to be but a very slight increase in the general absorption of the 310 solutions as compared with the previous observations; the bands 1, 2 and 3 were narrower than the corresponding bands in the 5 solution, the band 2 being the most marked in this respect, whilst bands 4 and 5 were slightly wider than the corresponding bands in the 5 solution.

The shift of the bands towards the red end in the 5 solution was distinct as compared with their positions in the 310 solutions.

The observations indicate that similar changes take place in the zinc permanganate solutions as in those of barium and potassium; and that light accelerates the changes.

General conclusions.

The results prove that, during the time of the observations, (1) in the stronger 5 solutions there did not appear to be any marked change in the ordinary colour nor in the absorption bands of the permanganates; (2) in the 154 solutions there were considerable changes in the colour which slowly turned to a rose red or reddish brown one accompanied by changes in the width of the absorption bands 1, 2, 3, 4 and 5 and an increase in the general absorption; (3) in the 310 solutions the changes in the colour were more marked and a brownish yellow colour was developed

with an almost complete obliteration of the absorption bands and a greatly increased general absorption; (4) a shift of the bands of the 5 solutions to the red end of the spectrum as compared with their positions in the 154 and 310 solutions, and this shift appeared to decrease with increased refrangibility of the absorbed rays.

In order to explain the different positions of the bands of the different solutions, the observations of H. Becquerel* show that such differences depend upon the concentration—the greater the concentration, the greater the shift of the bands towards the red end of the spectrum.

With regard to the other changes, attention is again called to the observations that the changes take place whether the solutions are subjected to the action of light or whether they are kept in the dark, although light accelerates the changes. One explanation is that the dissolved gases, the atmospheric gases, particularly carbon dioxide, may have been the influencing causes. Against this view is the fact that carbon dioxide usually converts manganates to permanganates very rapidly; and oxygen would scarcely act as a reducing influence, whilst nitrogen is completely inert. Nor could the changes have been caused by any dissolved organic substances, for the solutions were made in distilled water, and from pure crystallised salts; and also, all the glass vessels were well washed before use with a warm mixture of hydrochloric and nitric acids. Nor could any organic substances have entered the solutions, for the latter were always kept well covered up. Nevertheless, in order to eliminate the action of any extraneous gases or dirt, another series of solutions of potassium permanganate was made, and the glass bottles were filled to overflowing. The contents were isolated from the air by well ground stoppers. The bottles of one series were filled in the dark, and they stood in cupboards to which no light could penetrate. A similar series of bottles stood in a well lighted room and they were exposed to the full action of light. The changes in the colour of the solutions were exactly the same as in the earlier observations; but those of the solutions exposed to the action of light were much quicker. Besides the colour changes, a small bubble of gas collected in the bottles. The absorption spectra were not observed.

The explanation which seems to account for the changes is that the dissociating force of the water slowly acted upon the dissolved permanganates, so that the ionic condition of $\overset{+}{R}$ and $\overset{-}{Mn}O_4$, where $\overset{+}{R}$ represents the metallic ion, broke down. One may imagine that the $\overset{-}{Mn}O_4$ ion undergoes further dissociation

* *Compt. Rend.* Vol. CII. p. 106.

into MnO_3 and O, or into MnO_2 and O_2. In such dilute solutions the tension between the molecules of water surrounding the ions must have been very considerable, and it may have been so great as to destroy the MnO_4 condition, so that further changes took place which became visible in changes of colour, in changes in the width of the bands and in the liberation of oxygen; and these changes would be accelerated by the vibrations of light. The intermediate stages of the changes might be represented by the narrowing of the bands 1, 2 and 3 and the widening of the bands 4 and 5, and these might correspond to changes of the MnO_4 ions into MnO_3 and O ions. The complete change in colour from the well known permanganate tint to a light brownish yellow colour in the more diluted solutions, the disappearance of the absorption bands, and the increased general absorption represented the complete change into MnO_2 and O_2. The MnO_2 would be dissolved in the colloidal condition, and it would be represented by the complete disappearance of the bands, and the increased general absorption, whilst the oxygen partly escaped.

DESCRIPTION OF PLATE XI.

It has not been possible to reproduce all the original photographs comparable with them in clearness and precision. The reproductions on the plate are from photographs of original negatives enlarged one and a half times. The shaded parts represent light issuing through the solutions; the light bands represent the absorption bands; and the lighter parts in the more refrangible regions represent general absorption.

1. Barium permanganate, 154 solution.
2. do. do. 5 solution.

These solutions were made in the dark, and they were kept out of contact with the light during the experiments. They correspond to paragraph G in the notes.

3. Barium permanganate, 154 solution.
4. do. do. 5 solution.

These solutions remained fully exposed to the action of sunlight. They correspond to paragraph B in the notes.

It will be noticed that the weak band at λ 570 is almost obliterated, although it can be easily seen in the original photographs and in the enlargements therefrom. The narrowing of the bands at $\lambda\lambda$ 546, 524 and the widening of those at $\lambda\lambda$ 504, 486 in the diluted solutions when compared with those in the stronger solutions are well marked.

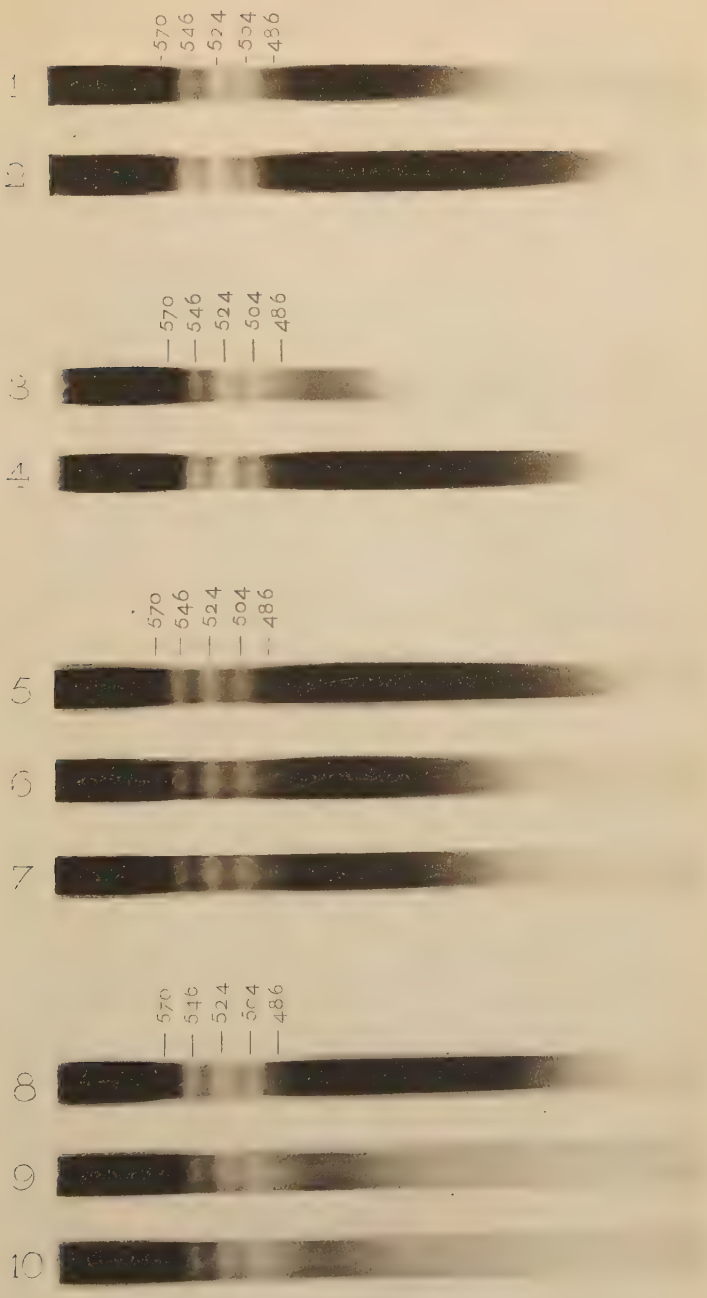
5. Zinc permanganate, 5 solution made in the dark and kept in the dark.
6. do. do. 154 do. do. do.
7. do. do. 154 do. fully exposed to the action of light.

These solutions correspond to paragraph L in the notes.

8. Zinc permanganate, 5 solution made in the dark and kept in the dark.
9. do. do. 310 do. do. do.
10. do. do. 310 do. fully exposed to the action of light.

These solutions correspond to paragraph M in the notes.

The band at λ 570 is only just visible; but the narrowing of the bands at $\lambda\lambda$ 546, 524 and the widening of those at $\lambda\lambda$ 504, 486 when compared with those in the stronger solutions are easily distinguishable. The general absorptions of the solutions 6 and 7 do not show very marked differences: but there is a little difference in those of the solutions 9 and 10.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Phenomena of X-Ray Transmission. (Preliminary Paper.) By Charles G. Barkla, M.A., D.Sc. (Communicated by Professor Sir J. J. Thomson.)*

[*Read* 17 May 1909.]

The principal phenomena accompanying the transmission of X-rays through matter may be classified under three heads—Absorption, Ionization, and Secondary Radiation.

Much that has been written on these subjects has been in the form of disconnected papers, in which little attempt has been made to collect and classify known experimental facts, or to give an explanation of observed phenomena in terms of laws previously discovered. As a consequence X-ray phenomena have been made to appear much more complicated than they actually are.

In the following paper it is intended to give, as briefly as possible and without entering into experimental details, a statement of facts already published, an account of the results of further experiments, and a survey of the whole subject of X-ray transmission, showing the connection between the various phenomena and accounting for apparent anomalies. As the phenomena of secondary radiation form the basis of classification they will be discussed first.

Secondary X-Radiation.

Secondary X-rays of two distinct types have been found to be emitted by substances subject to X-rays. Of these the scattered X-radiation—one of the same penetrating power as the primary—has invariably been found to be emitted during the transmission of a primary beam through matter.

All the experiments that have been made on this radiation have strikingly verified the theory of scattering, as given by Sir J. J. Thomson†, many of the results being foretold as an immediate consequence of this theory.

The second type of secondary X-radiation emitted by many elements, and probably by all, is a homogeneous radiation characteristic simply of the element emitting it. Unlike the scattered radiation this is distributed uniformly in all directions, and gives no evidence of polarization in a primary beam where such polarization exists. This radiation was shown by Barkla and

* The expenses of these researches were partially covered by a Government Grant through the Royal Society.

† *Conduction of Electricity through Gases.*

Sadler* to be emitted only when the exciting primary radiation was of more penetrating type. Also it was shown that beyond a certain penetrating power of the primary, the intensity of this secondary radiation from a given mass of substance was proportional to the ionization produced by this primary beam in a thin film of air. As we have no exact method of comparing the intensities of beams differing in penetrating power, we are unable to determine accurately the relative amounts of energy of different beams reappearing as secondary radiation of this type. If, however, we assume as approximately true that the ionizations produced in a thin film of air by beams of equal intensity are proportional to the absorptions of these beams in air, we may also say that the intensity of this secondary radiation, beyond a certain penetrating power of the primary radiation, is proportional to the coefficient of absorption of the primary radiation in aluminium. The relation between the intensity of secondary homogeneous radiation and the absorbability of the primary radiation may thus be approximately represented by curve II in fig. 4. Whatever the accuracy of the assumption, the general features of the curve remain unquestionable, i.e. first, no appreciable intensity until the primary is more penetrating; secondly, a rapid rise of intensity as the primary is made more penetrating; and thirdly, a decline of intensity, approximately, if not accurately proportional to the absorbability of the primary radiation. Mr Sadler† has recently investigated the curves in detail on the same assumption, using a series of homogeneous beams as primary radiations, and has found the same type of relation in all cases investigated.

The secondary radiation from a large number of elements has been studied, and the percentage absorption of the radiation by a sheet of aluminium ·01 cm. thick, as measured by the diminution of ionization in an electroscope 7 or 8 cms. from the radiating elements, has been published by the writer‡.

When subject to a beam of X-rays of only moderate penetrating power, the radiation reaching the electroscope from elements of atomic weight between those of hydrogen and sulphur was almost entirely a scattered radiation, none of the homogeneous type appearing. Later analysis of the radiations from other elements by Barkla and Sadler showed that the radiations from the elements from Cr to Ag were practically homogeneous radiations, producing in general, ionizations several hundred times the ionization produced by the scattered radiation from the equal masses of the light elements. A weak scattered radiation was mixed with these.

* "Homogeneous Secondary Röntgen Radiations," *Phil. Mag.*, Oct. 1908, pp. 550—584.

† "Transformations of Röntgen Rays," *Phil. Soc. Lond.*, April 23, 1909.

‡ "Secondary Röntgen Radiations," *Phil. Mag.*, June 1906.

The writer has recently investigated more closely the radiations from Sn, Sb, I (which have been recorded as elements emitting a radiation of variable penetrating power). It has been found that these consist of a very easy absorbed radiation and a very penetrating homogeneous radiation superposed. The absorptions of the penetrating portions of the beams from each element are shown in fig. 1 on curve *B*. The percentage absorptions of the soft radiations from these elements have not yet been determined, but they are roughly indicated on curve *A* in fig. 1. Though a full analysis of the radiations from W, Pt, Au, Pb, Bi, etc., has not yet been made, there is strong evidence that the observed radiations from these elements are also principally homogeneous radiations characteristic of the elements emitting them.

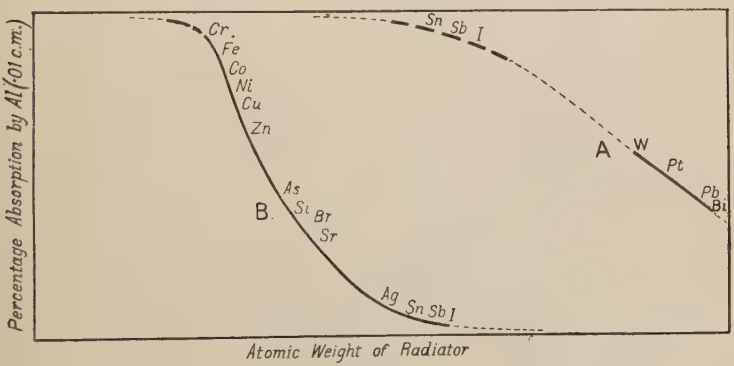


Fig. 1.

NOTE: ——— Continuous lines connect points which have been accurately determined.

— — — Discontinuous lines pass through *approximate* positions of points.

..... Dotted lines are hypothetical.

A knowledge of these characteristic radiations is sufficient to account for the variations in the intensity and in the character of the secondary rays emitted by different substances, and for what have been regarded as anomalies in secondary radiation phenomena by a number of investigators. Thus the intense radiation from ethyl bromide when subject to a beam of ordinary penetrating power is that given by the element bromine—an element in the group of substances which emit homogeneous rays in considerable intensity when subject to an ordinary primary radiation. The variable behaviour of tin bichloride and of methyl iodide may also be explained in terms of the results for tin and iodine already published. The variations are due to the fact that a beam of X-rays of ordinary penetrating power contains only a small

proportion of rays of penetrating power higher than that of the penetrating rays from Sn, Sb, and I; so that only a small proportion of the more penetrating radiation is emitted with the softer characteristic radiation. As the heterogeneous primary radiation is made more penetrating a greater proportion of the beam excites the penetrating secondary radiation. Thus the intensity of secondary radiation and the average penetrating power of this radiation increase rapidly.

By an examination of the curves connecting the absorption of the various characteristic radiations with the atomic weight of the radiating substance as shown in fig. 1 we are led to several important conclusions. These characteristic secondary radiations may be divided into several groups, the radiation belonging to each group becoming more penetrating with an increase in the atomic weight of the radiating substance. Thus in fig. 1 we show groups *A* and *B*.

If elements of low atomic weight emit characteristic secondary radiations belonging to groups *A* and *B*, these radiations must be exceedingly soft, and as a consequence must emerge from only a very thin surface layer of the radiating substance. Owing to this fact and to the absorption of the radiation in air before reaching the electroscope, the effects of these radiations must be very small. Such a soft radiation has been found to be emitted by Ca mixed with the scattered radiation, and there have been indications of such radiations from S and from Al. These, however, have not yet been examined carefully.

As the atomic weight of the element increases, the characteristic radiations become more penetrating and produce much bigger ionizations in the detecting electroscope, almost completely swamping the effect of the scattered radiation. As the atomic weight of the element becomes higher still, the characteristic radiation becomes so penetrating that only the most penetrating constituent in an ordinary primary beam is able to excite it. The intensity of the secondary thus diminishes, and finally the radiation becomes inappreciable. But before this has happened the radiation belonging to another group has appeared in appreciable intensity, and this ultimately becomes the most important secondary radiation. It appears very probable that there are other groups similar to these, for even when the primary radiation is too "soft" to excite the radiations *A* and *B* it is absorbed and it produces ionization. And these secondary radiation phenomena, as we shall see later, are connected with the phenomena of absorption and ionization. The hypothetical groups of radiations softer than those in group *A* we shall denote by the letter *X*.

Absorption.

Experiments on absorption of X-rays have been made by a number of experimenters, but owing to lack of knowledge of secondary X-rays, and to heterogeneity in the beams experimented upon, little regularity of behaviour could be observed from the experimental results.

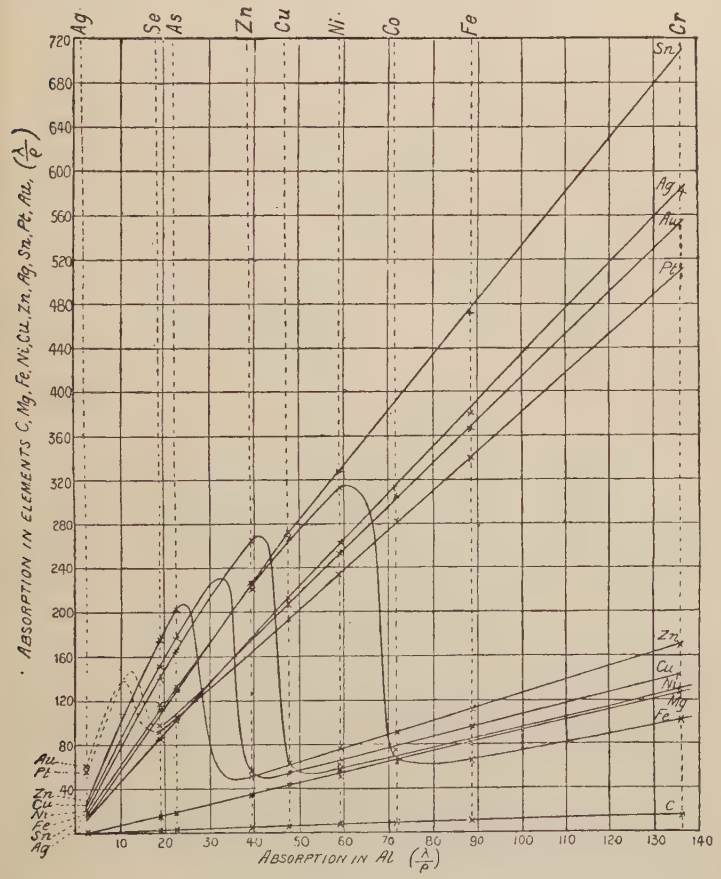


Fig. 2.

The connection between the absorption of X-rays and the secondary radiation emitted from the absorbing substance was first shown by Barkla and Sadler in a paper on "Homogeneous Secondary Röntgen Radiations." The laws governing the absorption of X-rays have recently been more fully investigated*. The

* "The Absorption of Röntgen Rays," *Phil. Mag.*, May 1909.

principal features brought to light by the investigation are exhibited in curve IV in fig. 4, in which the absorptions by a given element are plotted as ordinates, and the absorptions by Al as abscissae.

In all the absorbing substances examined—C, Mg, Fe, Cu, Zn, Ag, Sn, Pt, Au—there is *approximate* proportionality between the coefficients of absorption in a substance R say and in Al through a wide range of penetrating power. This approximate proportionality holds until the primary radiation is made more penetrating than the homogeneous radiation characteristic of the absorbing substance R. Then the absorption begins to increase more and more rapidly until, after reaching a maximum, it begins to approximate again to proportionality with the absorption in Al, the ratio of absorption coefficients being now much higher than before. The increase in the absorption was thus shown to commence at the same point as that at which the secondary homogeneous radiation began to be emitted, the connection between absorption and intensity of secondary radiation being shown by the curves IV and II. As the range of penetrating powers experimented upon was that from the radiation characteristic of Cr $\left(\frac{\lambda}{\rho} \text{ in Al} = 136\right)$ to that characteristic of Ag $\left(\frac{\lambda}{\rho} \text{ in Al} = 2{\cdot}5\right)$, it will be seen by comparison with fig. 2* that all elements, one of whose characteristic radiations is between these limits, showed the same feature in the absorption curve. All those which did not emit a characteristic radiation of penetrating power between these limits gave absorption curves without any special deviation from the approximate proportionality. It is important, however, to notice several features of the curves. The shape of the curve for primary radiations is considerably more penetrating than that necessary to produce the characteristic secondary radiation in any substance approximated to the shape just previous to the sudden rise in the absorption, showing the possibility of a similar rise in the absorption having occurred with a much softer radiation.

Again—and this is a most important point—the mass absorption coefficients for a given radiation are in the order of atomic weight of the absorbing substance when the corresponding radiations are excited.

Thus using the very soft radiation from Cr, $\frac{\lambda}{\rho}$ in C, Mg and Al are in order of atomic weight, but there is a step back for the elements Fe, Ni, Cu, Zn, Ag, Sn because in these the radiation of group *B* is not excited, and there is a further step back for Pt and

* Reproduced from paper on "The Absorption of Röntgen Rays," Barkla and Sadler.

Au because in these the radiation A is not excited. When, however, the primary radiation is sufficiently penetrating to excite the radiation B in Fe, Ni, Cu, Zn, these take their proper place. A radiation sufficiently penetrating to excite radiation B in Ag, Sn, Pt, Au was not used, but when the radiation was

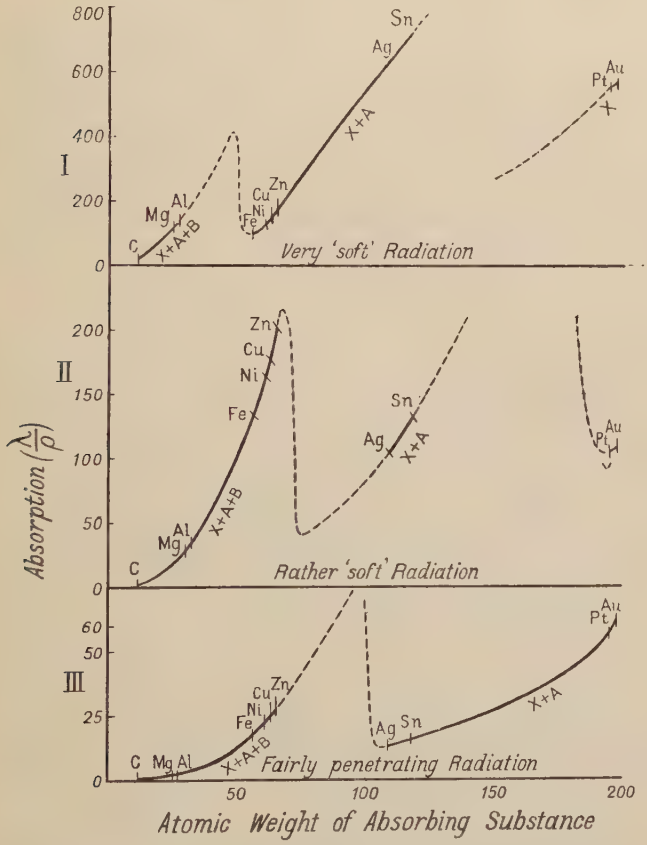


Fig. 3.

Note: Letters X, A and B denote the characteristic secondary radiations excited (see fig. 1).

sufficiently penetrating to excite the radiation A in Pt and Au, these took their proper places with respect to Ag and Sn.

Thus in fig. 3, curves I, II and III exhibit the relation between the absorption in different elements and the atomic weight of the absorbing element for three different radiations. Curve I was obtained by using the very soft homogeneous radiation

from Cr $\left(\frac{\lambda}{\rho} \text{ in Al} = 136\right)$; Curve II by the fairly soft homogeneous radiation from As $\left(\frac{\lambda}{\rho} \text{ in Al} = 22{\cdot}5\right)$; Curve III by the fairly penetrating radiation from Ag $\left(\frac{\lambda}{\rho} \text{ in Al} = 2{\cdot}5\right)$.

We thus see from these results that the absorption of a homogeneous radiation by elements of various atomic weights is a periodic function, but in general an increasing function of the atomic weight. Though observations have not been made on a sufficient number of elements to obtain the accurate relation between absorption and atomic weight of the absorbing substance for any one particular homogeneous beam, different portions of similar curves have been obtained from observation of the absorption of other homogeneous beams, and their shapes reproduced in the discontinuous curves in the figure.

Ionization.

An examination of the results obtained by different experimenters on the ionizations produced in gases by X-rays showed that these phenomena were in all probability also connected with secondary X-ray phenomena, and that the numerous apparent irregularities might be explained in terms of laws similar to those found for absorption. The writer was therefore led to attempt an investigation of the ionizations produced in different gases by homogeneous beams varying considerably in penetrating power. The radiations used were the homogeneous radiations from Fe, Cu, Zn, Br, Sr, Ag, Sn, Sb, varying in absorbability as shown by curve B in fig. 1.

The gases and vapours experimented upon were Air, O, CO_2, SO_2, C_2H_5Br, $SnCl_2$, CH_3I.

The results of preliminary experiments may be briefly stated as follows:

Gas	*Range of penetrating power of radiation*	*Results*
O	Fe radiation to Ag radiation	Ionization approximately proportional to ionization in air
CO_2	,, ,, ,,	,, ,, ,, ,,
SO_2	,, ,, ,,	,, ,, ,, ,,
C_2H_5Br	Fe radiation to Br radiation	,, ,, ,, ,,
	Br radiation to Sn radiation	Relative ionization increased approximately tenfold
$SnCl_2$	Fe radiation to Sn radiation	Ionization approximately proportional to ionization in air
	I radiation	Relative ionization considerably greater
CH_3I	Fe radiation to Ag radiation	Ionization approximately proportional to ionization in air [only slight gradual change, if any]

These results must be connected with the results of experiments on secondary radiation and absorption. Carbon, Nitrogen, Oxygen and Sulphur, elements in the group in which an appreciable homogeneous radiation within this range of penetrating power has not been detected, and in which there is proportionality of absorption within this range of penetrating power, here exhibit proportionality of ionization also.

Ethyl bromide and Tin bichloride exhibit a like proportionality in ionization until the radiation becomes more penetrating than the radiations characteristic of Br and Sn respectively. For more penetrating radiations the relative ionization increased just as the secondary radiation in Br and Sn began to be excited and as the absorption began to increase.

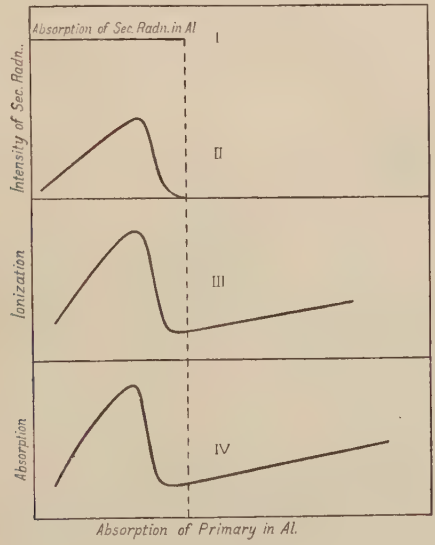


Fig. 4.

Methyl Iodide, which was not subjected to a radiation of more penetrating type than that characteristic of Iodine, also exhibited approximately the same proportionality of ionization.

We are thus led to the conclusion that as in the case of the secondary radiation and absorption, the turning point in the curves showing the relation between ionization and the penetrating power of the primary radiation is just where the primary radiation becomes more penetrating than the homogeneous radiation characteristic of the substance traversed, or of one of the elementary constituents of the substance traversed.

General Discussion.

The various phenomena may therefore be connected as shown in the diagram fig. 4.

Line I represents the absorbability of the secondary radiation characteristic of an element R say.

Curve II shows approximately the relation between intensity of secondary radiation from R, and absorbability of the primary radiation (measured in Al).

Curve III shows the relation between ionization in R and absorbability of the primary radiation (measured in Al).

Curve IV shows the relation between absorption in R and absorbability of the primary radiation (measured in Al).

Curves II and III are based on the assumption that the ionization in air is proportional to the absorption in air for beams differing in penetrating power. The important features are beyond doubt; the exact shape, however, depends on the accuracy of this assumption.

There is thus an intimate connection between the absorption of a primary radiation in a substance, the ionization in the absorbing substance (when in the gaseous state), and the intensity of secondary radiation from the absorbing substance. These are periodic functions of the penetrating power of the primary radiation, the three rising and falling together. Though two complete periods have not yet been obtained in any one substance, there is very strong evidence from the behaviour of different substances that there are at least several maxima and minima to be obtained by using a primary radiation of sufficient range in penetrating power. The periodicity in intensity of secondary radiation is not one of intensity alone, for each fresh period brings a characteristic radiation of different penetrating power.

Sufficient experiments have not yet been made in order to determine if the periodicity is a true periodicity in the behaviour of *one* system in the atom, or is due to different systems behaving in a similar manner when subject to primary radiations of different penetrating powers. Be that as it may, the experiments on secondary radiation show that there are various characteristic radiations; the experiments on ionization show that there are various ionizations, only a part of the total ionization being connected with one radiation; the experiments on absorption show that there are various processes of absorption, only a part of the total absorption being connected with one radiation.

It should perhaps be pointed out that it is owing to the fact that absorption, ionization and secondary radiation connected with one substance are periodic functions of the absorbability of the primary radiation, that these three quantities are also periodic

functions of the atomic weight of the substance subjected to a fixed primary beam.

Many experiments remain to be performed before any theory regarding the exact processes taking place during the transmission of X-rays through matter can be established. It is, however, worth while pointing out that if an electromagnetic pulse in which the electric force is uniform from front to back of the pulse passed over a number of independent electrons with a definite period of vibration, the absorption would be a periodic function of the thickness of the primary pulse. The absorption coefficient might be written

$$\lambda = \frac{\mu N e^2}{\pi m} \frac{\delta^2}{d} \left[1 - \cos 2\pi \frac{d}{\delta} \right],$$

where d is the thickness of primary pulse and δ the wave-length of the radiation emitted by the freely vibrating electrons, N the number of electrons in unit volume, e and m the charge and mass of an electron. This would give an absorption curve of somewhat the same form as those shown, and would also show periodicity. As we have no method of measuring the thickness of the pulse, and do not know anything of the force distribution in the pulse, we cannot strictly compare the experimental curve with this.

It should also be pointed out that with a primary pulse in which the force gradually increases from zero to a maximum and gradually again to zero, the periodicity and indeed the absorption of thicker pulses would practically disappear. In such a case the periodicity would have to be explained by assuming the existence of different absorbing, radiating and ionizable systems in the same atom.

The connection between absorption, ionization and intensity of secondary radiation might be easily accounted for. It would only be after such an absorption of energy that electrons would be hurled out of an atom, and ionization be produced—(both as a primary and secondary or subsequent effect of this ejection). The disturbance in the atom would set up an electromagnetic pulse characteristic of the atom.

It seems very improbable for several reasons that the subsequent bombardment of other atoms by the ejected electrons produces an appreciable secondary X-radiation. The speed of ejection has been found to depend on the penetrating power of the primary radiation causing that ejection; so also would the character of the secondary X-radiation produced by these. This is contrary to experimental fact. Again, the connection between the penetrating power of the primary radiation necessary to produce a certain secondary radiation and the penetrating power of that radiation would be very difficult to explain.

Lastly, preliminary experiments on the secondary radiation from alloys appears to indicate that the bombardment of a second substance by the ejected electrons produces no change in the character of the secondary radiation produced. The most difficult point to explain is the fact that the primary radiation must be *just* more penetrating than the homogeneous secondary radiation characteristic of the exposed substance for that radiation to be excited. This law appears to be an extension of Stokes' Law of Fluorescence to which the phenomenon of the secondary characteristic radiation is akin.

The exactness with which the law is obeyed, and the striking similarity between the various absorption curves indicates a definiteness in the structure of the pulses, and the absence of anything corresponding to variation in pulse structure.

As absorption of energy must precede the expulsion of electrons, we might expect to find the intensity of corpuscular radiation connected with the absorbability of the primary radiation in the same way as is the X-radiation, except that the velocity would vary with the penetrating power of the primary beam, as has been found by Innes.

A detailed account of these experiments and a fuller discussion will be published later.

Liverpool.

The Emission of Röntgen Rays from Thin Metallic Sheets. By G. W. C. Kaye, B.A., D.Sc., Trinity College, Cambridge. (Communicated by Prof. Sir J. J. Thomson.)

[*Received* 4 August 1909.]

Prof. Bragg and Dr Madsen* have, from the point of view of the neutral-pair theory of the Röntgen and γ rays, made measurements of the secondary corpuscular radiation which goes forward and backward when γ rays are incident on thin metal sheets. They find in most cases a large want of symmetry in the distribution of these secondary rays—there is more "emergence" than "incidence" radiation: especially is this so for metals of low atomic weight. For example, the ratio (R) of the emergence intensity to the incidence is, in the case of soft γ rays, about 13 for carbon, 7 for aluminium, 2 for the metals of the copper group, 1·1 for tin and lead: with hard γ rays these numbers become 20, 7, 3 and 1 respectively.

Madsen† obtained similar results in his investigation of the distribution of the secondary γ radiation produced by γ rays under analogous conditions. For zinc R (defined as before) is about 5, for lead 7. He showed also that the γ rays in passing through matter are softened as well as scattered. The distribution of the scattered radiation depends on the quality of the exciting γ rays and also upon the nature of the scattering medium. The qualities of the incidence and emergence radiations are not always identical.

Cooksey (*Nature*, Ap. 2, 1908) worked in like manner with the corpuscular secondary radiations from Röntgen rays, and brought his results into parallelism with those for γ rays; although the asymmetry was very much less pronounced, R varying from 1·1 to 2 for different elements.

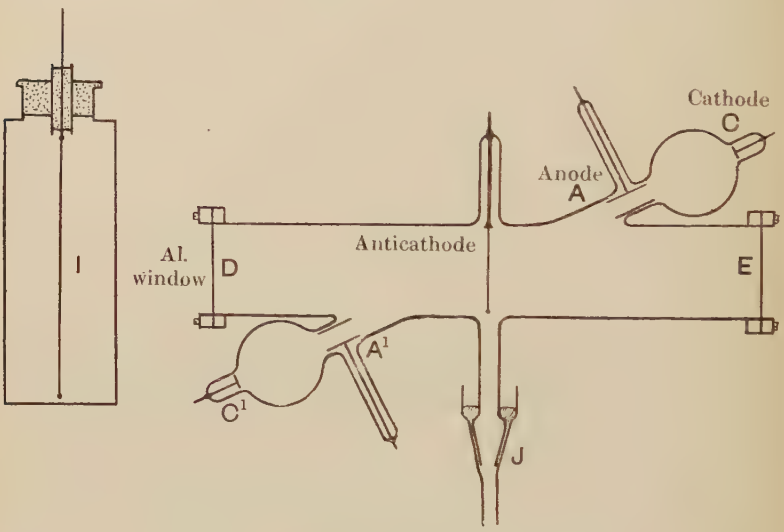
It remained for Bragg and Glasson‡ to find a corresponding lack of symmetry in the distribution of the secondary X rays produced by the transmission of X rays through thin metal sheets. The ratio R ranges from about 3 for Al to 1·3 for Pt.

Quite recently Madsen§ has extended the enquiry to β rays and found very similar results. The greatest value of R is about 9 for Al and 4·5 for gold. He found that as the thickness of the metal screen is increased, the total emergent scattered radiation increases rapidly to a maximum (at ·013 cm. Al and ·0008 cm. Au) and afterwards decreases.

With some of the foregoing results in mind the problem of

* Bragg and Madsen, *Phil. Mag.* 16, 1908, p. 918.
† Madsen, *Ib.* 17, 1909, p. 423.
‡ Bragg and Glasson, *Ib.* 17, 1909, p. 855.
§ Madsen, *Trans. Roy. Soc. S. Australia*, April, 1909.

the distribution of the Röntgen rays produced by the impact of cathode rays on a thin metal target invited attack. The simplest experimental conditions would be realised by the use of as thin a metal anticathode as can be obtained, and by employing cathode rays of uniform velocity. This latter condition is being satisfied in experiments now in progress, but as the cathode rays in soft coil-driven Röntgen ray tubes are not very heterogeneous it seemed worth while to carry out preliminary experiments in which the condition of homogeneity of the cathode rays was waived. Moreover it has to be remembered that the Röntgen rays emitted will be diluted to some extent with secondary Röntgen rays produced by the action of the primary X ray beam on the material of the anticathode. It remained to see what effect this would have on the distribution of the primary rays.



Apparatus.

The apparatus is sufficiently indicated by the figure. An anticathode of thin metal leaf was mounted centrally in the tube *DE*, so that it could receive a pencil of cathode rays from either side at an angle not far removed from the normal. The X rays produced could pass out normally by the aluminium windows *D* and *E* which, needless to say, were equally thick (·01 cm.). The idea in providing two cathodes *C* and *C′* was to guard against any possible want of symmetry in the apparatus. Plane cathodes had to be employed; such thin leaves as were used for anticathodes puncture at once if any attempt is made to focus the cathode rays. The tube was coil-driven.

The intensity of the X rays was measured by an ionisation chamber I (with thin aluminium bounding walls), and a Wilson electroscope: the usual insulation and shielding precautions were taken. If the rays were cut down by a screen, it was, of course, inserted between the aluminium window and the ionisation chamber.

The anticathode was earthed and put in metallic contact with the tube-anode A or A'. The whole of the discharge tube could be slewed round by means of the mercury-trapped ground-joint J, which led also to the pump. Thus the X rays proceeding from either window could be measured by the one ionisation chamber: one had only to rely on the constancy of the discharge.

Results.

Anticathode	Thickness of Anticathode	Spark Gap	Thickness of Al. Screen	$R = \frac{\text{Emergence Rad.}}{\text{Incidence Rad.}}$
Gold	·00001 cm.	·1 cm.	None	1·15
		,,	·022 cm.	1·25
		,,	·033 ,,	1·35
		,,	·044 ,,	1·50
		·5 cm.	None	1·30
		·8 ,,	,,	1·40
	·00002 cm.	·5 cm.	None	1·00
		,,	·047 cm.	1·00
		1·5 cm.	None	1·25
		,,	·022 cm.	1·35
		,,	·047 ,,	1·50
	·00003 cm.	·5 cm.	None	0·90
		1·5 ,,	None	1·05
Aluminium	·00001 cm.	·1 cm.	None	1·30
		·2 ,,	,,	1·90
		,,	·022 cm.	2·00
		·7 cm.	None	3·20
		,,	·047 cm.	3·10
		1·7 cm.	None	2·50
		,,	·047 cm.	2·00
	·00002 cm.	·7 cm.	None	2·80
		1·7 ,,	None	3·10
Copper	·000042 cm.	·2 cm.	None	1·10

In these preliminary experiments, aluminium, copper, gold and platinum leaf were used for anticathodes. A few typical measurements of the ratio R of the intensity of the emergent to the incident radiation are appended. Platinum behaves like gold in the values that it yields for the ratio under different conditions. The speed of the cathode rays, being dependent on the potential between the cathode and anode, is sufficiently indicated by the length of the alternative spark gap (between large brass balls).

These and other results indicate the following points:—

1. Generally speaking the emergence Röntgen radiation from a thin metal anticathode exceeds the incidence in intensity. This is most marked in the case of aluminium. Furthermore, as evidenced by the phosphorescence of the glass walls of the tube, the accompanying emergence secondary cathode radiation from the anticathode is more intense than the incidence cathode radiation. As far as could be judged "R" for the cathode rays seems to follow any variation of "R" for the X rays. Madsen's results (above) for β rays may be noted.

2. With heterogeneous cathode rays the incidence beam of Röntgen rays is softer than the emergence.

3. The ratio (R) of the emergence intensity to the incidence, increases with the speed of the impinging cathode rays, provided the anticathode is thick enough. This is shown both by increasing the potential on the tube, and by sifting out (with screens) the softer X rays produced by the more slowly moving constituents of the heterogeneous bundle of cathode rays.

4. For each thickness of anticathode there is a certain speed of cathode ray which gives a maximum value to the ratio R. The ratio drops in amount for greater or less speeds than this.

5. As the thickness of anticathode increases, the emergence intensity (for the same speed of cathode ray) first increases to a maximum and then dies away; the incidence intensity meanwhile gradually increases to a constant value. The resemblance to Madsen's results for β rays (above) will again be noticed.

A discussion of the results is reserved until the measurements are concluded.

I wish to thank Sir J. J. Thomson for his interest in these experiments.

On the Scattering of the β-rays from Radium by Air. (Preliminary Note.) By J. A. CROWTHER, M.A., St John's College.

[*Received* 18 August 1909.]

In a previous paper* I have described some experiments on the scattering of a beam of the β-rays from Uranium by thin sheets of various solid substances. It was shewn that a very thin sheet of metal was sufficient to produce almost complete scattering in a narrow pencil of β-rays; and further that the diminution in intensity of a definite pencil of rays due to the scattering of the rays, might be expressed by an exponential law.

The present paper contains a brief preliminary account of some experiments, still in progress, on the scattering of a beam of β-rays from Radium during its passage through Air.

A pencil of β-rays from Radium is spread out into a magnetic spectrum, a portion of which, consisting of rays having a definite small range of velocities, falls on a small aperture in a lead screen. The rays passing through this aperture are further limited by a second screen, so as to form a narrow divergent pencil of small angle consisting of rays of very nearly the same velocity. The intensity of the β-radiation passing through the geometrical cross-section of this pencil, at various distances from the origin, is measured by the ionization produced in an ionization chamber. It has been found that if I_0 is the initial intensity of the pencil of β-rays, and I the intensity passing through the geometrical cross-section of the beam at a distance d from the origin, then

$$I = I_0 e^{-\sigma . d},$$

where σ is a constant, which may be called the coefficient of scattering.

The following table gives the values of σ for ordinary air

Velocity (cm./sec.)	σ (cm.$^{-1}$)	$v.\sqrt[4]{\sigma}$	$(1-\beta^2)^{-\frac{1}{4}}.v\sqrt[4]{\sigma}$
$2{\cdot}26 \times 10^{10}$	·255	$1{\cdot}61 \times 10^{10}$	$1{\cdot}98 \times 10^{10}$
2·50	·134	1·51	2·03
2·74	·072	1·42	2·23
2·84	·040	1·27	2·23

* *Proc. Roy. Soc.* A, Vol. 80, p. 186. 1908.

for various velocities of the β-rays. The first column of the table gives the velocity of the rays employed, as calculated from their magnetic deflection; the second column the corresponding value of σ, the coefficient of scattering for air at normal temperature and pressure. The third column gives the value of the product $v\sqrt[4]{\sigma}$, where v is the velocity of the rays employed. It will be seen that the coefficient of scattering of the rays varies rather more rapidly than the inverse fourth power of their velocity. This is probably due to the increase in the electrical mass of the β-corpuscles with increasing velocity. Prof. Sir J. J. Thomson* has shewn that the coefficient of scattering is roughly proportional to $1/v^4m^2$, where v is the velocity and m the mass of the β-corpuscle. Assuming the Lorentz formula for the change in mass of an electron with change of velocity, we have

$$m = m_0\{1-\beta^2\}^{-\frac{1}{2}},$$

where β is the ratio of the velocity of the corpuscle to the velocity of light. Taking into account, therefore, this change of mass with velocity we should have

$$(1-\beta^2)^{-\frac{1}{4}} . v . \sqrt[4]{\sigma} = \text{constant}.$$

The last column of the table, which gives the values of this quantity, shews how far this is the case.

* See *Conduction of Electricity through Gases*, p. 377. 1906.

On some general Properties of Mixed Solutions. By L. VEGARD, Universitets-stipendiat of Christiania University.

[*Received* 9 June 1909.]

INTRODUCTION.

§ 1. The object of the following paper will chiefly be to generalise some results concerning properties of solutions, which I have given in a previous paper*. The results found in this paper were, in short, the following:

I. Determination of the variation of concentration in a binary solution exposed to any field of gravity.

(*a*) Applying a purely mechanical equilibrium condition, it was proved that the concentration in the equilibrium state must be constant along an equipotential surface and have its greatest slope in the direction of the force.

(*b*) The concentration gradient was found from the condition that a small volume element acted on by gravity shall be in thermodynamic equilibrium. The result found holds good for any concentration and without regard to the volatility of the compounds.

(*c*) In the case in which the osmotic pressure is known as a function of concentration, the concentration gradient can be found. Numerical calculations were carried out for cane sugar under the assumption that the osmotic pressure follows the gas laws. In the case of electrolytes, the equation had to be slightly modified, and the calculation was carried out for potassium hydrate.

II. (*a*) Using the conception of osmotic pressure, some very interesting relations were found connecting the concentration gradient with the variation of osmotic pressure with hydrostatic pressure. These relations were:

$$(1) \qquad \rho - \rho_0 = \frac{1}{K}\left(\frac{\partial \pi}{\partial c}\right)_p \frac{dc}{dn} + \rho \frac{\partial \pi}{\partial p},$$

$$(2) \qquad \rho - \rho_0 = \frac{1}{K}\left(\frac{\partial \pi}{\partial c}\right)_{p_0} \frac{dc}{dn} + \rho_0 \frac{\partial \pi}{\partial p_0},$$

$$(3) \qquad \frac{\partial \pi}{\partial p_0} = \frac{\partial \pi}{\partial p}\left(1 + \frac{\partial \pi}{\partial p_0}\right);$$

ρ is the density of the solution at pressure p; ρ_0 that of the solvent at p_0; $\pi = p - p_0 =$ osmotic pressure, p and p_0 being the

* "Beiträge zur Theorie der Lösungen." *Christiania Vid. Selsk. Skr.* No. 8, 1906. *Phil. Mag.* [6] 13, p. 589, 1907.

pressure on the solution and solvent respectively. The relations (1, 2, 3) are quite general for binary solutions. They hold for any concentration without regard to volatility of the compounds, and the fluids may be compressible.

(*b*) As a corollary from the three equations it was found that in general $\left(\frac{\partial \pi}{\partial c}\right)_p$ and $\left(\frac{\partial \pi}{\partial c}\right)_{p_0}$ must be different, or the variation of osmotic pressure with concentration will be different according as the pressure during the variation is kept constant on the solution or on the solvent.

The problem we shall deal with in the following is to find the distribution in a solution containing an arbitrary number of substances acted on by any field of gravity, provided equilibrium has set in and the temperature is constant all through the system.

We shall treat the problem in two ways: in one we shall use the thermodynamic potentials, in the other we shall make use of the conception of osmotic pressure. But as the osmotic pressure up to the present has only been defined for binary solutions, it will be necessary to extend it to solutions in general.

Part I.—*Solution of the problem by means of thermodynamic potentials.*

§ 2. *Some properties of thermodynamic potential functions.*—Let the solution be composed of $(r+1)$ substances 0, 1, 2 ... r, with molecular masses $M_0, M_1 \ldots M_r$, and let the concentration be measured by the number of gram-molecules in unit volume $n_0, n_1, n_2 \ldots n_r$, which in the case considered will vary from one point to another. In the following the concentrations as well as their first derivates will be considered as continuous functions of the coordinates. In the case of varying concentration, the thermodynamic potentials of the solution will no longer be homogeneous functions with respect to the total mass of each component: but we must be able to assume, considering the thermodynamic potential $\Delta\omega$ for a volume element Δv, that when Δv approaches zero $\Delta\omega$ will approach the value it would have if the element had contained a homogeneous solution with concentrations equal to those at any point inside the element. Thus we can put

$$\Delta\omega = H(\Delta m_0, \Delta m_1, \Delta m_2 \ldots \Delta m_r, p, T),$$

where $\Delta m_0, \Delta m_1 \ldots \Delta m_r$ are the masses inside the element and H *is the same function as in the case of a homogeneous solution.* T is the temperature, which is supposed constant, p the pressure on the element. Putting $\Delta m_i = M_i n_i \Delta v$:

$$\operatorname{Lim} \frac{\Delta\omega}{\Delta v} = \omega' = H(M_0 n_0, M_1 n_1 \ldots M_r n_r, p, T);$$

ω' is the thermodynamic potential per unit volume at the point considered. The thermodynamic potential for the whole system

$$(4) \qquad \omega = \iiint \omega' dx dy dz.$$

ω' being a homogeneous function of the first degree*,

$$(5) \qquad \begin{aligned} \omega' &= f_0 M_0 n_0 + f_1 M_1 n_1 + \dots f_r M_r n_r \\ &= \phi_0 n_0 + \phi_1 n_1 + \dots \phi_r n_r. \end{aligned}$$

$f_0, f_1 \dots f_r$ are the thermodynamic potentials per unit mass of the various components.

$\phi_0, \phi_1 \dots \phi_r$ are the thermodynamic potentials per gram-molecule of the various components, and

$$(6) \qquad M_i f_i = \phi_i = \frac{\partial \omega'}{\partial n_i};$$

ϕ_i being homogeneous of degree 0 with regard to $n_0, n_1 \dots n_r$,

$$(7a) \qquad \sum_{s=0}^{s=r} \frac{\partial \phi_i}{\partial n_s} n_s = 0,$$

and since

$$(7b) \qquad \frac{\partial \phi_i}{\partial n_s} = \frac{\partial \phi_s}{\partial n_i},$$

$$(7c) \qquad \sum_{s=0}^{s=r} \frac{\partial \phi_s}{\partial n_i} n_s = 0.$$

Let ω be the thermodynamic potential for constant pressure. Then

$$(8a) \qquad \frac{\partial \omega'}{\partial p} = (\text{volume}) = 1,$$

or by means of (5)

$$(8b) \qquad \sum_{0}^{r} \mu_i n_i = 1,$$

where μ_i is the molecular volume of the component (i) under the conditions present. Let $\delta n_0, \delta n_1 \dots \delta n_r$ be simultaneous variations of $n_1, n_2 \dots n_r$. Then from (8*b*) and (7*a*)

$$(9a) \qquad \sum_{0}^{r} \mu_i \delta n_i = 0.$$

The expression to the left in equation (8*b*) is a function of (xyz), and its total differential must be equal to zero, but as (xyz) are independent variables, the partial differentials with regard to

* See Duhern, *Mécanique Chimique*, tome III. livre VI.

x, y and z must be equal to zero. Then differentiating equation (8b) with regard to x, and remembering that $\frac{\partial^2 \omega}{\partial p^2} = 0$,

$$(9b) \qquad \sum_0^r \mu_i \frac{dn_i}{dx} = 0.$$

Taking the variation of the equation (9b),

$$\sum_0^r \mu_i \delta\left(\frac{dn_i}{dx}\right) + \sum_{i=0}^{i=r} \frac{dn_i}{dx} \sum_{s=0}^{s=r} \frac{\partial \mu_i}{\partial n_s} \delta n_s = 0 \ ;$$

but the last term is equal to

$$\sum_{s=0}^{s=r} \delta n_s \frac{\partial}{\partial n_s} \sum_{i=0}^{i=r} \mu_i \frac{dn_i}{dx} = 0,$$

then

$$(9c) \qquad \sum_0^r \mu_i \delta\left(\frac{dn_i}{dx}\right) = 0.$$

§ 3. *Thermodynamic equilibrium conditions.*

As is well known, the condition for thermodynamic equilibrium at constant temperature can be expressed

$$(10) \qquad \begin{cases} (\delta\psi)_{a_1} = (\delta A)_{a_1}, \\ (\delta\psi)_{a_2} = (\delta A)_{a_2}, \\ \cdots\cdots\cdots\cdots \\ (\delta\psi)_{a_r} = (\delta A)_{a_r}; \end{cases}$$

ψ is the internal thermodynamic potential $= I + ST$, where I is the internal energy, S the entropy. δA is the work done during the variation by the external forces acting on the system. The equations (10) express that for any independent variation the system may suffer the change in ψ must equal the work done.

The equilibrium of our fluid system is not disturbed if we assume that a closed surface is laid inside the fluid and this surface becomes rigid. Moreover, the equilibrium is maintained if we imagine that an element of the surface is replaced by a movable piston pressing against the fluid with a pressure P equal to that existing in the fluid at the element considered. Thus we are quite free as regards choice of bounding surface.

The field of gravity to which the system is exposed we shall assume depending on a potential U, where U, as well as its first derivates, are supposed to be continuous in the space occupied by the system.

Now we get

$$\delta(A) = -P\delta \iiint dx\,dy\,dz - \delta \iiint \rho U dx\,dy\,dz,$$

and by means of (4) the equilibrium condition can be expressed

$$\delta \iiint (\psi' + \rho U + P)\, dx dy dz = 0,$$

or introducing the thermodynamic potential for constant pressure ω'

$$\delta \iiint (\omega' + \rho U + P - p)\, dx dy dz = 0.$$

The variation must be subject to the condition that no mass leaves the boundary. Putting

$$(11) \qquad \omega' + \rho U + P - p = W,$$

we get the equilibrium conditions

$$(12a) \qquad \delta \iiint W\, dx dy dz = 0,$$

$$(12b) \qquad \delta \iiint n_i\, dx dy dz = 0, \qquad i = 0, 1, 2 \ldots r.$$

$$(12c) \qquad \delta \iiint \rho\, dx dy dz = 0.$$

(12c) is of course a consequence of (12b).

§ 4. *Solution of the problem of variation.*

We shall now specify our bounding surface; and, as we have seen, this can be done without restricting the generality of the solution. We shall consider the fluid inside a parallelopipedic element (Δx, Δy, Δz), and, for the sake of simplicity, one of the corners may be at the origin of the coordinates. Further, we shall let the element have a needle form, or we put

$$\Delta y = a_1 \Delta x^{n_1}, \quad \Delta z = a_2 \Delta x^{n_2},$$

where a_1 and a_2 are finite quantities and n_1 and n_2 are positive numbers, which can be given any values we like, and thus we can always be able to put

$$n_1 \gtreqless 2 \text{ and } n_2 \gtreqless 2.$$

The two ends of the element we shall denote by (0) and (1). The element is supposed to have rigid boundaries except at the end, which consists of the piston exerting a pressure P.

We shall specify the variation, which consists in a relative displacement of masses inside the element, by supposing that the masses are suffer a translation parallel to the principal axis of the element. Under these conditions the equilibrium conditions take the form

$$(13)\qquad \begin{cases} \delta\int_0^{\Delta x} W\,dx = 0 = W_1\delta(\Delta x) + \int_0^{\Delta x}\delta W\,dx, \\ \delta\int_0^{\Delta x} n_i\,dx = 0 = n_i'\delta(\Delta x) + \int_0^{\Delta x}\delta n_i\,dx, \quad i = 0, 1, 2 \ldots r. \\ \delta\int_0^{\Delta x}\rho\,dx = 0 = \rho_1\delta(\Delta x) + \int_0^{\Delta x}\delta\rho\,dx. \end{cases}$$

W_1, n_i', ρ_1 are the values of W, n_i and ρ at the end (1) of the element. We develop the functions W, n_i and ρ after Maclaurin's formula, and, forming the variations, we get

$$\delta W = \delta W_0 + \delta\left(\frac{dW}{dx}\right)_0 x + \text{etc.},$$

$$\delta n_i = \delta n_i^0 + \delta\left(\frac{dn_i}{dx}\right)_0 x + \text{etc.},$$

$$\delta\rho = \delta\rho_0 + \delta\left(\frac{d\rho}{dx}\right)_0 x + \text{etc.}$$

These values put into (13), and integrating

$$(14a)\qquad W_1\frac{\delta(\Delta x)}{\Delta x} + \delta W_0 + \tfrac{1}{2}\delta\left(\frac{dW}{dx}\right)_0\Delta x + \ldots = 0,$$

$$(14b)\qquad n_i'\frac{\delta(\Delta x)}{\Delta x} + \delta n_i^0 + \tfrac{1}{2}\delta\left(\frac{dn_i}{dx}\right)_0\Delta x + \ldots = 0, \quad i = 0, 1, 2 \ldots r.$$

$$(14c)\qquad \rho_1\frac{\delta(\Delta x)}{\Delta x} + \delta\rho_0 + \tfrac{1}{2}\delta\left(\frac{d\rho}{dx}\right)_0\Delta x + \ldots = 0.$$

To find $\delta(\Delta x)$ we shall apply equation (14*b*); we multiply with μ_i' and add

$$(15a)\qquad \frac{\delta(\Delta x)}{\Delta x}\sum_0^r \mu_i' n_i' + \sum_0^r \mu_i'\left(dn_i^0 + \tfrac{1}{2}\delta\left(\frac{dn_i}{dx}\right)_0\Delta x \ldots\right) = 0.$$

μ_i' is the value of μ_i at the end (1),

$$\mu_i' = \mu_i^0 + \left(\frac{d\mu_i}{dx}\right)_0\Delta x + \ldots.$$

Putting the expression for μ_i' into (15*a*), and using the equations (8*b*), (9*a*) and (9*c*),

$$\frac{\delta(\Delta x)}{\Delta x} + \Delta x\sum_0^r\left(\frac{d\mu_i}{dx}\right)_0\left(\delta n_i^0 + \tfrac{1}{2}\left(\frac{dn_i}{dx}\right)_0\Delta x + \ldots\right) = 0.$$

Using equation (14*b*)

$$(15b)\qquad \frac{\delta(\Delta x)}{\Delta x}\left(1 - \Delta x\sum_0^r\frac{d\mu_i}{dx}n_i'\right) = \alpha\Delta x^2 + \ldots,$$

where α is a quantity containing some linear relation of the form $a_1 \delta \frac{d^p n_i}{dx^p}$, $p \geqq 2$. We may at once remark that if the value for $\delta(\Delta x)$ is put into (14) and we go to the limit $\Delta x = 0$, we find that the term depending on $\delta(\Delta x)$ disappears.

From equation (11) we get

$$\delta W_0 = \delta \omega_0' + U_0 \delta \rho_0,$$

$$\delta \left(\frac{dW}{dx}\right)_0 = \delta \left(\frac{\delta \omega'}{dx}\right) + \left(\frac{dU}{dx}\right)_0 \delta \rho_0 + U_0 \delta \left(\frac{d\rho}{dx}\right)_0 - \delta \left(\frac{dp}{dx}\right)_0;$$

p_0 being the pressure at the bounding surface is not altered by the variation. Putting these expressions into (14*a*) and using (14*c*) and (15)

$$(16) \quad \delta \omega_0' + \tfrac{1}{2} \left\{ \delta \left(\frac{d\omega'}{dx}\right)_0 + \left(\frac{dU}{dx}\right)_0 \delta \rho_0 - \delta \left(\frac{dp}{dx}\right)_0 \right\} \Delta x + \text{etc.} = 0,$$

where etc. is of the form $\alpha_1 \Delta x^q + \alpha_2 \Delta x^{q+1} + \ldots$ $q \geqq 2$.

From equation (5)

$$\delta \omega_0' = \sum_0^r \phi_i^0 \delta n_i^0,$$

$$\delta \left(\frac{d\omega'}{dx}\right)_0 = \delta \left(\frac{dp}{dx}\right)_0 + \sum_0^r \phi_i^0 \delta \left(\frac{dn_i}{dx}\right)_0 + \sum_0^r \left(\frac{dn_i}{dx}\right)_0 \delta \phi_i^0.$$

Putting the expressions for $\delta \omega_0'$ and $\delta \left(\frac{d\omega'}{dx}\right)_0$ into (16), and using (14*b*) the equilibrium conditions take the form

$$\left(\frac{dU}{dx}\right)_0 \delta \rho_0 + \sum_0^r \left(\frac{dn_i}{dx}\right)_0 \delta \phi_i^0 + \epsilon_1 \Delta x + \ldots = 0,$$

or letting Δx converge towards zero

$$(17) \quad \frac{dU}{dx} \delta \rho + \sum_0^r \frac{dn_i}{dx} \delta \phi_i = 0.$$

We drop the index (0), and remember that all values refer to the same point

$$\delta \rho = \sum_0^r M_s \delta n_s,$$

$$\delta \phi_i = \sum_{s=0}^{s=r} \frac{\partial \phi_i}{\partial n_s} \delta n_s.$$

These values put into (17) give

$$(18) \quad \sum_{s=0}^{s=r} \left(M_s \frac{dU}{dx} + \sum_{i=0}^{i=r} \frac{\partial \phi_i}{\partial n_s} \frac{dn_i}{dx} \right) \delta n_s = 0,$$

or putting for brevity's sake

$$M_s \frac{dU}{dx} + \sum_{i=0}^{i=r} \frac{\partial \phi_i}{dn_s} \frac{dn_i}{dx} = Q_s,$$

and remembering equation (9*a*)

$$(19) \quad \begin{cases} Q_0 \delta n_0 + Q_1 \delta n_1 + \dots Q_r \delta n_r = 0, \\ \mu_0 \delta n_0 + \mu_1 \delta n_1 + \dots \mu_r \delta n_r = 0. \end{cases}$$

The equations (19) must be fulfilled for any variation δn_i consistent with the two equations, then

$$(20) \quad \frac{Q_0}{\mu_0} = \frac{Q_1}{\mu_1} = \dots = \frac{Q_r}{\mu_r} = \sum_0^r Q_s n_s.$$

Forming by means of the expression for Q_s the sum to the right,

$$\sum_0^r Q_s n_s = \rho \frac{dU}{dx}.$$

The equations (20) and (9*b*) then give

$$(21) \quad \begin{cases} \frac{\partial \phi_0}{\partial n_0} \frac{dn_0}{dx} + \frac{\partial \phi_1}{\partial n_0} \frac{dn_1}{dx} + \dots \frac{\partial \phi_r}{\partial n_0} \frac{dn_r}{dx} - \frac{dU}{dx} (\rho \mu_0 - M_0) = 0 = E_0, \\ \frac{\partial \phi_0}{\partial n_1} \frac{dn_0}{dx} + \frac{\partial \phi_1}{\partial n_1} \frac{dn_1}{dx} + \dots \frac{\partial \phi_r}{\partial n_1} \frac{dn_r}{dx} - \frac{dU}{dx} (\rho \mu_1 - M_1) = 0 = E_1, \\ \dots\dots\dots\

Thus we see that also in the general case the concentration in the equilibrium state must be constant along a potential surface, and for the same concentration pressure and temperature the concentration gradient will be proportional to the force intensity at the point considered.

The equations (21) determining the concentration fall in any direction are perfectly symmetrical with regard to the $(r+1)$ components. The solution found holds regardless of compressibility of the solution and of the aggregate form of the single components, and for any concentration whatever, that may have physical existence.

§ 5. *Transformations.*

During the theoretical development we have assumed that the concentration is defined as the number of gram-molecules per unit volume. There are, however, several other ways of expressing the concentrations, and it may be useful to find the expression of our equation system corresponding to the most usual forms of concentration.

1. The concentration is defined as the mass of each component contained in unit volume. Let us call these concentrations $C_0, C_1 \dots C_r$, then

$$C_i = M_i n_i. \qquad i = 0, 1, 2 \dots r.$$

Introducing $\phi_i = M_i f_i,$

and $$\frac{\mu_i}{n_i} = q_i,$$

$$(23)\quad \begin{cases} \dfrac{\partial f_i}{\partial C_0}\dfrac{dC_0}{dx} + \dfrac{\partial f_i}{\partial C_1}\dfrac{dC_1}{dx} + \dots \dfrac{\partial f_i}{\partial C_r}\dfrac{dC_r}{dx} = \dfrac{dU}{dx}(\rho q_i - 1), \quad i = 0, 1, 2 \dots r. \\ q_0\dfrac{dC_0}{dx} + q_1\dfrac{dC_1}{dx} + \dots q_r\dfrac{dC_r}{dx} = 0. \end{cases}$$

q_i is the volume which unit mass of the component (i) occupies in solution under the conditions present.

2. The concentration is given as the amount of mass of each component contained in unit mass of solution. Call these concentrations $\sigma_0, \sigma_1 \dots \sigma_r$, then

$$\sigma_i = \frac{C_i}{\rho} = \frac{C_i}{\sum_0^r C_s}.$$

Employing well-known rules for transformation

$$(24)\quad \begin{cases} \dfrac{\partial f_i}{\partial \sigma_0}\dfrac{d\sigma_0}{dx} + \dfrac{\partial f_i}{\partial \sigma_1}\dfrac{d\sigma_1}{dx} + \dots \dfrac{\partial f_i}{\partial \sigma_r}\dfrac{d\sigma_r}{dx} = \dfrac{dU}{dx}(\rho q_i - 1), \quad i = 0, 1, 2 \dots r. \\ \dfrac{d\sigma_0}{dx} + \dfrac{d\sigma_1}{dx} + \dots \dfrac{d\sigma_r}{dx} = 0. \end{cases}$$

3. We choose one of the components, *e.g.*, the first one as the principal component, and we express the concentrations of the others by the ratio between their masses and that of the principal component, all masses being united in the same volume element, or

$$c_1 = \frac{C_1}{C_0}, \quad c_2 = \frac{C_2}{C_0} \dots c_r = \frac{C_r}{C_0}.$$

We have

$$\frac{dC_i}{dx} = c_i \frac{dC_0}{dx} + C_0 \frac{dc_i}{dx},$$

$$\frac{\partial f_i}{\partial C_0} = \frac{\partial f_i}{\partial c_1}\frac{\partial c_1}{\partial C_0} + \frac{\partial f_i}{\partial c_2}\frac{\partial c_2}{\partial C_0} + \dots \frac{\partial f_i}{\partial c_r}\frac{\partial c_r}{\partial C_0},$$

$$\frac{\partial f_i}{\partial C_s} = \frac{\partial f_i}{\partial c_s}\frac{\partial c_s}{\partial C_s}. \qquad s = 1, 2 \dots r.$$

Forming the expression in equation (23) and putting

$$q_i \rho - 1 = \alpha_i,$$

we get the following system

$$(25) \qquad \sum_{s=1}^{s=r} \frac{\partial f_i}{\partial c_s}\frac{dc_s}{dx} = \alpha_i \frac{dU}{dx}. \qquad i = 0, 1, 2 \dots r.$$

In this case there are only r concentrations, and consequently r unknown. If we drop the first of the $(r+1)$ equations (25), we get the following solution

$$(26) \qquad \begin{vmatrix} \frac{\partial f_1}{\partial c_1} & \frac{\partial f_1}{\partial c_2} & \dots & \frac{\partial f_1}{\partial c_i} & \dots & \frac{\partial f_1}{\partial c_r} \\ \frac{\partial f_2}{\partial c_1} & \frac{\partial f_2}{\partial c_2} & \dots & \frac{\partial f_2}{\partial c_i} & \dots & \frac{\partial f_2}{\partial c_r} \\ \vdots & \vdots & & \vdots & & \vdots \\ \frac{\partial f_i}{\partial c_1} & \frac{\partial f_i}{\partial c_2} & \dots & \frac{\partial f_i}{\partial c_i} & \dots & \frac{\partial f_i}{\partial c_r} \\ \vdots & \vdots & & \vdots & & \vdots \\ \frac{\partial f_r}{\partial c_1} & \frac{\partial f_r}{\partial c_2} & \dots & \frac{\partial f_r}{\partial c_i} & \dots & \frac{\partial f_r}{\partial c_r} \end{vmatrix} \frac{dc_i}{dx} = \frac{dU}{dx} \begin{vmatrix} \frac{\partial f_1}{\partial c_1} & \frac{\partial f_1}{\partial c_2} & \dots & \frac{\partial f_1}{\partial c_{i-1}} & \alpha_1 & \frac{\partial f_1}{\partial c_{i+1}} & \dots & \frac{\partial f_1}{\partial c_r} \\ \frac{\partial f_2}{\partial c_1} & \frac{\partial f_2}{\partial c_2} & \dots & \frac{\partial f_2}{\partial c_{i-1}} & \alpha_2 & \frac{\partial f_2}{\partial c_{i+1}} & \dots & \frac{\partial f_2}{\partial c_r} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \frac{\partial f_i}{\partial c_1} & \frac{\partial f_i}{\partial c_2} & \dots & \frac{\partial f_i}{\partial c_{i-1}} & \alpha_i & \frac{\partial f_i}{\partial c_{i+1}} & \dots & \frac{\partial f_i}{\partial c_r} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \frac{\partial f_r}{\partial c_1} & \frac{\partial f_r}{\partial c_2} & \dots & \frac{\partial f_r}{\partial c_{i-1}} & \alpha_r & \frac{\partial f_r}{\partial c_{i+1}} & \dots & \frac{\partial f_r}{\partial c_r} \end{vmatrix}$$

To find $\frac{dc_i}{dx}$ we must know the r quantities $\alpha_1, \alpha_2 \dots \alpha_r$, and, besides, a number of quantities derived from the thermodynamic potentials. As we have

$$(27) \qquad \frac{\partial f_i}{\partial c_s} = \frac{\partial f_s}{\partial c_i},$$

the determinant to the left is symmetrical, and the number of thermodynamic quantities reduces to $\frac{1}{2}r(r+1)$. For a binary solution we get the very simple result

$$\frac{\partial f_0}{\partial c_1}\frac{dc_1}{dx} = (\rho q_0 - 1)\frac{dU}{dx}.$$

In the earlier paper* the quantity q_0 is found for binary solutions

$$\frac{\mu_0}{M_0} = q_0 = \frac{1}{\rho^2}\left\{\rho + c_1(1+c_1)\frac{\partial \rho}{\partial c_1}\right\},$$

and

$$\frac{\partial f_0}{\partial c_1}\frac{dc_1}{dx} = \frac{1}{\rho}c_1(1+c_1)\frac{\partial \rho}{\partial c_1}\frac{dU}{dx},$$

which is the same result as that found in the previous paper.

Part II.—*Extension of the conception osmotic pressure to solutions in general.*

§ 6. In the previous development we have seen that the thermodynamic potentials are very apt for a general mathematical treatment. The thermodynamic quantities appearing in the equations are mathematically very simple, all of them being derived from a single function by means of a simple mathematical operation. In spite of their mathematical simplicity, the thermodynamic potentials have their disadvantages when we are trying to get a clear idea of what they contain physically, and their interpretation is in fact the important thing when we are going to utilise them in any special case. Here it is that the importance of the osmotic pressure comes in; and even if we consider the thermodynamic potential to be the basis for the theory of solutions, the generalisation of the conception of osmotic pressure will still be of importance for a general treatment of solutions. The equations connecting the osmotic pressure with the thermodynamic potentials are from one point of view to be regarded as equations defining the osmotic pressure, but will at the same time give a physical interpretation to the thermodynamic functions.

The conception of osmotic pressure may be generalised in a number of ways; it will be our task to find the most useful. Let the solution, as before, consist of $(r+1)$ substances, and suppose that we have a membrane permeable to (i) of them but impermeable to the rest. The solution may be supposed to be contained in a cylinder and the membrane to have the form of a piston. We imagine that there is at first no fluid above the piston. If we apply a pressure sufficiently high, some substance of the (i)

* *Phil. Mag.* [6] 13, p. 599.

components will pass through, forming above the membrane a solution, which we may call the secondary solution of the osmotic system. By diminishing the pressure, the current through the membrane will be less, until for a certain pressure an equilibrium will set in. There will be (i) equations determining the equilibrium state; from these the $(i-1)$ concentrations of the secondary solution, as well as the osmotic pressure, will be determined. When the osmotic pressures are derived in this way we shall say that they are *formed upon a given solution.*

The number of osmotic systems that can be formed upon a solution is

$$N = \sum_{i=1}^{i=r} \frac{(r+1)!}{i!\,(r-i+1)!}.$$

For a binary solution ..... $N = 2$.

For a solution containing three components... $N = 6$.

For a solution containing four components ... $N = 14$.

For a solution containing five components ... $N = 30$.

The most important of these systems are those we get for $i = 1$ and $i = r$.

§ 7. *The partial osmotic pressure.*

When the membrane is permeable to all but one component, we get an osmotic pressure which is analogous to the partial pressure in a mixture of gases; we may call it the partial osmotic pressure for the component considered.

Let the membrane be impermeable for the component (r), and let $c_1' = \frac{m_1'}{m_0'}$, $c_2' = \frac{m_2'}{m_0'} \ldots c_{r-1} = \frac{m'_{r-1}}{m_0'}$ be the concentrations at the secondary solution. The equilibrium conditions are expressed by the equations

$$(28) \qquad f_i = f_i'', \qquad i = 0, 1, 2 \ldots (r-1).$$

$f_i = F_i(c_1, c_2 \ldots c_r, P)$ = thermodynamic potential per unit mass of component (i) in primary solution.

$f_i'' = F_i(c_1', c_2' \ldots c'_{r-1}, 0, p_r)$ = thermodynamic potential per unit mass of component (i) in secondary solution.

P the pressure on the primary, p_r that on the secondary solution. If $c_1, c_2 \ldots c_r, P$ are supposed to be given, the r equations (28) determine the r unknown $c_1', c_2' \ldots c'_{r-1}, p_r$. What interests us is the osmotic pressure $Q_r = (P - p_r)$.

In order to get a nearly exact expression for this pressure we shall assume that in the state of equilibrium the differences $c_i - c_i'$ are small quantities; or, that the substances for which the

membrane is permeable have nearly the same mutual proportion on both sides of the membrane. On this supposition, neglecting quantities of second order

$$f_i'' = f_i' + \sum_{s=1}^{s=r-1} \frac{\partial f_i'}{\partial c_s}(c_s' - c_s),$$

where $$f_i' = F(c_1, c_2 \dots c_{r-1}, 0, p_r).$$

We multiply the equations (28) with $1, c_1, c_2 \dots c_{r-1}$ in succession and add them together, and inserting the value for f_i''

$$(29) \qquad \sum_{0}^{r-1}(f_i - f_i')c_i = \sum_{s=1}^{s=r-1}(c_s' - c_s)\sum_{i=0}^{i=r-1}\frac{\partial f_i}{\partial c_s}c_i,$$

where we have to remember that $c_0 = 1$. From equation $(7a)$ we get

$$\sum_{i=0}^{i=r-1}\frac{\partial f_i'}{\partial c_s}c_i = 0.$$

From a well-known property of thermodynamic potential $\frac{\partial f_i'}{\partial p} = q_i'$ where q_i is the volume occupied by unit mass of the component (i) in a solution of the same substances as the secondary solution, but with concentrations equal to those of the primary one $c_1, c_2 \dots c_{r-1}$. Then

$$F_i(c_1, c_2 \dots c_{r-1}, 0, P) - F_i(c_1, c_2 \dots c_{r-1}, 0, p_r) = \int_{p_r}^{P} q_i' dp.$$

$$F_i(c_1, c_2 \dots c_{r-1}, c_r, P) - F_i(c_1, c_2 \dots c_{r-1}, 0, P) = \int_{0}^{c_r}\frac{\partial f_i}{\partial c_r}dc_r.$$

Adding these two equations together

$$f_i - f_i' = \int_0^{c_r}\frac{\partial f_i}{\partial c_r}dc_r + \int_{p_r}^{P} q_i' dp.$$

Inserting into (29)

$$(30) \qquad \int_0^{c_r}\left(\sum_{i=0}^{i=r-1}\frac{\partial f_i}{\partial c_r}c_i\right)dc_r = -\int_{p_r}^{P}\left(\sum_{i=0}^{i=r-1} q_i' c_i\right)dp,$$

but

$$\sum_{i=0}^{i=r-}\frac{\partial f_i}{\partial c_r}c_i = -c_r\frac{\partial f_r}{\partial c_r},$$

$$\sum_{0}^{r-1} q_i' c_i = \frac{V'}{(m_0)''},$$

where V' is the volume of a solution with concentrations $c_1, c_2 \dots c_{r-1}$ and $(m_0)''$ the amount of mass of the component (0) contained in the same solution.

Equation (30) then takes the form

$$\frac{(m_0)''}{\int_{p_r}^{P} V' dp} = \frac{m_0}{\int_0^{m_r} m_r \frac{\partial f_r}{\partial m_r} dm_r}.$$

In general we have

$$\frac{(m_i)''}{\int_{p_r}^{P} V' dp} = \frac{m_i}{\int_0^{m_r} m_r \frac{\partial f_r}{\partial m_r} dm_r}. \quad i = 0, 1, 2 \ldots (r-1).$$

Taking the sum of these equations from $i = 0$ to $i = r-1$,

$$\frac{\sum_0^{r-1} (m_i)''}{\int_{p_r}^{P} V' dp} = \frac{\sum_0^{r-1} m_i}{\int_0^{m_r} m_r \frac{\partial f_r}{\partial m_r} dm_r},$$

but $\frac{1}{\rho'} = \frac{V'}{\Sigma (m_i)''}$ = specific volume of a solution of concentrations $c_1, c_2 \ldots c_{r-1}$, and introducing

$$K_r = \frac{m_r}{m_0 + m_1 + \ldots m_{r-1}},$$

the osmotic pressure will be defined by the equation

$$(31a) \qquad \int_{p_r}^{P} \frac{1}{\rho'} dp = \int_0^{K_r} K_r \frac{\partial f_r}{\partial K_r} dK_r.$$

If $\frac{1}{\rho_m'}$ is the mean value of $\frac{1}{\rho'}$ in the interval $P - p_r$ we get for the osmotic pressure Q_r

$$(31b) \qquad Q_r = \rho_m' \int_0^{K_r} K_r \frac{\partial f_r}{\partial K_r} dK_r.$$

This expression is well known for binary solutions, and we see that with a proper modification of the quantities ρ_m, K_r and f_r *the same expression holds for the partial osmotic pressure.*

§ 8. *Osmotic pressure of the first order.*

When the secondary solution only contains one substance, we shall designate the corresponding osmotic pressure to be of the first order. Let π_i denote the osmotic pressure, when the membrane is permeable to the substance (i). From the condition of equilibrium we get in the usual way

$$(32) \qquad \frac{\pi_i}{\rho_{mi}} = \int_{p_i}^{P_i} \frac{1}{\rho_i} dp = g_i - f_i.$$

f_i = thermodynamic potential per unit mass of component (i) of the solution at pressure P_i.

g_i = thermodynamic potential per unit mass of pure substance (i) at pressure P_i.

ρ_i is the density of the pure component (i), and ρ_{mi} its mean value between P_i and p_i.

Partial derivation of (32) with respect to c_s gives

$$(33) \qquad \frac{\partial F_i(c_1, c_2 \dots c_r, P_i)}{\partial c_s} = -\frac{1}{\rho_{mi}} \left(\frac{\partial \pi_i}{\partial c_s}\right)_{P_i}.$$

Assuming that the pressures on the solution P_i are the same for all osmotic systems and equal to p, *the equation* (33) *will give an expression in terms of osmotic pressure for all the thermodynamic quantities appearing in equation* (26).

From the equation (27) we get the following relation

$$(34) \qquad \frac{1}{\rho_{mi}} \left(\frac{\partial \pi_i}{\partial c_s}\right)_p = \frac{1}{\rho_{ms}} \left(\frac{\partial \pi_s}{\partial c_i}\right)_p.$$

This interesting reciprocal relation between the osmotic pressures of the first order, which is a simple corollary from their connections with the thermodynamic potential, would have been very difficult to obtain from a separate consideration of the osmotic pressures themselves.

Part III.—*The effect of gravity upon solutions found by means of the osmotic pressures of the first order.*

§ 9. The osmotic pressure of the first order enables us to generalise equations (1) to solutions in general. The method used in the previous paper only needs to be slightly modified. Instead of only one, we have to form (r) systems, which we get by supposing the solution successively in osmotic equilibrium with the r substances $1, 2 \dots r$, which, at the temperature considered, all are supposed to be in the liquid state. Moreover, we must assume that in the state of equilibrium the pressure in the solution is the same for all osmotic systems.

Using the same way of reasoning as before*, we get

$$(35) \qquad \sum_{s=1}^{s=r} \left(\frac{\partial \pi_i}{\partial c_s}\right)_p \frac{dc_s}{dx} = \left(\rho_i - \rho + \rho \frac{\partial \pi_i}{\partial p}\right) \frac{dU}{dx}, \qquad i = 1, 2 \dots r.$$

$\frac{dU}{dx}$ and $\frac{dc_s}{dx}$ mean here the variation of U and c per unit length

* *Phil. Mag.* [6] 13, pp. 607, 608.

from a point on the surface of the membrane and along some direction in this surface, but as the direction of the surface is quite arbitrary, the increment dx may have any direction whatever.

From equation (32) we get

$$\frac{\partial \pi_i}{\partial p} = 1 - \rho_i q_i .$$

Putting the value for $\frac{\partial \pi_i}{\partial p}$ into (35)

$$(36) \qquad \sum_{s=1}^{s=r} \frac{1}{\rho_i} \left(\frac{\partial \pi_i}{\partial c_s}\right)_p \frac{dc_s}{dx} = (1 - \rho q_i) \frac{dU}{dx} . \qquad i = 1, 2 \ldots r.$$

If we transform this system of (r) linear equations by means of equation (33) we get the result expressed in equation (25), showing that the osmotic method leads to the same result as that in which we used the thermodynamic potentials.

There is, however, one difference between the two solutions of the problem. In order to apply the osmotic pressure method, we had to assume that the r components were in a liquid state; the treatment with thermodynamic potentials only requires that the *solution* is fluid at the temperature considered, and the components may have any aggregate form whatever.

§ 10. *Summary of results.*

(1) Applying the general thermodynamic equilibrium condition, the variation of concentration caused by any field of gravity is found for a solution containing any number of substances, and the result is expressed in terms of thermodynamic potentials.

(2) The conception of osmotic pressure is extended to a solution containing any number of substances.

(3) Simple expressions in terms of thermodynamic potentials are found for the partial osmotic pressure and for the osmotic pressure of the first order.

(4) By means of the osmotic pressure of the first order we obtained a physical interpretation of the thermodynamic quantities $\frac{\partial f_i}{\partial c_s}$. From their connection with the thermodynamic potentials a reciprocal relation was found for the osmotic pressure of the first order.

(5) The osmotic pressure of the first order gave us a simple way of finding the influence of gravity upon a solution.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1908—1909.

ANNUAL GENERAL MEETING.

October 26th, 1908.

In the Optical Lecture Room.

Dr Hobson, President, in the Chair.

The following were elected officers for the ensuing year:

President:

Prof. Sedgwick.

Vice-Presidents:

Mr S. Ruhemann.
Prof. Thomson.
Dr Hobson.

Treasurer:

Mr H. F. Newall.

Secretaries:

Mr A. E. Shipley.
Dr Barnes.
Mr A. Wood.

Other Members of the Council:

Mr F. G. Hopkins.
Mr A. Harker.
Prof. Larmor.
Dr Duckworth.
Mr W. G. Fearnsides.
Dr Sell.
Mr W. E. Dixon.
Prof. Wood.
Prof. Hopkinson.
Prof. Seward.
Dr Fenton.
Mr G. H. Hardy.

The following was elected a Fellow of the Society:

G. N. Watson, B.A., Trinity College.

The following Communications were made:

1. Note on Russo's attempt to show differentiation of sex in the ovarian ova of the Rabbit. By W. Heape, M.A., Trinity College.

2. A further note on the eggs of the hermaphrodite *Angiostomum nigrovenosum*. By S. A. M^{c}Dowall, M.A., Trinity College.

3. Plemelj's Canonical Form. By J. Mercer, B.A., Trinity College.

4. On Monotone Sequences of Continuous Functions. By Dr Young.

5. The Operator Reciprocants of Sylvester's Theory of Reciprocants. By Major P. A. MacMahon.

November 9th, 1908.

In the Cavendish Laboratory.

Professor Sedgwick, President, in the Chair.

The following were elected Fellows of the Society:

J. Pope, M.A., Professor of Chemistry.
A. Henry, M.A., Reader in Forestry.

The following Communications were made:

1. The positive ions given out by hot wires. By Professor Thomson.

2. The weight of a corpuscle on the electrical theory of gravitation. By Professor Thomson.

3. Note on the distribution of electric force along the striated discharge. By Professor Thomson.

4. Note on the Radio-activity of Rhubidium. By N. R. Campbell, M.A., Trinity College.

5. The free pressure under osmosis. By L. Vegard. (Communicated by Professor Thomson.)

6. The Laws of Mobility and Diffusion of the Ions formed in Gaseous Media. By E. M. Wellisch. (Communicated by Professor Thomson.)

November 23rd, 1908.

In the New Medical Schools.

PROFESSOR SEDGWICK, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

V. H. Mottram, M.A., Trinity College.
G. R. Mines, B.A., Sidney Sussex College.

The following Communications were made:

1. The relationship between human and bovine tuberculosis. By Professor WOODHEAD.

2. The transmission of Trypanosoma lewisi by fleas and lice. By Professor NUTTALL.

3. The presence of anticoagulin in the salivary glands of *Argas persicus*. By Professor NUTTALL.

4. The mode of action of specific substances. By W. E. DIXON, M.A., Downing College, and P. HAMILL, B.A., Trinity College.

5. The action of specific substances in toxaemia. By W. E. DIXON, M.A., Downing College, and W. H. HARVEY, B.A., Christ's College.

6. Therapeutic Inoculation for generalised bacterial infections. By L. NOON, M.A., Trinity College.

7. A simple method for examining leucocytes. By C. PONDER, M.A., Emmanuel College.

8. The mode of growth of bacteria. By Dr GRAHAM-SMITH.

9. The radiation of neon in a strong magnetic field. By J. E. PURVIS, M.A., St John's College.

10. On the effect of Pressure on the ionization produced by Röntgen Rays in different gases and vapours. By J. A CROWTHER, B.A., St John's College.

11. On the variation of the relative ionization produced by Röntgen Rays in different gases with the hardness of the rays. By J. A. CROWTHER, B.A., St John's College.

12. Waves in a stream of viscous liquid. By W. J. HARRISON, M.A., Clare College.

January 25th, 1909.

In the Cavendish Laboratory.

PROFESSOR SIR J. J. THOMSON, VICE-PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

J. E. Littlewood, B.A., Trinity College.

The following Communications were made:

1. A String Electroscope. By T. H. Laby, B.A., Emmanuel College.

2. Secondary Röntgen Radiation. By J. A. Crowther, B.A., St John's College. (Communicated by Professor Sir J. J. Thomson.)

3. Interference Fringes with Feeble Light. By G. I. Taylor, B.A., Trinity College. (Communicated by Professor Sir J. J. Thomson.)

4. The solution of Linear Differential Equations by means of Definite Integrals. By H. Bateman, M.A., Trinity College.

February 8th, 1909.

In the Chemical Laboratory.

Mr S. Ruhemann, Vice-President, in the Chair.

The following were elected Fellows of the Society:

W. L. Balls, M.A., St John's College.
W. H. Harvey, B.A., Christ's College.
H. C. Haslam, M.B., Gonville and Caius College.

The following Communications were made:

1. Further studies on Dihydroxymaleic Acid. By Dr Fenton and W. A. R. Wilks, B.A., Gonville and Caius College.

2. Homologues of Furfural. By Dr Fenton and F. Robinson, B.A., Peterhouse.

3. Action of Urethane on Esters of Organic Acids and Mustard Oils. By S. Ruhemann, M.A., Gonville and Caius College, and J. G. Priestley.

4. The absorption spectrum of a solid alkyl derivative of picene. By Miss A. Homer and J. E. Purvis, M.A., St John's College.

5. The absorption spectra of mesitylene and 3-chloromesitylene. By J. E. Purvis, M.A., St John's College.

6. The absorption spectra of concentrated and diluted solutions of chlorophyll. By J. E. Purvis, M.A., St John's College.

7. A coloured thio-oxalate. By H. O. Jones, M.A., Clare College, and H. S. Tasker, B.A., Emmanuel College.

8. Note on some double Fluorides of Sodium. By W. A. R. Wilks, B.A., Gonville and Caius College. (Communicated by Dr Fenton.)

9. On a configuration of twenty-seven hyper-planes in four dimensional spaces. By W. Burnside, M.A., Pembroke College.

February 22nd, 1909.

In the Optical Lecture Room.

PROFESSOR SEDGWICK, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

J. A. Crowther, B.A., St John's College.
H. H. Paine, B.A., Trinity College.
F. Shillington Scales, M.A., Jesus College.

The following Communications were made:

1. On the alleged influence of lecithin on the determination of sex in rabbits. By R. C. PUNNETT, M.A., Gonville and Caius College.

2. Observations on the Changes in the Common Shore Crab caused by *Sacculina*. By F. A. POTTS, M.A., Trinity Hall.

3. On a so-called "sexual" method of forming spores in Bacteria. By C. C. DOBELL, B.A., Trinity College.

4. On the migration of the thread cells of *Moerisia*. By C. L. BOULENGER, B.A., King's College.

5. A note on a specimen of *Pelagothuria* from the Seychelles. By J. C. SIMPSON. (Communicated by Professor Sedgwick.)

6. The Study of Discontinuous Phenomena. By N. R. CAMPBELL, M.A., Trinity College.

March 8th, 1909.

In the Optical Lecture Room.

PROFESSOR SEDGWICK, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

G. Dixon, M.A., Trinity College.

The following Communications were made:

1. On the Nature of Anthocyanin. By Miss M. WHELDALE. (Communicated by Professor Bateson.)

2. An Experiment on Ionisation with γ Rays. By L. VEGARD. (Communicated by Professor Sir J. J. Thomson.)

3. The Nature of the Ionisation produced in a Gas by γ Rays. By R. D. KLEEMAN, B.A. (Communicated by Professor Sir J. J. Thomson.)

4. On Uniform Oscillation. By Dr YOUNG.

5. On the parametric representation of the co-ordinates of points on a cube surface in space of four dimensions. By H. W. Richmond, M.A., King's College.

6. The irreducible concomitants of two quadratics in n variables. By H. W. Turnbull, B.A. (Communicated by Dr Barnes.)

May 3rd, 1909.

In the Botany School.

Dr Hobson, Vice-President, in the Chair.

The following Communications were made:

1. On a specimen of the cone *Calamostachys binneyana* Carruthers. By H. H. Thomas, B.A., Downing College. (Communicated by Mr E. A. N. Arber.)

2. Note on two new leeches from Ceylon. By W. A. Harding, M.A., Peterhouse.

3. Note on an abnormal pair of appendages in *Lithobius*. By L. Doncaster, M.A., King's College.

4. On a property of summable functions. By Dr A. C. Dixon.

May 17th, 1909.

In the Cavendish Laboratory.

Professor Sir J. J. Thomson, Vice-President, in the Chair.

The following Communications were made:

1. Phenomena of X-ray Transmission. By C. G. Barkla, M.A., King's College. (Communicated by Professor Sir J. J. Thomson.)

2. Phenomena of the Cathode Discharge. By J. A. Orange, B.A., Trinity College. (Communicated by Professor Sir J. J. Thomson.)

3. Some fatigue effects of the Cathode in a discharge tube. By R. Whiddington, B.A., St John's College. (Communicated by Professor Sir J. J. Thomson.)

4. The influence of dilution on the color and the absorption spectra of various permanganates. By J. E. Purvis, M.A., St John's College.

5. Note on the Histology of the 'Giant' and ordinary Forms of *Primula sinensis*. By R. P. Gregory, M.A., St John's College.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On the Oscillations of Superposed Fluids. By H. J. PRIESTLEY, M.A. [Communicated by W. Welsh, M.A.]

[*Received* 28 October 1909.]

[*Read* 8 November 1909.]

In his classical research "On the Theory of Oscillatory Waves" Stokes* has shewn that the first order period equation for waves at the free surface of a liquid is modified when we take into consideration the third order terms. The present paper is undertaken with the object of finding the corresponding modification for waves at the common surface of two fluids; and ascertaining the effect of the small terms on the stability when the upper fluid has a stream velocity. It is found that the range of wave lengths for which oscillations are possible is greater than that given by the first order equation. When the upper fluid has no stream velocity the period given by this equation is too long in the case of propagated waves and too short in the case of standing waves.

For the propagated waves the method adopted is a modification of that used by Stokes in his Supplement† to the paper quoted above. This method is found unsuitable for the standing waves, and it is necessary to use direct approximations from the ordinary equations of motion. In each case the origin is taken in the undisturbed surface, with the axis of y vertically upwards. The sign given to the velocity potential is that adopted by Professor Lamb, so that the velocity along the x axis is $-\dfrac{\partial \phi}{\partial x}$.

* *Math. and Phys. Papers*, Tome I. pp. 197 *et seq.*

† *Ibid.* pp. 317 *et seq.*

I. Propagated Waves.

Reducing the problem to one of steady motion by the usual artifice we express the coordinates of any point in terms of the velocity potential and stream function by means of the equations

$$x_1 + \iota y_1 = -(\phi_1 + \iota\psi_1)/U_1 + \iota \sum_{m=0}^{m=\infty} A_m e^{\iota m k(\phi_1+\iota\psi_1)/U_1} \quad \ldots\ldots (1)$$

for the lower liquid, and

$$x_2 + \iota y_2 = -(\phi_2 + \iota\psi_2)/U_2 + \iota \sum_{m=0}^{m=\infty} B_m e^{-\iota m k(\phi_2+\iota\psi_2)/U_2} \ldots\ldots (2)$$

for the upper.

$A_1, A_2, A_3 \ldots$ etc., $B_1, B_2, B_3 \ldots$ etc. are two series of quantities in descending order of magnitude.

The conditions to be satisfied at the common surface are

$$x_1 + \iota y_1 = x_2 + \iota y_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3),$$

and the pressure condition

$$\rho_1/S_1 + 2g\rho_1(y_1 + C_1) = \rho_2/S_2 + 2g\rho_2(y_2 + C_2) \quad \ldots\ldots (4),$$

where

$$1/S = \left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2,$$

and C_1 and C_2 are constants.

We have yet to choose our origin for ψ and we select the common surface as the line $\psi = 0$.

Writing $\vartheta = \phi/U$, condition (3) gives

$$\vartheta_2 - \vartheta_1 = \iota \sum_{0}^{\infty} (B_m e^{-\iota m k \vartheta_2} - A_m e^{\iota m k \vartheta_1}).$$

We proceed to find relations between the A's and B's from this equation.

1st Approximation.

$$\vartheta_2 - \vartheta_1 = \iota(B_0 - A_0),$$

whence $A_0 = B_0$

and $\vartheta_1 = \vartheta_2$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$(A).

We take as the common first approximation to ϑ_1 and ϑ_2 the mean of the two exact values which we denote by ϑ.

2nd Approximation.

$$\vartheta_2 - \vartheta_1 = \iota(B_0 - A_0) + (A_1 + B_1)\sin k\vartheta + \iota(B_1 - A_1)\cos k\vartheta,$$

whence

$$\left.\begin{aligned} A_1 &= B_1 = \beta \text{ (say)} \\ \vartheta_2 - \vartheta_1 &= 2\beta \sin k\vartheta \\ \therefore\ \vartheta_2 &= \vartheta + \beta \sin k\vartheta \\ \vartheta_1 &= \vartheta - \beta \sin k\vartheta \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(B).}$$

3rd Approximation.

$$\Im_2 - \Im_1 = \iota (B_0 - A_0) + \iota e^{-\iota k\beta \sin k\Im} [B_1 e^{-\iota k\Im} - A_1 e^{\iota k\Im}] + \iota [B_2 e^{-2\iota k\Im} - A_2 e^{2\iota k\Im}],$$

whence we obtain, by equating coefficients of different harmonic terms in the imaginary part,

$$\left.\begin{aligned} B_0 - A_0 &= k\beta^2 \\ A_2 - B_2 &= k\beta^2 \\ A_1 &= B_1 \end{aligned}\right\} \quad \text{.....(C),}$$

while the real part gives

$$\Im_2 = \Im + \beta \sin k\Im + \tfrac{1}{2}(A_2 + B_2) \sin 2k\Im,$$
$$\Im_1 = \Im - \beta \sin k\Im - \tfrac{1}{2}(A_2 + B_2) \sin 2k\Im.$$

[β may now differ from its former value by a small term of order $k\beta^2$.]

4th Approximation.

$$\Im_2 - \Im_1 = \iota (B_0 - A_0) + \iota e^{-\iota k[\beta \sin k\Im + \frac{1}{2}(A_2 + B_2) \sin 2k\Im]} [B_1 e^{-\iota k\Im} - A_1 e^{\iota k\Im}] + \iota e^{-2\iota k\beta \sin k\Im} [B_2 e^{-2\iota k\Im} - A_2 e^{2\iota k\Im}] + \iota [B_3 e^{-3\iota k\Im} - A_3 e^{3\iota k\Im}].$$

Equating coefficients of the different harmonic terms in the imaginary part we obtain

$$\left.\begin{aligned} B_0 - A_0 &= k\beta^2 \\ B_1 - A_1 &= \tfrac{3}{2}\beta k (A_2 + B_2) \\ B_2 - A_2 &= -k\beta^2 \\ B_3 - A_3 &= -\tfrac{3}{2}\beta k (A_2 + B_2) \end{aligned}\right\} \quad \text{.....(D).}$$

Pressure Condition.

To determine completely the quantities A and B we require another set of relations between them. These relations are found from the pressure equation (4). To use this equation we must find $1/S$ in terms of sines and cosines of multiples of $\Im$ and this we proceed to do.

Value of $1/S_1$.

$$S_1 = \left\{ \left| \frac{dz_1}{dw_1} \right| \right\}^2, \text{ where } \begin{aligned} z_1 &= x_1 + \iota y_1, \\ w_1 &= \phi_1 + \iota \psi_1. \end{aligned}$$

From (1)

$$U_1 \frac{dz_1}{dw_1} = -1 - k [A_1 e^{\iota k w_1 / U_1} + 2A_2 e^{2\iota k w_1 / U_1} + \ldots]$$
$$= -[(1 + kA_1 \cos k\Im_1 + 2kA_2 \cos 2k\Im_1 + \ldots) + \iota (kA_1 \sin k\Im_1 + 2kA_2 \sin 2k\Im_1 + \ldots)],$$

when $\psi_1 = 0$.

Thus $S_1 = U_1^{-2} [1 + 2k\Sigma m A_m \cos mk\Im_1 + k^2 \Sigma m^2 A_m^2 + 2k^2 \Sigma mn A_m A_n \cos (m - n) k\Im_1],$

the last summation being taken once for each pair of values m and n,

$$= U_1^{-2}[1 + k^2A_1^2 + 2k\{A_1 \cos k\vartheta_1 + \ldots + 3A_3 \cos 3k\vartheta_1\} + 2k^2\{2A_1A_2 \cos k\vartheta_1\}],$$

neglecting terms of order higher than $k^3\beta^3$.

Expanding $1/S_1$ by the Binomial Theorem and neglecting terms of order higher than $k^3\beta^3$ we have

$$1/S_1 = U_1^2\left[\begin{array}{l} 1 - k^2A_1^2 - 2k\{A_1 \cos k\vartheta_1 + 2A_2 \cos 2k\vartheta_1 \\ \qquad\qquad + 3A_3 \cos 3k\vartheta_1\} \\ - 2k^2A_1\{2A_2 \cos k\vartheta_1\} \\ + 4k^2\{A_1^2 \cos^2 k\vartheta_1 + 4A_1A_2 \cos k\vartheta_1 \cos 2k\vartheta_1\} \\ + 4k^3A_1^3 \cos k\vartheta_1 - 8k^3A_1^3 \cos^3 k\vartheta_1 \end{array}\right].$$

We pass to the corresponding value of $1/S_2$ by changing the sign of k and using B coefficients.

Since the origin is in the undisturbed surface the pressure equation is satisfied over the line $y = 0$ when there is no wave motion. This gives

$$\rho_1U_1^2 + 2g\rho_1C_1 = \rho_2U_2^2 + 2g\rho_2C_2.$$

Using this condition we obtain the following relations from the pressure equation.

1st Approximation.

$$2g\rho_1A_0 = 2g\rho_2B_0,$$

which, with $A_0 = B_0$ (equations A), gives

$$A_0 = 0,$$
$$B_0 = 0.$$

2nd Approximation.

$$\rho_1U_1^2(-2kA_1 \cos k\vartheta_1) + 2g\rho_1(A_0 + A_1 \cos k\vartheta_1) = \rho_2U_2^2(2kB_1 \cos k\vartheta_2) + 2g\rho_2(B_0 + B_1 \cos k\vartheta_2).$$

Remembering $A_0 = B_0$, $A_1 = B_1 = \beta$, (B), and

$$\vartheta_1 = \vartheta_2, \text{ (A)},$$

this gives $A_0 = B_0 = 0$,

and $g(\rho_1 - \rho_2)/k = \rho_1U_1^2 + \rho_2U_2^2$,

the ordinary period equation*.

3rd Approximation.

$$\rho_1U_1^2(-2kA_1 \cos k\vartheta_1 - 4kA_2 \cos 2k\vartheta_1 - k^2A_1^2 + 4k^2A_1^2 \cos^2 k\vartheta_1) + 2g\rho_1[A_0 + A_1 \cos k\vartheta_1 + A_2 \cos 2k\vartheta_1]$$
$$= \rho_2U_2^2(2kB_1 \cos k\vartheta_2 + 4kB_2 \cos 2k\vartheta_2 - k^2B_1^2 + 4k^2B_1^2 \cos^2 k\vartheta_2) + 2g\rho_2[B_0 + B_1 \cos k\vartheta_2 + B_2 \cos 2k\vartheta_2].$$

* Lamb, *Hydrodynamics*, § 224, ed. 1895.

Expressing the harmonic terms in terms of Θ instead of Θ_1 and Θ_2, and neglecting terms of order $k^3\beta^3$ we obtain, by equating coefficients of different harmonic terms to zero,

$$2(\rho_1 A_0 - \rho_2 B_0) = -(\rho_1 + \rho_2) k\beta^2,$$
$$g(\rho_1 - \rho_2)/k = \rho_1 U_1^2 + \rho_2 U_2^2 = K, \text{ (say)},$$
$$\rho_1 U_1^2 [3k^2\beta^2 - 4kA_2] + 2g\rho_1 [A_2 - \tfrac{1}{2}k\beta^2]$$
$$= \rho_2 U_2^2 [3k^2\beta^2 + 4kB_2] + 2g\rho_2 [B_2 + \tfrac{1}{2}k\beta^2].$$

These equations, combined with (C) above, give

$$\left.\begin{aligned} A_0 &= -\tfrac{1}{2}k\beta^2 \\ B_0 &= \tfrac{1}{2}k\beta^2 \\ A_2 &= k\beta^2 \rho_1 U_1^2/K \\ B_2 &= -k\beta^2 \rho_2 U_2^2/K \end{aligned}\right\} \quad \text{.....(E).}$$

We note that, as in the case of waves on a free surface, the period equation is not changed at this stage of the approximation.

4th Approximation.

In proceeding to the next approximation we adopt the same method as before. The period equation is obtained from the coefficient of $\cos k\Theta$ in the reduced pressure condition. This coefficient gives

$$\rho_1 U_1^2 [-2kA_1 - \tfrac{1}{2}k^2\beta(A_2 + B_2) + \tfrac{1}{4}k^3\beta^3]$$
$$+ 2g\rho_1 [A_1 + \tfrac{1}{4}k\beta(A_2 + B_2) - \tfrac{1}{8}k^2\beta^3 + k\beta A_2]$$
$$= \rho_2 U_2^2 [2kB_1 - \tfrac{1}{2}k^2\beta(A_2 + B_2) - \tfrac{1}{4}k^3\beta^3]$$
$$+ 2g\rho_2 [B_1 - \tfrac{1}{4}k\beta(A_2 + B_2) - \tfrac{1}{8}k^2\beta^3 - k\beta B_2].$$

Remembering that

$$gk^{-1}(\rho_1 - \rho_2) = \rho_1 U_1^2 + \rho_2 U_2^2$$

to the order $k\beta$ and

$$B_1 - A_1 = \tfrac{3}{2}\beta k(A_2 + B_2) = \tfrac{3}{2}k^2\beta^3(\rho_1 U_1^2 - \rho_2 U_2^2)/(\rho_1 U_1^2 + \rho_2 U_2^2),$$

we obtain for the period equation

$$gk^{-1}(\rho_1 - \rho_2) - K = -k^2\beta^2(\rho_1^2 U_1^4 + \rho_2^2 U_2^4)/(\rho_1 U_1^2 + \rho_2 U_2^2) \quad \text{...(5).}$$

Before discussing this result we proceed to find the wave profile. To do this we require the values of the A's and B's to the order $k^2\beta^3$.

The coefficients of the absolute term and $\cos 2k\Theta$ in the pressure equation are the same as the corresponding coefficients in the third approximation. Hence A_0, B_0, A_2, B_2 have values already given (E).

A_3 and B_3 are found from the coefficient of $\cos 3k\Theta$, which, with the help of (D), gives

$$A_3 = \tfrac{3}{4}\rho_1 U_1^2 (2\rho_1 U_1^2 - \rho_2 U_2^2) K^{-2}\beta^3 k^2,$$
$$B_3 = \tfrac{3}{4}\rho_2 U_2^2 (2\rho_2 U_2^2 - \rho_1 U_1^2) K^{-2}\beta^3 k^2.$$

Equation (1) gives us the transcendental equation of the free surface in the form

$$x = -\vartheta_1 - A_1 \sin k\vartheta_1 - A_2 \sin 2k\vartheta_1 - A_3 \sin 3k\vartheta_1,$$
$$y = A_0 + A_1 \cos k\vartheta_1 + A_2 \cos 2k\vartheta_1 + A_3 \cos 3k\vartheta_1.$$

From these we find

$$\vartheta_1 = -x + A_1 \sin kx + (A_2 - \tfrac{1}{2}kA_1^2) \sin 2kx,$$

and, using values of coefficients already found,

$$y = [A_1 + \tfrac{3}{8}k^2\beta^3 (4\rho_1 U_1^2 K^{-1} - 1)] \cos kx$$
$$+ \tfrac{1}{2}k\beta^2 (\rho_1 U_1^2 - \rho_2 U_2^2) K^{-1} \cos 2kx$$
$$+ \tfrac{3}{8}k^2\beta^3 (1 - 6\rho_1 U_1^2 . \rho_2 U_2^2 . K^{-2}) \cos 3kx.$$

Writing b for the amplitude of the principal harmonic term we have, to order $k^2\beta^3$,

$$y = b \cos kx + \tfrac{1}{2}kb^2 (\rho_1 U_1^2 - \rho_2 U_2^2) K^{-1} \cos 2kx$$
$$+ \tfrac{3}{8}k^2 b^3 [1 - 6\rho_1 U_1^2 . \rho_2 U_2^2 . K^{-2}] \cos 3kx \quad \ldots\ldots(6).$$

Both this equation of the wave profile and the period equation reduce to Stokes' equations for waves on a free surface on putting $\rho_2 = 0$.

In discussing these results we shall denote the wave velocity by c and the stream velocity of the upper fluid by u, so that

$$U_1 = -c; \quad U_2 = u - c.$$

Writing the period equation in the form

$$gk^{-1} (\rho_1 - \rho_2) - K = -k^2\beta^2 [K - 2\rho_1 U_1^2 . \rho_2 U_2^2 . K^{-1}],$$

and remembering that K may be put equal to $gk^{-1} (\rho_1 - \rho_2)$ in the small terms, we notice that, corresponding to a given wave length, the small terms have their greatest importance when $U_2 = 0$ or $c = u$ and their least importance when $\rho_1 U_1^2 = \rho_2 U_2^2$.

The first case gives

$$c^2 = gk^{-1} (1 - s) [1 + k^2\beta^2], \text{ where } s = \rho_2/\rho_1.$$

Turning to the equation of the wave profile we see that the small terms have their greatest importance for this value of c.

The equation takes the form

$$y = b \cos kx + \tfrac{1}{2}kb^2 \cos 2kx + \tfrac{3}{8}k^2 b^3 \cos 3kx,$$

which is the form Stokes found for waves on a free surface. The expression for the velocity only differs from Stokes' free surface velocity by the factor $(1 - s)$.

The second case gives

$$2\rho_1 c^2 = K,$$

whence

$$c^2 = \tfrac{1}{2} gk^{-1} (1 - s) (1 + \tfrac{1}{2}k^2\beta^2) \quad \ldots\ldots(7),$$

and

$$u = \pm c (1 \pm s^{\frac{1}{2}})/s^{\frac{1}{2}} \quad \ldots\ldots(8).$$

For this case the correction to the wave profile has its least value. (Note. The correction in the $\cos 3kx$ term has not its numerically smallest value.) The wave form is given by

$$y = b\cos kx - \tfrac{3}{16}k^2b^3\cos 3kx.$$

[This wave is traced for the value $kb = \cdot 5$.]

Thus waves travelling with velocity given by (7) are least affected by the small terms. The corresponding stream velocities in the upper liquid are given by (8).

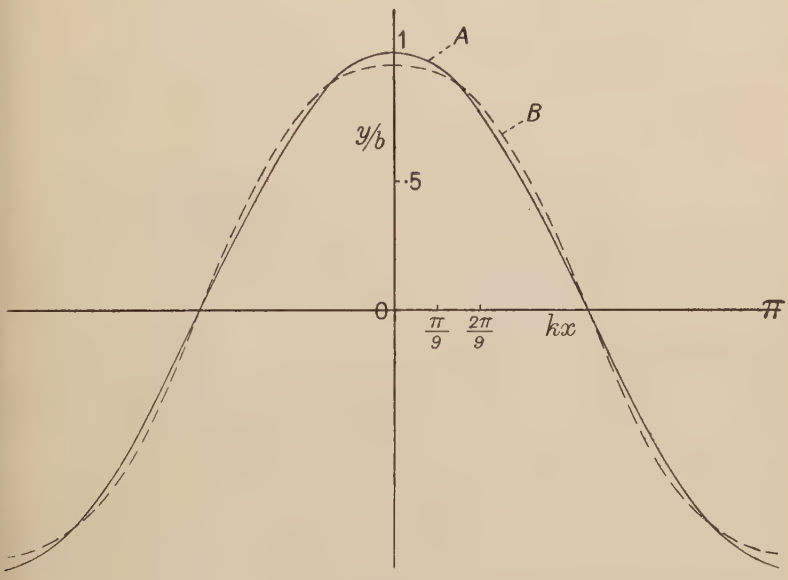


Fig. 1.

$c = [\frac{1}{2}gk^{-1}(1-s)]^{\frac{1}{2}}$.

A, wave form as given by first approximation.

B, wave form corrected for terms of order k^2b^3.

Stability of waves for a given value of u. The period equation is

$$\rho_1 U_1^2 + \rho_2 U_2^2 = gk^{-1}(\rho_1 - \rho_2) + k^2\beta^2(\rho_1^2 U_1^4 + \rho_2^2 U_2^4)/(\rho_1 U_1^2 + \rho_2 U_2^2).$$

In the small term we can take the first order values of c and hence that term may be considered a known function of k and u. The period equation therefore gives as a quadratic equation for c

$$c^2(\rho_1 + \rho_2) - 2cu\rho_2 + \rho_2 u^2 = gk^{-1}(\rho_1 - \rho_2) + Pk^2\beta^2,$$

where $$P = (\rho_1^2 U_1^4 + \rho_2^2 U_2^4)/(\rho_1 U_1^2 + \rho_2 U_2^2),$$

whence

$$c = u\rho_2/(\rho_1+\rho_2) \pm [gk^{-1}(\rho_1-\rho_2)/(\rho_1+\rho_2) + Pk^2\beta^2 - u^2\rho_1\rho_2/(\rho_1+\rho_2)^2]^{\frac{1}{2}}.$$

Thus c is real if

$$gk^{-1}(1-s)/(1+s) > u^2 s/(1+s)^2 - Pk^2\beta^2.$$

Hence the range of unstable wave lengths is smaller than that given by the first order equation and the corrections for small terms tend to make the system more stable.

As in the first order case the shortest possible wave travels with a velocity $us/(1+s)$.

II. Standing Waves.

The method of the first part of the paper is unsuitable for the discussion of standing waves so we proceed as follows:

With the origin in the common surface and the y axis vertically upwards we assume for the lower liquid

$$\phi_1 + \iota\psi_1 = \Sigma A_m e^{-\iota mk(x+\iota y)},$$

and for the upper

$$\phi_2 + \iota\psi_2 = \Sigma B_m e^{\iota mk(x+\iota y)}.$$

The condition to be satisfied at the common surface is

$$\rho_1\dot{\phi}_1 - g\rho_1 y - \tfrac{1}{2}\rho_1 q_1^2 = \rho_2\dot{\phi}_2 - g\rho_2 y - \tfrac{1}{2}\rho_2 q_2^2 - F(t),$$

where

$$q_1^2 = k^2\Sigma m^2 A_2^2 e^{2mky} + 2k^2\Sigma mn A_m A_n e^{(m+n)ky}\cos(m-n)kx$$
$$= k^2A_1^2(1+2ky) + 4k^2A_1A_2\cos kx$$

(to third power of the amplitude),

and $F(t)$ is a function of the time.

This condition gives the equation of the common surface which we will write

$$U = 0.$$

The condition that no part of either liquid shall flow across the common surface leads to the equations

$$\left.\begin{aligned} \frac{dU}{dt} + u_1\frac{dU}{dx} + v_1\frac{dU}{dy} &= 0 \\ \frac{dU}{dt} + u_2\frac{dU}{dx} + v_2\frac{dU}{dy} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1),$$

where (u_1, v_1), (u_2, v_2) are the x and y components of the velocities in the two liquids.

We proceed to find the values of the A's and B's by successive approximations from the conditions (1).

1*st Approximation.*

The equation of the free surface is

$$g(\rho_1 - \rho_2)\, y = (\rho_1 \dot{A}_1 - \rho_2 \dot{B}_1) \cos kx + F(t).$$

The origin being taken in the undisturbed surface, we have $F(t) = 0$ and

$$g(\rho_1 - \rho_2)\, y = (\rho_1 \dot{A}_1 - \rho_2 \dot{B}_1) \cos kx.$$

The surface conditions give

$$\rho_1 \ddot{A}_1 - \rho_2 \ddot{B}_1 + g(\rho_1 - \rho_2)\, kA_1 = 0,$$
$$\rho_1 \ddot{A}_1 - \rho_2 \ddot{B}_1 - g(\rho_1 - \rho_2)\, kB_1 = 0,$$

whence $A_1 = -B_1 = p\beta k^{-1} \sin pt$, (say),

and $p^2 = gk(1-s)/(1+s)$, where $s = \rho_2/\rho_1$.

Using the values of A_1 and B_1, we have, as equation of the surface

$$y = \beta \cos pt \cos kx.$$

2*nd Approximation.*

The equation of the free surface is

$$\begin{aligned} g(\rho_1 - \rho_2)\, y = {} & \rho_1 \dot{A}_1 (1 + ky) \cos kx - \rho_2 \dot{B}_1 (1 - ky) \cos kx \\ & + \rho_1 \dot{A}_2 \cos 2kx - \rho_2 \dot{B}_2 \cos 2kx \\ & - \tfrac{1}{2} \rho_1 k^2 A_1^2 + \tfrac{1}{2} \rho_2 k^2 B_1^2 + F(t); \end{aligned}$$

whence, using first approximations to A_1, B_1 and y in the small terms

$$\begin{aligned} g(\rho_1 - \rho_2)\, y = {} & (\rho_1 \dot{A}_1 - \rho_2 \dot{B}_1) \cos kx + (\rho_1 \dot{A}_2 - \rho_2 \dot{B}_2) \cos 2kx \\ & + (\rho_1 - \rho_2)\, p^2 \beta^2 \cos^2 pt \cos^2 kx \\ & - \tfrac{1}{2} (\rho_1 - \rho_2)\, p^2 \beta^2 \sin^2 pt + F(t). \end{aligned}$$

The velocities, required for the surface conditions, are given by

$$u_1 = -\frac{\partial \phi_1}{\partial x} = kA_1 \sin kx, \quad \text{(to first order)},$$

$$\begin{aligned} v_1 = -\frac{\partial \phi_1}{\partial y} &= -kA_1 \cos kx\,(1 + ky) - 2kA_2 \cos 2kx \\ &= -kA_1 \cos kx - 2kA_2 \cos 2kx \\ &\quad - pk\beta^2 \sin pt \cos pt \cos^2 kx, \quad \text{(to second order)}, \end{aligned}$$

with similar values of u_2 and v_2.

The surface conditions give

$$\begin{aligned} & (\rho_1 \ddot{A}_1 - \rho_2 \ddot{B}_1) \cos kx + (\rho_1 \ddot{A}_2 - \rho_2 \ddot{B}_2) \cos 2kx \\ & \quad - (\rho_1 - \rho_2)\, p^3 \beta^2 \sin 2pt \cos^2 kx \\ & \quad - \tfrac{1}{2} (\rho_1 - \rho_2)\, p^3 \beta^2 \sin 2pt + F'(t) \\ & \quad - (\rho_1 + \rho_2)\, p^3 \beta^2 \sin^2 kx \sin pt \cos pt \\ & \quad + g(\rho_1 - \rho_2) [kA_1 \cos kx + 2kA_2 \cos 2kx \\ & \qquad + kp\beta^2 \cos kx \cos pt \sin pt] = 0, \end{aligned}$$

with a similar equation from the other liquid.

Equating the coefficients of the absolute term, $\cos kx$ and $\cos 2kx$ to zero in these two equations, we have the following sets of relations :

$$F'(t) - (\rho_1 - \rho_2)\, p^2\beta^2 \sin 2pt = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2),$$

(from both equations),

$$\left.\begin{aligned} \rho_1 \ddot{A}_1 - \rho_2 \ddot{B}_1 + g(\rho_1 - \rho_2)\, kA_1 = 0 \\ \rho_1 \ddot{A}_1 - \rho_2 \ddot{B}_1 - g(\rho_1 - \rho_2)\, kB_1 = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3),$$

$$\left.\begin{aligned} \rho_1 \ddot{A}_2 - \rho_2 \ddot{B}_2 + 2p^2(\rho_1 + \rho_2)\, A_2 = -\rho_2 p^3\beta^2 \sin 2pt \\ \rho_1 \ddot{A}_2 - \rho_2 \ddot{B}_2 - 2p^2(\rho_1 + \rho_2)\, B_2 = \quad \rho_1 p^3\beta^2 \sin 2pt \end{aligned}\right\} \ldots\ldots (4),$$

(on making use of the period equation to eliminate g).

From (3) we see that A_1 is still equal to $-B_1$; we shall still give them the value $p\beta k^{-1} \sin pt$. The period equation will be the same as before.

From (4) $\quad A_2 = -\frac{1}{2}\rho_2/(\rho_1 + \rho_2)\, p\beta^2 \sin 2pt,$

$$B_2 = -\tfrac{1}{2}\rho_1/(\rho_1 + \rho_2)\, p\beta^2 \sin 2pt,$$

and from (2)

$$F(t) = -\tfrac{1}{2}(\rho_1 - \rho_2)\, p^2\beta^2 \cos 2pt + C,$$

where C is a constant of integration.

To keep the origin in the undisturbed surface we take $C = 0$.

The equation of the surface takes the form

$$y = \beta \cos pt \cos kx + \tfrac{1}{2}k\beta^2 (1 - s)/(1 + s) \cos^2 pt \cos 2kx.$$

3rd Approximation.

$$\begin{aligned} g(\rho_1 - \rho_2)\, y = {} & \rho_1 \dot{A}_1 (1 + ky + \tfrac{1}{2}k^2y^2) \cos kx \\ & - \rho_2 \dot{B}_1 (1 - ky + \tfrac{1}{2}k^2y^2) \cos kx \\ & + \rho_1 \dot{A}_2 (1 + 2ky) \cos 2kx \\ & - \rho_2 \dot{B}_2 (1 - 2ky) \cos 2kx \\ & + (\rho_1 \dot{A}_3 - \rho_2 \dot{B}_3) \cos 3kx \\ & - \tfrac{1}{2}\rho_1 k^2 [A_1^2 (1 + 2ky) + 4A_1 A_2 \cos kx] \\ & + \tfrac{1}{2}\rho_2 k^2 [B_1^2 (1 - 2ky) + 4B_1 B_2 \cos kx] \\ & + F(t). \end{aligned}$$

On using above values for y, A's and B's in the small terms and writing

$$\chi(t) = F(t) + \tfrac{1}{2}(\rho_1 - \rho_2)\, p^2\beta^2 \cos 2pt,$$

this becomes

$$\begin{aligned} g(\rho_1 - \rho_2)\, y = {} & \chi(t) + \cos kx\, [\rho_1 \dot{A}_1 - \rho_2 \dot{B}_1] + \cos 2kx\, [\rho_1 \dot{A}_2 - \rho_2 \dot{B}_2] \\ & + \cos 3kx\, [\rho_1 \dot{A}_3 - \rho_2 \dot{B}_3] + \tfrac{1}{2}(\rho_1 - \rho_2)\, p^2\beta^2 \cos 2kx \cos^2 pt \\ & + \tfrac{1}{32} kp^2\beta^3/(\rho_1 + \rho_2) \left[\begin{array}{l} \cos kx \left\{ \begin{array}{r} (13\rho_1^2 + 13\rho_2^2 - 46\rho_1\rho_2) \cos 3pt \\ + (7\rho_1^2 + 7\rho_2^2 - 10\rho_1\rho_2) \cos pt \end{array} \right\} \\ + \cos 3kx \left\{ \begin{array}{r} (3\rho_1^2 + 3\rho_2^2 - 34\rho_1\rho_2) \cos 3pt \\ + (9\rho_1^2 + 9\rho_2^2 - 38\rho_1\rho_2) \cos pt \end{array} \right\} \end{array} \right]. \end{aligned}$$

To apply surface conditions (1) we shall require the u-velocities to the second order and the v-velocities to the third order.

These are given by

$$\begin{aligned} u_1 = -\frac{\partial \phi_1}{\partial x} &= kA_1 \sin kx\,(1+ky) + 2kA_2 \sin 2kx \\ &= kA_1 \sin kx + 2kA_2 \sin 2kx \\ &\quad + kp\beta^2 \sin kx \cos kx \sin pt \cos pt, \\ v_1 = -\frac{\partial \phi_1}{\partial y} &= -kA_1 \cos kx\,(1+ky+\tfrac{1}{2}k^2y^2) \\ &\quad - 2kA_2 \cos 2kx\,(1+2ky) \\ &\quad - 3kA_3 \cos 3kx \\ &= -kA_1 \cos kx - 2kA_2 \cos 2kx - 3kA_3 \cos 3kx \\ &\quad - pk\beta^2 \sin pt \cos pt \cos^2 kx \\ &\quad - \tfrac{1}{8}pk^2\beta^3/(\rho_1+\rho_2) \,.\, \cos^2 pt \sin pt\,[(5\rho_1 - 15\rho_2)\cos kx \\ &\qquad + (3\rho_1 - 17\rho_2)\cos 3kx], \end{aligned}$$

with similar values for u_2 and v_2.

Writing these values in the surface conditions (1), using known approximations to coefficients wherever possible and eliminating g by means of the period equation in all terms except those of the first order, we obtain the following sets of relations:

$$\chi'(t) = 0 \quad \text{.....(5),}$$

$$\begin{aligned} &\rho_1\ddot{A}_1 - \rho_2\ddot{B}_1 + g(\rho_1-\rho_2)kA_1 \\ &\quad = \tfrac{1}{32}kp^3\beta^3/(\rho_1+\rho_2)\Big[\begin{matrix}(40\rho_1^2 + 44\rho_2^2 - 132\rho_1\rho_2)\sin 3pt \\ + (8\rho_1^2 + 12\rho_2^2 - 4\rho_1\rho_2)\sin pt\end{matrix}\Big], \\ &\rho_1\ddot{A}_1 - \rho_2\ddot{B}_1 - g(\rho_1-\rho_2)kB_1 \\ &\quad = \tfrac{1}{32}kp^3\beta^3/(\rho_1+\rho_2)\Big[\begin{matrix}(44\rho_1^2 + 40\rho_2^2 - 132\rho_1\rho_2)\sin 3pt \\ + (12\rho_1^2 + 8\rho_2^2 - 4\rho_1\rho_2)\sin pt\end{matrix}\Big] \\ &\qquad \text{.....(6),} \end{aligned}$$

$$\left.\begin{aligned} \rho_1\ddot{A}_2 - \rho_2\ddot{B}_2 + 2p^2(\rho_1+\rho_2)A_2 &= -\rho_2 p^3\beta^2 \sin 2pt \\ \rho_1\ddot{A}_2 - \rho_2\ddot{B}_2 - 2p^2(\rho_1+\rho_2)B_2 &= \rho_1 p^3\beta^2 \sin 2pt \end{aligned}\right\} \quad \text{.....(7),}$$

$$\begin{aligned} &\rho_1\ddot{A}_3 - \rho_2\ddot{B}_3 + 3p^2(\rho_1+\rho_2)A_3 \\ &\quad = -\tfrac{1}{8}kp^3\beta^3\rho_2/(\rho_1+\rho_2)\Big[\begin{matrix}(21\rho_1 - 9\rho_2)\sin 3pt \\ + (5\rho_1 - 9\rho_2)\sin pt\end{matrix}\Big], \\ &\rho_1\ddot{A}_3 - \rho_2\ddot{B}_3 - 3p^2(\rho_1+\rho_2)B_3 \\ &\quad = -\tfrac{1}{8}kp^3\beta^3\rho_1/(\rho_1+\rho_2)\Big[\begin{matrix}(21\rho_2 - 9\rho_1)\sin 3pt \\ + (5\rho_2 - 9\rho_1)\sin pt\end{matrix}\Big] \\ &\qquad \text{.....(8).} \end{aligned}$$

(5) and (7) shew that $F(t)$, A_2, B_2 are not changed by this closer approximation.

If we write

$$A_1 = \quad p\beta k^{-1} \sin pt + C_1,$$
$$B_1 = -\, p\beta k^{-1} \sin pt - D_1,$$

we obtain, from (6),

$$C_1 - D_1 = -\tfrac{1}{8} pk\beta^3 (\rho_1 - \rho_2)/(\rho_1 + \rho_2) \,.\, (\sin pt + \sin 3pt).$$

The part of C_1 and D_1 depending on $\sin pt$ is indeterminate as any addition to that part simply means a change in the amplitude of the principal oscillation.

We take

$$C_1 = \tfrac{1}{8} pk\beta^3 \rho_2/(\rho_1 + \rho_2) \,.\, \sin pt + C_2 \sin 3pt,$$
$$D_1 = \tfrac{1}{8} pk\beta^3 \rho_1/(\rho_1 + \rho_2) \,.\, \sin pt + D_2 \sin 3pt.$$

If we use these values in the two equations (6) and equate the coefficients of $\sin pt$ and $\sin 3pt$ to zero we have four equations. Two of these are the same and lead to the new period equation,

$$p^2 = gk\,(1-s)/(1+s) \,.\, [1 - \tfrac{1}{4} k^2\beta^2 \,(1+s^2)/(1+s)^2] \ldots\ldots (9),$$

the other two give equations for C_2 and D_2, the solutions of which are

$$C_2 = -\tfrac{1}{32} pk\beta^3 \,(5\rho_1^2 + \rho_2^2 - 12\rho_1\rho_2)/(\rho_1 + \rho_2)^2 \,.\, \sin 3pt,$$
$$D_2 = -\tfrac{1}{32} pk\beta^3 \,(\rho_1^2 + 5\rho_2^2 - 12\rho_1\rho_2)/(\rho_1 + \rho_2)^2 \,.\, \sin 3pt.$$

Equations (8) give

$$A_3/pk\beta^3 = \tfrac{1}{8}\rho_2 (3\rho_2 - \rho_1)/(\rho_1 + \rho_2)^2 \sin 3pt + \tfrac{1}{16}\rho_2 (9\rho_2 - 5\rho_1)/(\rho_1 + \rho_2)^2 \sin pt,$$
$$B_3/pk\beta^3 = -\tfrac{1}{8}\rho_1 (3\rho_1 - \rho_2)/(\rho_1 + \rho_2)^2 \sin 3pt - \tfrac{1}{16}\rho_1 (9\rho_1 - 5\rho_2)/(\rho_1 + \rho_2)^2 \sin pt.$$

If we now denote the amplitude of the principal oscillation by b we find for the equation of the wave profile

$$y = [b \cos pt - \tfrac{3}{16} k^2 b^3 \,(\rho_1^3 + \rho_2^3)/(\rho_1 + \rho_2)^3 \,.\, \cos 3pt] \cos kx + \frac{kb^2}{2} (\rho_1 - \rho_2)/(\rho_1 + \rho_2) \cos^2 pt \cos 2kx + \tfrac{1}{8} k^2 b^3 (\rho_1 - 3\rho_2)\,(3\rho_1 - \rho_2)/(\rho_1 + \rho_2)^2 \cos^3 pt \cos 3kx \quad \ldots (10).$$

Comparison with propagated wave.

Wave Profile.

Putting $u = 0$ in equation (6) of section I we obtain, as the equation of the profile of the propagated wave when the upper liquid has no stream velocity

$$y = b \cos kx + \tfrac{1}{2} kb^2 \,(\rho_1 - \rho_2)/(\rho_1 + \rho_2) \,.\, \cos 2kx + \tfrac{3}{8} k^2 b^3 \,(\rho_1^2 + \rho_2^2 - 4\rho_1\rho_2)/(\rho_1 + \rho_2)^2 \cos 3kx.$$

Comparing this with the standing wave given by (10) [section II] we see that the forms of the propagated and standing waves are the same if we neglect the second power of the ratio of the amplitude to the wave length but are different if we proceed to a higher approximation.

Period.

We have seen that the period equations are not altered by terms of order $k\beta$. When we proceed to the next approximation the period of the standing wave is given by (9) [section II] while that of the propagated wave is found from

$$p^2 = gk\,(1-s)/(1+s)\,[1 + k^2\beta^2\,(1+s^2)/(1+s)^2]$$

[(5) section I].

We see that the period $2\pi/p$ given by the first order equation is too long for the propagated waves and too short for the standing ones. We proceed to tabulate the periods for waves of length 10 ft. and 100 ft. We take

$$k\beta = \cdot 5 \text{ and } g = 32{\cdot}16 \text{ f.s.s.}$$

s	10 feet			100 feet		
	1st order	Propagated	Standing	1st order	Propagated	Standing
	sec.	sec.	sec.	sec.	sec.	sec.
·98	13·91	13·04	14·12	43·98	41·23	44·67
·9	6·093	5·711	6·188	19·27	18·06	19·57
·8	4·193	3·927	4·260	13·26	12·42	13·92
·6	2·796	2·610	2·842	8·840	8·794	8·987
·4	2·135	1·977	2·175	6·752	6·253	6·877
·2	1·712	1·557	1·751	5·414	4·925	5·536
0	1·398	1·223	1·441	4·421	3·868	4·558

Discontinuities in Light Emission. By NORMAN CAMPBELL, M.A., Trinity College.

[*Read* 8 November 1909.]

SUMMARY.

1. Introduction.
2. The ideas underlying the experiments.
3, 4. The theory of the method of experiment.
5. General nature of the experiments.
6—10. Mathematical theory of the interpretation of the observations.
11—15. The apparatus:
    11. The source of light.
    12. The photo-electric cells.
    13. The measuring instruments.
    14. The high resistance.
    15. Measurement of the instrumental constants.
16. Numerical calculations showing the possibility of the experiment.
17. Combination of the observations.
18—20. The experimental difficulties which have prevented the attainment of results.

§ 1. The following pages are the account of an experiment which failed. The occasional publication of a description of such failures may be justified by two reasons, both of which appear to me sufficient in the present case. In the first place, the theory of the experiment may be of interest: and, according to the view which is taken of the causes of failure, others better equipped for the task may be induced to repeat the attempt, or others, to whom the same idea may occur, may be saved from wasting their time. In the second place, the methods of experiment, if they are novel, though unsuccessful in their immediate object, may have other applications.

§ 2. The ideas on which the experiment was based are not easy to describe in a brief title: a little explanation is needed.

According to all modern theories of radiation, the emission of light is discontinuous in time. Whatever may be the nature of the radiators from which the light proceeds, there is little doubt that their vibration is not continuous. They are set into vibration by some external disturbance, such as the impact of electrons, and continue vibrating until the energy so acquired is dissipated by the damping due to radiation: they are then quiescent until the occurrence of a fresh disturbance.

But there are at least two theories of radiation which suggest that light emission is discontinuous in space as well as in time.

These theories were proposed by J. J. Thomson* and by Planck†. They both suggest (at least if we interpret Planck's theory in the sense given to it by Stark‡) that the light from a single radiator is not emitted equally in all directions, but is concentrated along a finite number of narrow tubes radiating from the radiator: they also suggest that the energy emitted by such a radiator in virtue of a single disturbance is not infinitely divisible, but only divisible into a finite number of equal parcels, the magnitude of which, according to Planck, depends on the frequency: further, this finite number is usually quite small under experimental conditions.

This view is capable of explaining many difficulties connected with the ionisation of gases and allied phenomena which are quite incomprehensible on the older theory that the light spreads out from the radiator as a spherical wave. But it also raises some new difficulties in connection with optical interference. If the light from a single radiator can only be split into a small number of parts, then, if the beam of light from an ordinary source, containing a great number of separate radiators, is divided by any of the ordinary interference methods, the light in one beam must, in general, come from radiators different from those which emit the light in the other. Two rays of light which come from different radiators will be termed 'independent.'

And here a distinction, which may be of importance for our purpose, must be made between the two methods of exciting interference. In the first method, of which Fresnel's mirrors are typical, the two beams which are made to interfere eventually are emitted at different angles from the source: they will be independent, in our present sense, if it be true that the light from a single radiator is emitted in one direction only, and this conclusion requires no assumptions as to the divisibility of the energy in the light from a single radiator. In the second method, exemplified by Michelson's interferometer, the two beams proceed from the source in the same direction. They will be independent, only if it is impossible to split up the energy from a single radiator into two parts, so that the beam reflected by the half-silvered mirror must come from different radiators from that transmitted by it. On Planck's theory of radiation there should be no distinction important for our purpose between the two interference methods, but on Thomson's theory it is possible that the two beams produced by the second method are dependent, while those produced by the first are independent.

Now it is not difficult to imagine why two independent beams

* See *Electricity and Matter*, London, 1904, p. 63, or *Proc. Camb. Phil. Soc.* xiv. 1907, p. 417.

† M. Planck, *Wärmestrahlung*, Leipzig, 1906.

‡ J. Stark, *Phys. Zeit.* x. 1909, p. 579.

should show interference, but it is difficult to explain why two beams should interfere when they come originally from a 'single source,' and should not interfere when they come originally from 'different sources.' For, on this view, in both cases they really come from different radiators. Accordingly it appeared that considerable value would attach to any experiment which should indicate, by means of observations other than those of optical interference, whether the two beams, into which a beam from a single source of light is divided by interference methods, are to be regarded as coming from the same or from different radiators.

§ 3. Now the theory recently given by Schweidler*, and applied to radioactive processes by Kohlrausch†, by Meyer and Regener‡, and by Geiger§, provides a very powerful method of investigating any discontinuous process. Schweidler showed that if any effect, of which the magnitude can be measured, is due to the random occurrence of a finite number of independent events, then the magnitude of the effect will not be constant, but will show fluctuations about a mean value, and that from the magnitude of the fluctuations the number of independent events can be calculated. Further, if the magnitude of the sum or difference of two effects so constituted is measured, the square of the mean fluctuation of the sum or difference is the sum of the squares of the mean fluctuations of the two effects, if, and only if, the events which constitute one event are wholly independent of those which constitute the other. If, on the other hand, there is complete correlation between the events constituting the two effects, the mean fluctuation of the difference will be zero. In a recent paper‖, which will be quoted frequently below, I have endeavoured to investigate closely the application of this theory to experiment.

§ 4. Suppose, then, that by some means we can measure the intensity of a beam of light in such a way that the total intensity is the sum of the intensities of the radiations from the isolated radiators of which the source is composed. Let us split up the beam from a 'single source' by one of the ordinary interference methods and measure in this way the difference in the intensity of the resulting beams, adjusting the arrangements so that the mean difference is zero. If the older theory of radiation is correct, there is complete correlation between the 'events' which constitute the two beams: when a radiator sends light into one beam, it also sends it into the other. But if the newer theory of the radiation is correct,

* Schweidler, *Congrès Internat. à Liège*, 1904.
† Kohlrausch, *Wien. Ber.* 1906, p. 673.
‡ Meyer and Regener, *An. Ph.* xxv. p. 757.
§ Geiger, *Phil. Mag.* April 1908, p. 539.
‖ Campbell, *Proc. Camb. Phil. Soc.* xv. 1909, p. 117.

the 'events' are independent, or very nearly so, for in general each radiator sends its radiation into one beam and not into both. On the first theory the mean fluctuation of the difference should be zero: on the second the square of the mean fluctuation should be the sum of the squares of the fluctuations of either beam separately.

§ 5. The application of these ideas to experiment was attempted in the following way. The intensity of the beams was measured by means of the photo-electric current which they excite in the alloy of sodium and potassium in a high vacuum. It seems reasonable at the outset to suppose that each train of radiation emitted by a single radiator, when it falls on the alloy, liberates a number of electrons which is, on the average, the same for all radiators and independent of the total intensity of the beam, i.e. the number of such trains emitted in a given time. If this assumption be true, then the difference between the photo-electric currents due to the two beams will be a function of the difference in the number of trains of radiation constituting the two beams, and measurements of the fluctuations of this difference will enable information to be deduced as to the independence of these trains. The fluctuations were measured by Meyer and Regener's method of observing the readings of an electrometer, the quadrants of which were connected to the two ends of a high resistance through which the current passed.

§ 6. Before proceeding to detail the experimental methods it will be well to consider the theory a little more closely: if this order had been adopted in the first instance much time would have been saved.

The main result of the previous paper * may be stated as follows. Let $X\tau$ be the average number of events which happen in a small time τ, and let x be the deviation of that number from its mean value during the time τ: let

$$\theta = f(t) \qquad \ldots\ldots(1)$$

represent the motion of the indicator of the measuring instrument at all times t subsequent to the happening of one of the events, and $\overline{\theta_T'^2}$ the square of the mean deviation of the indicator from its mean position for all times T from the moment of starting the observations, where T is a time long compared with the time constants of the measuring instrument. Then it is shown that

$$\overline{\theta_T'^2} = \overline{x^2}/\tau \cdot \int_0^T f^2(t)\, dt \qquad \ldots\ldots(2).$$

§ 7. We have first to calculate $\overline{x^2}$. The 'events' in our case are the liberation of individual electrons. Our fundamental

* Campbell, *loc. cit.*

assumption states that the number of electrons liberated is proportional (at any rate for a given intensity of the light) to the number of light disturbances falling on the cell. If N is the average number of such disturbances striking the cell in unit time, ω the average number of electrons liberated by each light disturbance, then

$$X\tau = N\omega\tau \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

The fluctuations of X, which are measured by $\overline{x^2}$ arise (1) from fluctuations of N and (2) from fluctuations in ω. The introduction of ω allows for all changes of the absorption of the light in different parts of the optical train, and for all differences in the state of the cell and the manner of the incidence of the light upon it. Then, if ξ and η are the deviations of $N\tau$ and ω respectively from their mean values during any very short time τ, we have

$$(X\tau + x) = (N\tau + \xi)(\omega + \eta) \quad \ldots\ldots\ldots\ldots\ldots(4),$$

and

$$x = N\tau\eta + \xi\omega + \xi\eta \quad \ldots\ldots\ldots\ldots\ldots\ldots(5),$$

and

$$\overline{x^2} = [\overline{N^2\tau^2\eta^2} + \overline{\xi^2\omega^2} + \overline{\xi^2\eta^2} + 2\overline{N\tau\eta\xi\omega} + 2\overline{N\tau\eta^2\xi} + 2\overline{\xi^2\omega\eta}] \ldots(6).$$

It will be assumed that ξ and η are wholly independent, hence $\overline{\eta\xi}$, $\overline{\eta^2\xi}$ and $\overline{\xi\eta^2}$ are all zero, since $\bar{\eta}$ and $\bar{\xi}$ are zero. Also, since τ tends to the limit zero, $N^2\tau^2$ is infinitesimal compared to ω.

Therefore

$$\overline{x^2} = \overline{\xi^2}(\omega^2 + \overline{\eta^2}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(7).$$

As was proved in the previous paper,

$$\overline{\xi^2} = N\tau \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8).$$

Hence

$$\overline{x^2} = N(\omega^2 + \overline{\eta^2})\tau \quad \ldots\ldots\ldots\ldots\ldots\ldots(9).$$

§ 8. (9) gives the result when we are considering a single beam falling on a single cell and measuring the fluctuations of that cell as Meyer and Regener* measured the fluctuations of a single source of α rays. Let us now consider a second cell (the quantities referring to which are distinguished by dashes) and measure the fluctuations in the quantity $X - X'$, where $\overline{X} = \overline{X'}$. In this case η and η' are independent and it will be assumed that $\overline{\eta^2} = \overline{\eta'^2}$; ξ and ξ' are dependent or independent according as the two beams of light are dependent or independent in the sense which has been discussed. Then it is easy to show that, if x now represents the fluctuations in $(X - X')\tau$,

$$\overline{x^2} = 2N(\omega^2 + \overline{\eta^2})\tau \ldots\ldots\ldots\ldots\ldots(10)$$

if the beams are independent, or

$$\overline{x^2} = 2N\overline{\eta^2}\,\tau \ldots\ldots\ldots\ldots\ldots\ldots(10')$$

if the beams are dependent.

* Meyer and Regener, *loc. cit.*

§ 9. We cannot estimate the value of $\overline{\eta^2}$ until we know (and we are not likely to know in the near future) the causes of the fluctuations of ω. But, from the general theory of probability, it is certain that $\frac{\overline{\eta^2}}{\omega^2}$ will diminish as ω increases. Further the same quantity will, almost certainly, diminish as the number of electrons which come under the influence of the light beam increases: for ω fluctuates because all those electrons are not in the same condition, and in a very large collection of electrons the distribution of electrons of different properties is likely to be very nearly the same. Accordingly, on the older theory of light, we should expect $\frac{\overline{\eta^2}}{\omega^2}$ to be very small: for any light disturbance is spread over the whole surface of the cell and of the optical train, and acts upon a vast number of electrons. But, on the theories of light which form the basis of this experiment, the area of the surface affected by any one light disturbance is very small and the number of electrons in that surface may possibly be as small as one. With these considerations in mind, let us consider how observations of the fluctuation of the measuring instrument might be applied to test the rival theories. (It must be remembered that, in any comparable series of experiments, $f(t)$ will be the same, so that $\overline{\theta_T'^2}$ is proportional to $\overline{x^2}/\tau$.)

In the first place we might compare the fluctuations of $X - X'$, i.e. the fluctuations of the balanced cells, (1) when we know that the two beams are independent and (2) in the case in which we wish to discover if they are independent. The ratio of the fluctuations in (1) to those in (2) should be 1 if the beams in (2) are independent, and $\frac{\omega^2 + \overline{\eta^2}}{\overline{\eta^2}}$ if they are dependent. If $\overline{\eta^2}$ were the same in both cases and $\frac{\overline{\eta^2}}{\omega^2}$ were large compared to 1, the two ratios might well be indistinguishable: it is for this reason important to note that $\frac{\overline{\eta^2}}{\omega^2}$ is certainly not likely to be large on the 'spherical wave' theory of light. If that theory be true we should expect the ratio in case (2) to be much greater than that in case (1); if the 'bundle of energy' theory be true we should expect the two ratios to be equal. The objection to this method of test is that it is difficult to ensure that the optical trains and the entire apparatus remain the same, when the change is made from the beams known to be independent (probably two different lamps) to the beams which it is desired to test.

In the second place, we may compare the fluctuations of

$X - X'$, with those of X or of X', by simply putting one of the two beams investigated out of action. If the two beams are independent the ratio of the fluctuations in the first case to those in the second should be 2, if they are dependent it should be $\frac{2\overline{\eta^2}}{\omega^2 + \overline{\eta^2}}$. Here again a distinction will be possible only if $\overline{\eta^2}$ for the dependent beams is small compared to ω^2. The objection to this method is that, as will be seen, it is not easy to exclude sources of fluctuation which affect the single cell, but do not affect the balanced cells.

§ 10. Let us now investigate briefly the quantity $\int_0^T f^2(t)\,dt$.

It can be shown by mere algebra that, if the form of $f(t)$ given in equation (2) of the previous paper is correct, involving the constancy of the capacity of the system and the proportionality of $f(t)$ to the charge communicated to the instrument, we have

$$\int_0^T f^2(t)\,dt = \frac{\epsilon^2 s^2}{C^2} \cdot \frac{\alpha\beta\,(p+\alpha+\beta)}{2p\,(\alpha+\beta)\,(p+\alpha)\,(p+\beta)} \quad \ldots\ldots(11),$$

or

$$= \frac{\epsilon^2 s^2}{C^2} \cdot \frac{(p+2a)(a^2+b^2)}{4ap\,(p+a^2+b^2)} \quad \ldots\ldots\ldots\ldots(11'),$$

according as α and β are real or imaginary*. Here ϵ is the charge on an electron, C the capacity of the system, R the resistance between the quadrants of the electrometer, $p = 1/RC$ the logarithmic constant of decay of the charge, α and β, or a and b, time constants determined by the period and damping of the electrometer, and s the sensitiveness of that instrument, i.e. the ratio of the deflection to the steady P.D. between the quadrants.

The following deductions from (11) and (11′) are important, if it is desired to make $\overline{\theta_T^2}$ as large as possible:

(1) It is desirable to decrease C as far as possible: but, taking into account the fact that p is a function of C and (4) below, C practically enters only as the first power and not as the square.

(2) s should be made as large as possible. Since s enters as its square its value is far more important than that of C.

(3) R should be made as large as possible. It will be seen that there are practical limits to the value of R.

(4) So long as p is small compared with α and β, or a and b, which are of the same order of magnitude in all ordinary instru-

* It should be remarked that the previous paper is deficient in its algebra. It was not noticed that equation (20) can be reduced to the form given here and, consequently, the conclusions of paragraph 10 as to the most desirable values of the instrumental constants are worthless. The conclusions given here are believed to be correct, subject to further considerations given below in paragraph 15.

ments, the value of α, β, a, b is of little importance. If $a = b$ and a/p is changed from infinity to 1, the value of $\overline{\theta_T^2}$ is only decreased in the ratio 5 to 3.

This conclusion may be surprising till it is remembered that $f(t)$ represents the motion of the needle after a sudden disturbance. The values of the time constants of the instrument, supposed smaller than that of the high resistance, affect only the initial part of the motion.

Accordingly it is clear so far that the sensitiveness of the instrument is the one thing that really matters. A Dolezalek electrometer will be much preferable to any form of electroscope with smaller capacity but smaller sensitiveness; for the capacity of the system apart from the electrometer will not be insignificant compared with that of the electrometer.

The actual experimental arrangements will now be described. Although no results have been obtained as yet, I think that, in all essentials, they have attained a final form. But since no results are claimed as yet, a very detailed account will not be necessary.

In all quantitative statements that follow, electrostatic units are employed.

§ 11. In the earlier experiments the source of light was a Nernst lamp of 50 watts heated by the town alternating supply which was the most constant source of current available. To indicate changes in the intensity of the light a small thermo-element in a vacuum was placed immediately under the lamp: changes of 1 in 10,000 could be detected with certainty. Since the constancy of the light proved insufficient an attempt was made to use an Osram lamp of 8 volts and 16 watts, but, though by the use of two such lamps some preliminary observations on the fluctuations of two independent sources have been made, the intensity of the light was insufficient for the main experiment. It is hoped in the near future to be able to run a Nernst lamp of 50 or 100 watts off accumulators of constant potential.

When continuous current was used to excite the lamp, its constancy was measured by making the filament one arm of a Wheatstone bridge, for which the exciting battery served as the source of current. A change of 1 in 10^5 of the resistance could be detected, or, since the illumination appears to vary approximately as the 10th power of the resistance, a change of 1 in 10^4 of the illumination.

The light from the lamp was rendered parallel by a simple lens of 7·5 cm. focal length and 4·5 cm. aperture. The resulting beam was split into two parts, either by interposing in its path a half-silvered mirror, or two full mirrors, one behind the other, of which the front mirror had a sharp edge; the plane of the mirrors

made an angle of approximately 45° with the direction of the light. The resulting two beams were suitably reflected by mirrors and brought to a focus on the surface of the alloy in the photo-electric cells by lenses similar to that previously mentioned. The angle of incidence of the light on the surface of the alloy was approximately 65°.

In order to adjust the currents through the two photo-electric cells to equality, a shutter moved by a fine screw was inserted in front of one of the cells. The light acting on the cells in the absence of the shutter was first adjusted as closely as possible to give equal currents, and the final adjustment made by means of

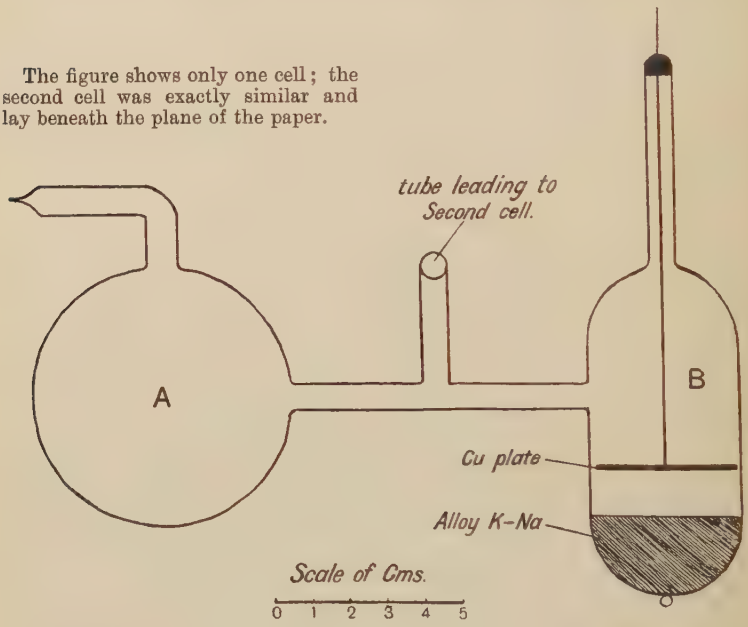

The figure shows only one cell; the second cell was exactly similar and lay beneath the plane of the paper.
tube leading to Second cell.
A
B
Cu plate
Alloy K–Na
Scale of Cms.
0 1 2 3 4 5

Fig. 1.

the shutter. The primary adjustment was made by varying the distance of the lamps from the cells, when two independent lamps were used: by moving the front mirror across the path of the beam, when the light from a single lamp was split by two full mirrors: and by altering the inclination of the half-silvered mirror to the path of the light, when the splitting was effected by the half-silvered mirror.

§ 12. As photo-electric cells an attempt was first made to use

rubidium cells made by the Polyphos Elek. Ges. of Munich: but, though these were more sensitive than any made subsequently, the differences between various cells were too great to permit of their use. It should be noted that these cells, unlike those made usually by the firm, were exhausted to the highest possible vacuum. It is necessary in these experiments to avoid the presence of any gas in the cells which might give rise to 'ionisation by collision,' for two reasons. (1) If there is ionisation by collision it is very improbable that the fundamental assumption of the theory given above, that the current through the cells is simply proportional to the number of light disturbances incident upon it, is fulfilled. (2) As was pointed out in the previous paper*, if the current is not saturated, the effective value R of the resistance between the quadrants of the electrometer varies with the intensity of the light: there is a portion of the conductivity which does not depend on the properties of the conductor inserted specially, the magnitude of which it is almost impossible to measure, and the constancy of which cannot be assured.

Cells were then made containing the alloy of sodium and potassium. The form ultimately adopted is shown in fig. 1†. After the vessel had been evacuated the alloy was sucked into the bulb A. When the evacuation had been completed it was allowed to flow into the vessel B by inclining the structure.

The difficulties which attend the use of the alloy have been discussed recently so fully by Elster and Geitel‡ that little more need be said. Of the two great difficulties which they found, the distillation of the alloy on to the other electrode and the contamination of its surface with particles of oxide, the former was of no importance in this work and the latter of but slight importance. The presence of oxide would only increase the value of η^2. But a method was found of getting rid of the small specks of oxide by a process much simpler than filtering. If, when the alloy is in its final position, one point at its edge is warmed slightly (the neighbourhood of the hand is sufficient) the change in the surface tension is sufficient to drive all impurities to the opposite wall of the cell, leaving a perfectly clean surface. The warm body must be kept in position while the experiment is in progress, but the most careful experiments failed to show any changes in the conductivity of the cells due to its presence.

The cells were exhausted by charcoal immersed in liquid air before being placed in position. A second tube of charcoal was always kept in liquid air while observations were in progress.

* Campbell, *loc. cit.* § 16.

† The tube joining A and B should have been drawn so that it joins B between the copper plate and the surface of the alloy.

‡ Elster and Geitel, *Phys. Zeit.* x. 1909, p. 457.

Fig. 2 gives the relation between the potential difference on one of the cells and the current through it. It will be seen that very complete saturation is attained: the point on the curve which is marked by the cross corresponds to the P.D. actually used in the observations.

The electrodes of the two photo-electric cells were connected with the battery of small accumulators, which provided the potential used, in such a way that the currents through them to the electrometer were opposite in sign. See fig. 3 which shows the connections of the measuring circuit.

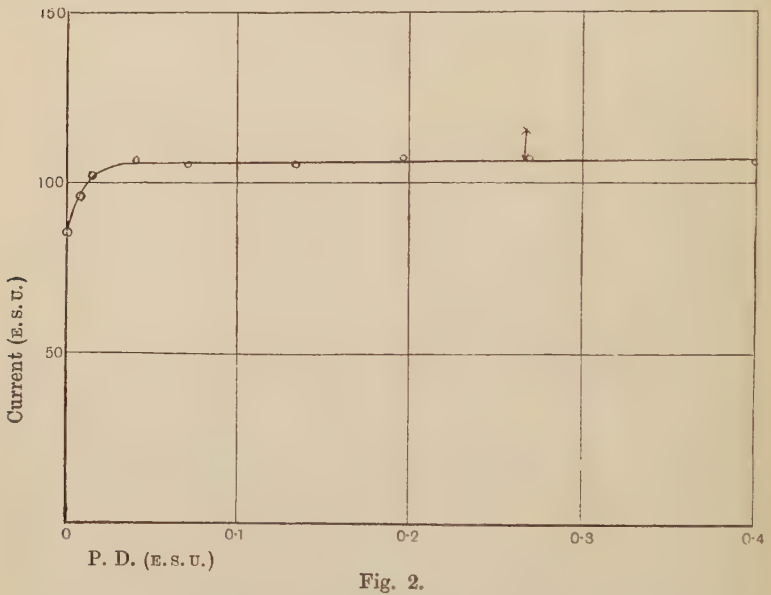


Fig. 2.

§ 13. The electrometer employed was of the Dolezalek pattern. The constants α and β (real for low potentials on the needle, and imaginary for high potentials) were changed by altering the potential on the needle. These constants were determined in the ordinary way by observing the motion of the needle when displaced from its zero position with the quadrants kept at constant potential. The sensitiveness in most cases was about 10^6, the scale being $1\frac{1}{2}$ metres distant and capable of readings to $\frac{1}{2}$ division.

A tilted electroscope was also connected to the common electrode of the cells. It was more convenient than the electrometer for some purposes, such as the measurement of the total current through either cell. It was for such measurements that the capacity box (9×10^2 to 9×10^5) shown in fig. 3 was employed.

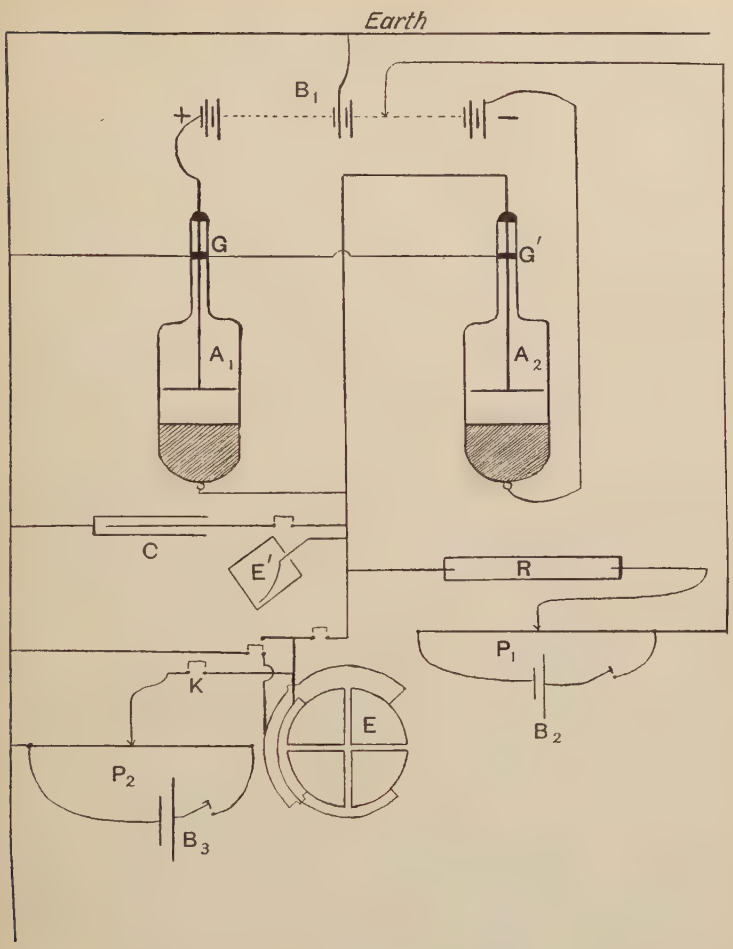


Fig. 3.

A_1, A_2	photo-electric cells.
B_1	small accumulators.
B_2, B_3	accumulators for potentiometer circuits P_1, P_2.
C	sub-divided capacity box (1 micro-farad).
E	electrometer.
E'	tilted electroscope.
R	Xylol-alcohol resistance.
G, G'	guard-rings.
K	key insulating electrode system.

§ 14. The resistance by which the electrometer was shunted was of the order of $\frac{1}{10}$ E.S.U. (9×10^{10} ohms). Such resistances are too low to be provided by ionised air in the manner now associated with the name of Bronson: moreover, such air resistances give rise to fluctuations, due to the finitude of the number of rays by which the air is ionised, which would be of the same order of magnitude as those that are here investigated. Several substances were tried as materials for the required resistance, including graphite, copper oxide, Phillips' conducting glass and various liquids. The latter eventually proved the most satisfactory. A mixture of Xylol and Ethyl Alcohol (from 30 to 5 parts of the former to 1 of the latter) was ultimately employed in a tube of such a form that the distance between the platinum electrodes could be varied. One end of the resistance was connected to the common electrode of the cells, the other to a potentiometer arrangement, the need for which will be explained later.

The insulation of the electrode system, of sulphur and amber, was such that its conductivity was certainly less than 0·01 of that of the artificial resistance: its conductivity is therefore neglected.

The value of R was determined directly by Ohm's Law. One of the cells being kept dark, the light falling on the other was adjusted to such an intensity as to give a convenient current through the cell. The magnitude of this current was measured by the electroscope and capacity box—the electrometer being taken out of the circuit. The electrometer was then connected again and such a potential applied to the potentiometer P_1 that the electrometer showed no deflection. Then the potential between the ends of the resistance was such that the current through the resistance was equal to that through the photo-electric cell. In this manner it could be shown that the resistances used obeyed Ohm's Law with an accuracy of 1 $^{\circ}/_{\circ}$ over a range of P.D. from 1·5 to 0·002 E.S.U. The polarisation E.M.F. of the liquid resistance was certainly less than 10^{-5} E.S.U.

§ 15. The only quantity in equation (11) the measurement of which presented any difficulty was the capacity. The capacity of the electrometer was a considerable part of that of the whole electrode system, and it is well known that this capacity is not constant, but varies with the deflection and, so far as I can make out, with the rate of movement of the needle. The most hopeful way of obtaining an effective value for the capacity of the electrode system appeared to be by measuring the value of R and of $p = 1/RC$: p was determined by observing the rate of decay of the deflection of the electrometer, when the electrode system was raised to a given potential and then insulated. (The initial part of the decay was neglected, because of the influence of the time

constants of the electrometer.) If the capacity of the system varies with the deflection, the resulting curve of deflection against time ought not to be exponential. As a matter of fact, it appeared to be perfectly exponential, but, on the other hand, the calculated value of the capacity increases regularly with a decrease in R. On decreasing R from 0·13 to 0·013 C appeared to increase from 380 to 500.

That this peculiarity was due to the electrometer was proved by the fact that, if the electrometer were disconnected and the decay observed by the electroscope, the capacity being varied by means of the capacity box, perfectly consistent results were obtained. A great deal of time was wasted in an inquiry into this matter and no conclusive information was obtained. The matter is not of first class importance, because in comparing fluctuations, care would, of course, be taken to keep the instrumental constants the same: and, in estimating the absolute value of $N(\omega^2 + \overline{\eta^2})$, the errors in the measurement of the fluctuations are likely to be as serious as the uncertainty in the value of the capacity.

§ 16. It will be well to consider here for a moment whether the magnitude of the fluctuations to be expected on the theory of light which is under discussion is such that it is likely to be capable of detection and measurement. On Planck's theory some estimate of this magnitude can be made.

According to that theory the quantity of energy (e) contained in one light disturbance is given by

$$e = 6{\cdot}5 \times 10^{-27}\nu, \text{ where } \nu \text{ is the frequency of the light.}$$

Hence if we know the amount of energy in each of the electrons shot off from the photo-electric substance, we can deduce the average number of electrons shot off by each light disturbance: i.e. the quantity ω. The application in this manner of Planck's theory to the photo-electric effect has been made by Einstein* and by Joffé†, the latter giving numerical calculations based upon the work of Ladenburg‡. It appears from these calculations that the energy given by the light to the electrons shot out from zinc is about one-third of that contained in each light disturbance: so that ω is about 3. For the alloy of sodium and potassium, which is more photo-electric, ω is certainly greater. But, since a minimum estimate is desired, we will take ω as 1, and will put $\overline{\eta^2} = 0$—though, if ω is as small as 1 it is probable that $\overline{\eta^2}$ is of the same order as ω.

* Einstein, *Ann. d. Phys.* xx. 1906, p. 199.
† Joffé, *Ann. d. Phys.* xxiv. 1907, p. 939.
‡ Ladenburg, *Phys. Zeit.* viii. 1907, p. 590.

The total current through either cell (i) is given by

$$i = \epsilon\omega N \quad \text{.....(12).}$$

The magnitudes of the various measured instrumental constants in the experiments which have been made are approximately as follows:

(These values are not the most favourable that can be obtained experimentally, so that our estimate is once more a minimum estimate.)

$$a = b = 0{\cdot}2, \quad R = 0{\cdot}2, \quad C = 500, \quad s = 10^6, \quad i = 100.$$

Assuming that the value of $\int_0^T f^2(t)\,dt$ can be calculated from (11′), it is found to be about $2 \times 10^8\epsilon^2$. Hence for the fluctuations of the balanced cells illuminated by independent sources

$$\overline{\theta_{T'}^2} = 2(\omega^2 + \overline{\eta^2})N \,.\, 2 \,.\, 10^8\epsilon^2$$

$$= 4 \,.\, 10^8 \,.\, \frac{\omega^2 + \overline{\eta^2}}{\omega}\,\epsilon i$$

$$= 16,$$

or $$\sqrt{\overline{\theta_{T'}^2}} = 4 \text{ (about).}$$

This value is just on the limit of the range within which detection is possible—it means that the electrometer spot will be on the average about 4 divisions distant from its mean position. But this is a minimum estimate: as a matter of fact such preliminary observations as have been made with two independent sources (two Osram lamps) indicate that, with these values for the instrumental constants, $\sqrt{\overline{\theta_{T'}^2}}$ is about 18—which is of the right order of magnitude.

§ 17. A brief explanation must be given of the manner in which the observations were taken and the value of $\overline{\theta_{T'}^2}$ deduced from them.

This quantity is the arithmetic mean of the squares of the deviation of the indicator from its mean position for all times after a period T from the moment that the electrode is insulated. As was pointed out in the previous paper, the most satisfactory way of estimating $\overline{\theta_{T'}^2}$ would be by recording the motion of the spot photographically and integrating the resulting curve (with the ordinates squared) graphically. But since, at least at present, no great accuracy has been required, the observations have been taken by eye at equidistant intervals of time (4″) indicated by a metronome. 200 such observations constitute one series, of which the probable error is, as has been shown, $1/\sqrt{200}$.

In almost all series there was, in addition to the fluctuations, a slow drift of the spot in one direction or the other. To allow for this drift, it was assumed to be linear so that

$$\overline{\theta_T} = u + vt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13).$$

It is easy to see that, if we treat the n observations made as readings from which the constants u and v are to be determined by a least square solution, $\overline{\theta_T'^2}$ is the mean of the square of the residuals, so that

$$\overline{\theta_T'^2} = \Sigma\theta_t^2 - \frac{(\Sigma\theta_t)^2}{n} - \frac{12}{n(n^2-1)}\left\{\frac{n-1}{2}\Sigma\theta_t - \Sigma(t.\theta_t)\right\}^2 \ldots(14).$$

Any series in which there appeared to be any suspicion of a change in the drift was rejected.

The arithmetical work in finding $\overline{\theta_T'^2}$ was heavy, but with practice and the aid of a calculating machine a single series could be solved in 20 minutes.

§ 18. At the beginning of the work a great deal of time was wasted because a complete theory of the experiment was not known. But once the influence of the various instrumental constants upon the fluctuations was discovered, all the experimental difficulties which had been anticipated were quickly overcome. They were such as attend all work upon electrostatic currents and no further reference need be made to them.

Blank experiments, made without the action of the light at all, showed that, when no light acted, there were no fluctuations whatever. For two hours or more the spot on the scale would keep perfectly steady or move constantly with a very slow drift in one direction. It is certain, therefore, that no trouble need be expected from such sources as changes in the potential of the battery used to send the current through the cells.

It was also important to show that the high resistance did not introduce disturbing fluctuations, such as would certainly have been introduced by a Bronson air resistance. For this purpose two similar high resistances were used and opposite potentials applied to their ends, so that the currents flowing through them were opposite in direction and equal in magnitude to each other and to the photo-electric current employed. No fluctuations whatever could be detected, though the steady drift of the spot was greater than in the former case, when practically no current was flowing through the resistance.

§ 19. But there were two outstanding difficulties. The first of these, the difficulty of determining the capacity, has been noted already. The second was far more serious and has prevented, up

to the present time, any results being obtained. It was found that the balance of the two currents through the cells was upset, if the total intensity of the light was changed. Whether the two currents were excited by different lamps lit by the same current or by the light from a single lamp split into two parts, a change in the current supplying the lamp or lamps was attended by a very serious displacement of the balance of the currents.

A great deal of work has been done in the effort to discover the cause of this defect and to remove it: but it has remained as mysterious as ever. It was thought at first that the cause must lie in some asymmetry of the photo-electric cells*.

That this explanation is not sufficient was shown by focussing the two beams, not on photo-electric cells, but on thermo-elements. Though these elements were much less sensitive than the cells, there was no doubt that the balance of the thermo-elements was upset in exactly the same way as the balance of the photo-electric cells by a change in the total illumination. It appears then that the effect is connected rather with the optical train employed than with the detectors: and additional evidence for this view was found in the fact that the sign of the change in the balance could be changed by comparatively trifling alterations in the mirrors and lenses. But in no case were attempts to reduce the change to zero successful.

The magnitude of the change in balance (measured relatively to the whole current passing through either cell) increases very rapidly with an increase in the total light. By placing diaphragms before the lamp and causing a given increase in the illumination by shunting a constant resistance in the lamp circuit, it was found that the change in the balance varied approximately as the cube of the total light.

It was thought at one time that the change in the balance might be due to the fact that there was a portion of the current passing to the electrode which was independent of the light. It is easy to see that, if there were such a portion, the balance between the currents would hold only for one intensity of the light. The experiments that have just been noted show that

* It was thought at one time that the reason might lie in the fact that the current through the cells was not strictly proportional to the intensity of the light acting upon them: and observations were made with the object of testing this proportionality. The variations in the intensity of the light were produced by passing it through a photographic lens in which diaphragms of known aperture could be inserted. It was found that the current was not proportional to the light, being relatively smaller for smaller intensities of the light. It was discovered subsequently that a similar result for ultra-violet light acting on zinc had previously been noted by Griffith (*Phil. Mag.* vi. 14, 1907, p. 297). But the matter is not very important for our purpose, because all that is necessary, in order that a change in the total light should not alter the balance, is that the current should be the same function of the light in the two cells.

such a cause cannot account for the whole of the effect: but it was found that by introducing a current independent of the light the magnitude of the effect could be decreased very considerably. Such a current was introduced by raising the end of the high resistance R distant from the electrode to a suitable potential, instead of connecting it to earth and the earthed quadrants of the electrometer. The magnitude of this potential had to be found by trial. But though the effect could be reduced greatly by this device, it could not be entirely abolished. The best state of affairs that could be produced was when, on changing the total light, the spot on the scale first moved rapidly a short distance in one direction and then gradually took up a final position on the other side of the zero corresponding to the new balance.

§ 20. The change of balance was of especial importance in the earlier experiments, because the most constant source of potential which was available for a Nernst lamp was subject to variations of as much as 5 %. But at the outset it was not even thought worth while to make special efforts to obtain a more constant source, until all possible investigations into the matter had been made. If there were any uncertainty whether the observed fluctuations were due to changes in the total intensity of the light or to changes in the relative intensity of the two beams, the experiments could be of no possible value. It was only when all efforts to get rid of the difficulty had failed that it seemed desirable to proceed further.

It appeared, then, that errors due to this source could be detected, if not eliminated, by comparing the fluctuations due to one of the cells with those due to both of the cells illuminated by totally independent sources. Changes in the total intensity of the light, even if they affect the latter experiment, must be much more serious in the former. And, if it were found that these changes were so small that the fluctuations in the former case were, as theory indicates, half those in the latter, then it would be certain that the fluctuations were due to changes in the relative intensity of the two sources and not to changes in the total intensity of the light.

In order to measure the fluctuations due to one source separately, the current through the photo-electric cell was balanced by Meyer and Regener's method of sending an equal and opposite current through the high resistance R by applying to its ends a P.D. by means of the potentiometer. It is not intended in this paper to give any actual results, but it has been found that the total light from two Osram lamps run from the same battery in parallel is so constant that the theoretical relation is fulfilled within the limits of error: the fluctuations of one

source are certainly less than those of two: and, accordingly, the main source of the fluctuations must lie in changes in the relative intensities of the lamps.

It is for this reason that the work that has been done appears of sufficient promise to justify publication. If results are ultimately obtained, the description of the methods and the narration of the results would be so lengthy that it appears desirable to divide thus the account into two parts.

The emission of positive rays from heated phosphorus compounds.
By FRANK HORTON, M.A., St John's College.

[*Read* 25 October 1909.]

Prof. Sir J. J. Thomson has shown that certain salts when heated give rise to a large positive ionisation*. Of the salts he experimented with the greatest effect was given by phosphates, and of these aluminium phosphate was found to be the most active. From this salt at a red heat Sir J. J. Thomson found that the emission was so great as to be easily measured with a galvanometer. The following experiments were made with the object of discovering whether there was any connection between this phenomenon and the "anode rays" of Gehrcke and Reichenheim. These experimenters have found† that when certain salts are used as anodes in a vacuum tube they give off positive rays which proceed at right angles to the surface of the anode and behave in a manner completely analogous to that of cathode rays.

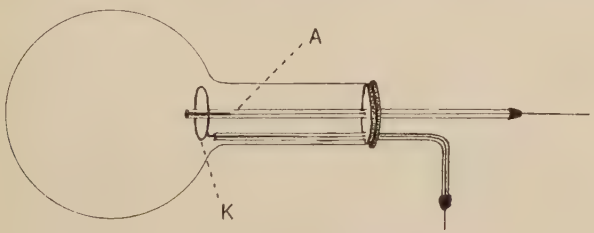


Fig. 1.

A, anode. *K*, cathode.

These anode rays consist of positively charged particles of atomic dimensions moving with a velocity of about 10^7 cms. per sec. Spectroscopic evidence has proved them to consist of atoms of the metal contained in the salt anode. Gehrcke and Reichenheim found that anode rays were most freely emitted by the halogen salts of the alkali metals, and they consider that in general the most suitable salts to use as anodes are those which are easily fusible and easily dissociated by heat. The halogen salts experimented on by Sir J. J. Thomson were found to give a small excess of positive electrification when heated, but nothing nearly so great as that given by the phosphates.

Aluminium phosphate was the salt first used in the present experiments, and tubes containing this were employed as anodes in an apparatus (see Fig. 1) similar to that described by Gehrcke

* J. J. Thomson, *Proc. Camb. Phil. Soc.* Vol. XIV. p. 105.
† Gehrcke and Reichenheim, *Ann. der Phys.* XXV. p. 861, 1908.

and Reichenheim. The discharge tube was a round-bottomed flask of some 500—1000 c.c. capacity. It was connected to a mercury pump and McLeod gauge, and had a charcoal tube attached for producing a low vacuum by means of liquid air. Owing to the high melting points of the salts used in these experiments, the tube A, which contained the anode, was of fused quartz, 2 mm. internal and 5 mm. external diameter. The aluminium phosphate to be used as anode was finely powdered and mixed with a little powdered graphite to render it conducting, and also with a little silver chloride to bind the mass together when heated. This mixture was rammed into the end A of the quartz tube to a length of 2—3 cms. The tube was then strongly heated in a blow-pipe flame. This caused the silver chloride to melt, and on cooling the mixture was firmly held in the tube. Electrical connection with this mixture was made by means of a copper wire introduced into the tube at the other end and pushed into the mixture while that was still hot. The cathode is the aluminium ring K at the end of an aluminium wire, the straight part of which is covered by a glass tube as shown in the diagram. These tubes, from the anode and cathode, pass through holes in an ebonite bung which closes the mouth of the flask, the joints being made air-tight with sealing-wax.

The discharge through the tube was sent from a large Marconi induction coil. It would have been better to have used a large Wimshurst machine (following the method of Gehrcke and Reichenheim), but there was not one available. A spark-gap and a valve of Lodge's pattern were usually placed in the secondary circuit of the coil in order to prevent, as far as possible, the current from passing in both directions.

Perhaps it may be of interest to mention here that for obtaining quickly the low vacuum required in these experiments, it was found to be of great advantage to have the tube containing the charcoal for cooling in liquid air made of fused quartz. During the preliminary pumping a blow-pipe flame was played directly on to the quartz tube, and in this way the charcoal was more quickly and more completely freed from occluded gas.

When the induction coil discharge passed through this tube (the gas pressure being so low that no luminous gas was seen in the bulb) the "torch" of light at the anode, described by Gehrcke and Reichenheim, was obtained. On examination, the spectrum of this light was found to contain the brightest silver lines, but no lines of aluminium could be detected. In a magnetic field part of the luminosity was deflected in the direction which would mean that the rays consisted of positively charged particles leaving the anode, and partly in the opposite direction, but I could never be quite sure that this was not due to the discharge from the coil

passing in both directions. The light at the anode never showed the fine bright pencils so characteristic of the anode rays, and the spectrum proved that if any anode rays were produced they were due to the silver chloride, and not to the aluminium phosphate.

Other methods of making the aluminium phosphate into a solid mass, capable of being used as an anode, without mixing with any substance likely to produce anode rays, were tried, but without success. It was therefore decided to use some more fusible phosphate, and a mixture of sodium and lithium pyrophosphates with a little graphite was heated in the quartz tube until it fused. This tube was then fitted up as the anode in a bulb similar to that described above. On evacuating and passing a discharge this tube behaved in a curious way. When the pressure was not too low, there was a torch of light surrounding the anode, in front of this a dark space, and then at about the centre of the bulb a little ball of luminosity, which probably corresponded to a striation in the positive column. On putting on a magnetic field transverse to the direction of the rays from the anode, the torch seemed to jump out nearer to the glowing ball, at the same time spreading out symmetrically about the axis of the anode, and curling round towards the cathode ring. The appearance was as though a stream of rays were suddenly brought up against a barrier. The effect produced seemed to be independent of the direction of the magnetic field, so long as the lines were at right angles to the direction of the discharge. That sodium was shot off from the anode could be seen by the gradual appearance of the yellow sodium light upon the cathode—appearing at first on the side nearer to the anode and gradually spreading all over it. The spectrum of the light at the anode showed the lines of sodium and lithium brilliantly, also the brightest mercury lines; including the three new lines in the orange the wavelengths of which are given by the author in a previous paper* as 6232, 6121, 6070.

In a more perfect vacuum the gas luminosity in the bulb disappeared, but the glow at the anode remained. There was however no sign of the fine pencils of light, with the accompanying phosphorescence of the bulb, which is so characteristic of the anode rays. In a magnetic field the glow at the anode was mostly deflected in the positive direction, but there was some which behaved as though it were negatively charged. In order to decide whether the rays deflected in the positive direction were really positively charged, or whether they were negatively charged rays travelling *towards* the anode, their behaviour in an electrostatic

* *Proc. Camb. Phil. Soc.* Vol. xiv. Pt 5.

field was investigated. For this purpose the apparatus diagrammatically represented in Fig. 2 was used. The rays leaving A were made to shoot through the hole in the earth-connected screen S, and then between the parallel plates P_1, P_2, which were about 3 cms. long and placed about 1 cm. apart. A strong electric field could be created between these by connecting them to the terminals of a battery of small accumulator cells. Z is a zinc-blende screen by means of the phosphorescence of which the deflection of the rays could be observed even when the illumination along their path was very faint.

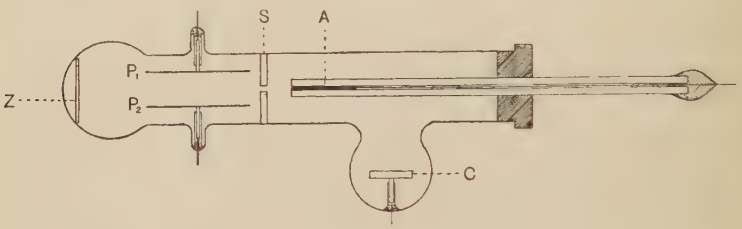


Fig. 2.

A, anode. S, earth-connected screen. Z, zinc-blende screen.
C, cathode. P_1, P_2, parallel aluminium plates.

As with the former apparatus it was found that there was no fine pencil of rays given off by the anode. The rays getting through the hole in the screen S were always deflected towards the positive plate by the electric field, showing that they were negatively charged. They were, no doubt, ordinary cathode rays formed by the reversals of the induction coil. The impossibility of obtaining anode rays from these phosphates seems to show that there is no connection between the anode rays and the positive electrification emitted by salts when heated, but it was thought worth while seeing whether anode rays could be obtained from other compounds of phosphorus, because these, as well as phosphorus itself, have the power of emitting positive ions when heated on the anode of a vacuum tube.

Some experiments were therefore made in which calcium phosphide mixed with a little graphite was used as the anode in the apparatus of Fig. 1. At a low pressure a distinct torch of rays was seen surrounding the anode, and, after a few minutes, from the midst of this torch there came a fine pencil of rays shooting right across the bulb and causing phosphorescence of the glass where they struck. This pencil seemed to have its origin at a spot on the surface of the anode which was hotter than the rest of the surface. It was not deflected in a weak magnetic field,

but was deflected in the positive direction in strong fields. These rays were evidently anode rays similar to those obtained by Gehrcke and Reichenheim. The spectrum showed some calcium lines and other lines which could not be identified in the short time during which the rays persisted. Thus we see that anode rays can be obtained from calcium phosphide, although they could not be produced from the phosphates experimented with.

Gehrcke and Reichenheim have connected the production of anode rays from the halogen salts of the alkalis and alkaline earths with their easy fusibility and their property of dissociating when strongly heated. The heated salt is supposed to be decomposed electrolytically by the passage of the current, the metal being liberated at the surface of the anode and travelling across the tube. Close to the anode the electric force is very strong on account of the anode fall of potential, and it is in traversing this region that the liberated metal atoms obtain their great velocity. Some time ago it was shown by Matthies* that the anode fall in a vacuum tube was greatly increased by the presence of a halogen vapour. In a recent paper Reichenheim† states that, at low pressures and with large currents, the anode fall in the halogens amounts to several thousand volts. This abnormally high value Reichenheim thinks may be explained on the supposition that the halogen vapours possess, in a higher degree than other gases, the power of absorbing negative electrons. If this be correct, other electro-negative vapours should behave in a similar manner, and Reichenheim shows that a large anode fall is also obtained in phosphorus vapour. In the experiments described in this paper it is probable that phosphorus vapour was present in the case of the calcium phosphide anode, owing to its decomposition when heated. There would thus be a large anode fall and a strong force near the anode which would give to the liberated atoms the energy to make them luminous anode rays. The phosphates, on the other hand, are extremely stable bodies, and in their case there would be no phosphorus vapour to cause an abnormal anode fall and consequently no anode rays. Thus these experiments seem to confirm the view of Gehrcke and Reichenheim as to the origin of anode rays, and to show that there is no connection between this phenomenon and the emission of positive electrification from heated salts.

The author wishes to acknowledge his indebtedness to the Government Grant Committee of the Royal Society for the means of purchasing some of the apparatus used in these experiments.

* Matthies, *Ann. der Phys.* xviii. p. 473, 1905.
† Reichenheim, *Verh. d. D. Phys. Ges.* p. 168, 1909.

On the shape of beams of canal-rays. By J. A. ORANGE, B.A., Senior Scholar of Trinity College. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 8 November 1909.]

In a recent paper* the writer stated certain conclusions with respect to beams of canal-rays obtained in connection with "sandwich-kathodes." It was maintained that the canal-*rays* are straight, the curvature of the boundaries of the beam being explained by supposing that they are merely the envelopes of such rays.

This point has been tested since by using a mica obstacle and observing the form of the shadow in the beam. The method of placing the obstacle will be evident from Fig. 1, where *m* is the mica slip, which is attached to the aluminium kathode-plates by means of water-glass. The beams of canal-rays were photo-

Fig. 1.

graphed as described in the former paper. The sharpness of the boundaries of the beams in the case of hydrogen was remarkable, and a considerable degree of sharpness was obtained in the photographs, the softening that did occur being attributable to slight changes in the form of the beams (due to changes of pressure) while an exposure was being made. [The time of exposure was of the order of 45 minutes.] Figs. 2 to 6 are copied from photographs and illustrate the point at issue.

The outlines of the beams and of the Crookes' dark space at a fairly high pressure are shown in Fig. 2. For clearness' sake, the shadows in the beams due to the three pins are omitted. The beams will be referred to by the letters shown in Fig. 2. The shadows due to the pins in Figs. 3 to 6 are in accordance with the explanation adopted.

With regard to the effect of the mica slip, it may be as well to say at the outset that the use of this is liable to serious objections. If the mica slip were an obstacle *only* to the *canal-rays*, the method would be much sounder; but since the mica is a local obstacle to the discharge in general, its introduction

* J. A. Orange, *Proc. Camb. Phil. Soc.* Vol. xv. Pt. 3, 1909, p. 217.

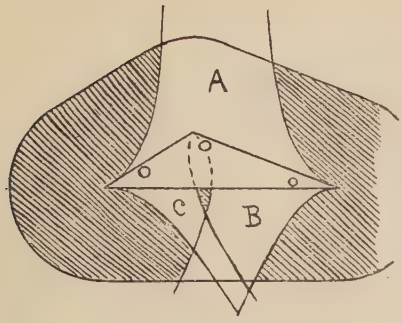


Fig. 2.

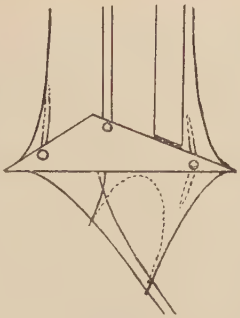


Fig. 3.

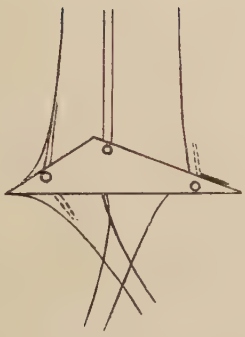


Fig. 4.

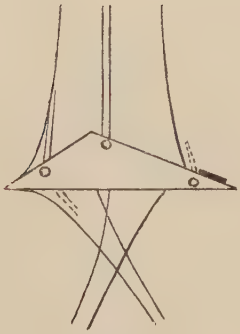


Fig. 5.

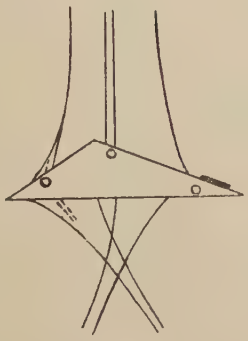


Fig. 6.

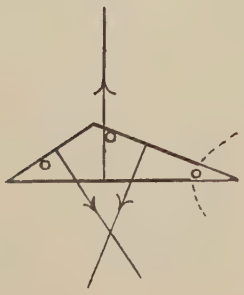


Fig. 7.

may produce great changes in the electrostatic field, which is known to depend largely on the distribution of ions in the kathode region.

Consider the effect of the slip in Fig. 3 in the light of this reservation. Beam A, which we hold to originate in the lower part of the region, (kathode-rays from the longest side of the triangle being the primary agent), contains a sharp shadow as shown. Here we can claim that the mica does not interfere at all with the discharge from the longest side of the triangle, and hence the beam A is formed normally, the mica simply blocking out part of the canal-rays when they have passed through the kathodic interspace.

But now consider the effect on beam B. Here we have to deal with canal-rays produced in the N.E. corner of the region, the kathode-rays from the medium side of the triangle being responsible. The slip, however, is placed so as to interfere considerably with the discharge (including the kathode-rays) from this side. Thus instead of getting a neat shadow in the beam, we get an ill-defined dark core which is vaguely indicated in Fig. 3 by a dotted line of elliptical form.

The details of Fig. 3 are typical of the appearances for various positions of the slip, as may be readily imagined. The effect of placing the slip in a more exceptional position is shown in Figs. 4, 5 and 6, which correspond respectively to decreasing pressures.

The general tendency is best shown at the lowest pressures, e.g. in Fig. 6. The beams are roughly of the forms which would be obtained if the right-hand corner of the kathode were removed altogether, as indicated by the dotted line in Fig. 7.

This right-hand corner is certainly fenced off by the mica and the pin so that it is prevented from acting as a double kathode; the forms of the beams would seem to be in accordance with this fact. The only other feature of interest is the fine pencil of canal-rays coming from the aperture between the mica and the pin, and visible in Figs. 4 and 5.

These observations would seem to support the view that the *constituent* canal-*rays* are straight.

I wish to thank Prof. Sir J. J. Thomson for his kindly interest in these experiments.

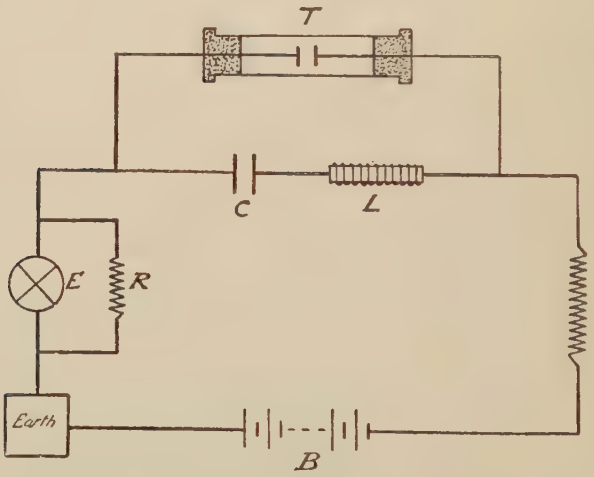
An Electric Detector for Electromagnetic Waves. By E. M. Wellisch, B.A., Emmanuel College. (Communicated by Professor Sir J. J. Thomson.)

[*Received* 6 November 1909.]

A series of experiments in connection with ionisation produced by collision was undertaken by the writer in order to test certain theories which he had previously advanced with regard to the passage of electricity through gases. The preliminary experiments were made with an ionisation chamber containing two parallel plane electrodes made of aluminium, and at a distance from one another of 2 cm.; one of these electrodes was connected to a source of potential, the other to a Dolezalek electrometer. During the course of these experiments it was observed that, when the gaseous pressure and the electric field were so chosen that the gas was on the verge of breaking down, a very feeble discharge of a Röntgen ray bulb placed in the neighbourhood was sufficient to produce an exceedingly large deflection of the electrometer needle which had previously been stationary; the direction of the deflection indicated the passage of a transient electric current through the gas. Further investigation of this electric charge produced within the gas showed, however, that the determining cause of the deflection lay not in the rays issuing from the Röntgen bulb, but in the electric oscillations set up by electric waves proceeding from the induction coil which worked the bulb. The Röntgen bulb was henceforth removed and a series of experiments was conducted in order to ascertain what degree of sensitiveness to electric waves could be obtained from similar arrangements. Several forms of ionisation chambers were used, and each of them proved sensitive as detectors of extraneous electric waves. The accompanying diagram represents the scheme of connections employed in one of the trials. In this case the detector consisted of a glass tube T (about 4 cm. in diameter); the electrodes were two plane parallel aluminium discs (each 2 cm. in diameter) at a distance apart of 1 cm. The tube was connected to a Töpler mercury pump and a MacLeod gauge so that the pressure of the gas (dry air) could be adjusted and measured. In the diagram B represents the battery, one of whose poles is earthed, E the electrometer, R a high resistance (consisting of conducting glass) shunted across the electrometer, C a variable capacity, and L a variable self-induction.

The sensitiveness of the detector depends naturally on a large variety of circumstances; in fact, in the preliminary trials, it was

found impossible to record the conditions present when a high degree of sensitiveness had been obtained, so that it was a matter of difficulty to reproduce these conditions *ad libitum*. However, as a rough example of one set of conditions, it was found that with the air at a pressure of ·76 mm., and with one electrode connected to the negative pole of a battery of 400 volts, the tube would respond to faint electric impulses provided the capacity, resistance, and self-induction were suitably adjusted; the electrometer needle, otherwise stationary, would be deflected on the arrival of the wave. The object of the high resistance R was to cause the needle to return quickly to its zero so that any succeeding impulse might be readily detected.



The effect of an extraneous electric impulse is probably to set up electric oscillations in the circuit including the vacuum tube; the electric force in these oscillations, when superposed on the electric field in the tube, may then suffice to produce a discharge in the gas. It is important to notice that this discharge need not be, and in most of the cases investigated was not, luminous; its occurrence is manifested merely by the galvanometer or electrometer deflection. In this respect the present detector differs from Zehnder's Trigger Tube* in which the induced electric oscillations precipitate an electric discharge from an auxiliary battery and thus produce a glow in the tube. Another method for detecting electric waves which depends upon the same principle is that due to Boltzmann†. In this method a battery is on the verge of charging

* *Wied. Ann.* Vol. XLVII. 1892, p. 77.
† *Wied. Ann.* Vol. XL. 1890, p. 399.

an electroscope through a very small air-gap; electric waves in the vicinity will be detected by the electroscope if they are able to excite oscillations of strength sufficient to break down the spark gap. In the Fleming Oscillation Valve induced electric oscillations are detected by their ability to impart unilateral conductivity to the space between a cold cylinder and a hot carbon filament, the conductivity being due to the negatively charged corpuscles emitted from the heated carbon under the action of a directional electric force in the oscillations.

Comparative tests have been roughly made with the electric detector and an iron filings coherer, and have proved quite favourable to the former; a short period electrometer (e.g. the string electrometer described by Laby in *Proc. Camb. Phil. Soc.*, vol. XV.) seems especially suited for use in connection with the present form of detector.

It was observed also that in ionisation chambers such as those described, distinct deflections would occur under certain conditions of pressure and voltage in the absence of any electric waves produced by the induction coil; as an example of one set of conditions it was found that with the air at a pressure of ·76 mm., and with one electrode connected to the negative pole of a battery of 480 volts, frequent deflections occurred. The frequency of these deflections varied considerably with the inductance in the circuit; in fact, a small variation of the inductance changed the frequency from about 100 to 12 deflections per minute. It is reasonable to suppose that these deflections are due also to electric oscillations set up in the circuit including the detector. The writer is making further experiments with regard to these oscillations, especially with the object of determining their origin.

Aldabra and neighbouring Islands. By J. C. F. Fryer, B.A., Gonville and Caius College.

[*Read* 22 November 1909.]

Plate XII.

[Abstract.]

Introduction.

The group of islands comprising Aldabra, Assumption, Cosmoledo and Astove is situated some 270 miles to the N.W. of Madagascar and thus lies in the extreme S.W. corner of the Indian Ocean. Aldabra has long been known as the last spot in the old world on which indigenous giant land tortoises continue to exist. It was also known to have a peculiar land avifauna; and these facts combined with the discovery of unusual percentages of iron, alumina and silica in guano from the locality pointed to a possible land connection with Madagascar in previous times.

In 1905 the Sealark expedition carried on investigations in the Indian Ocean relating partly to the former land connection between India and Africa and partly to the formation of the coral islands of this ocean. The Aldabra group however was not visited and therefore from both points of view a further expedition was desirable. Certain of the more general results of this latter expedition are set forth in this paper.

Topography.

Aldabra is an atoll 24 miles long and from 4—10 miles broad. The land-rim is very perfect and is divided by narrow passes into four large islands called respectively Picard, Polymnie, Malabar, and Main or South islands. Round the outside of the atoll is a narrow fringing reef which is divided by channels from the passes and is indefinite or non-existent on the extreme east.

To landward the fringing reef is bounded by rocky cliffs 10—20 ft. in height which are overhanging and show evident signs of wave erosion.

The highest point on the land-rim (20 ft. above sea-level) is always situated near the sea-cliffs and there is a gradual slope to the lagoon which is fringed with dense mangrove swamps. Small overhanging cliffs are usually found at high tide mark in the swamp and give striking evidence as to the rapidity of lagoon erosion. The lagoon itself is very shallow except near the passes; it is muddy towards the outsides and sandy in the centre. Small islands and rocks are numerous round its outskirts and all show much loss due to erosion.

The general topography of the larger islands is as follows:

Picard is noted as possessing on its westward shore a narrow stretch of sand on which is built the settlement and which also constitutes the only arable soil on the atoll. In the centre of the island is a rocky plain with a flat pavement-like surface: it is known as Plain Cabris and is the locality from which were obtained certain peculiar rocks to be referred to later. The rest of Picard is more typical of the atoll as a whole; the surface of the ground has been broken up by rain-water denudation into points, pinnacles and sharp ridges, and the whole is covered with a dense scrub of Pemphis acidula. This combination of rock and jungle, known as 'champignon' country, forms the great bar to exploration in Aldabra.

Polymnie, Malabar, and the north of Main island consists of this champignon country. In the S.E. of Main island however another plain of flat pavement-like rock exists. It is covered by a somewhat open and varied jungle, and within its boundaries are found the only fresh-water springs on the atoll. These are situated at Takamaka and consist of pits in the rock about 5 ft. deep. Sand dunes 60 ft. high occur along the south of Main island and have been formed by the action of the prevalent winds (S.E. Trades) which blow up sand from the reef and pile it in mounds on the shore. The west of Main island is again entirely of the champignon type of country.

Leaving now the general topography of the atoll a more detailed account of its structure may be given, and this will be dealt with under three heads, (*a*) structure of land, (*b*) structure of fringing reef, (*c*) structure of lagoon.

Structure.

(*a*) The land is entirely rocky and the main rock component is in all cases coral though complications have been introduced by the presence of a quantity of calcium phosphate derived from guano. The rock of the land-rim may be divided into three classes. The first forms a zone round the outside of the land-rim and is marked by the fact that its component corals are very perfect and are almost all in position of growth. This point is most important as it proves that the atoll is an elevated reef and that it has not been formed by wave piling on a subsiding base. The second class is the pavement-like rock before referred to: it is composed of broken and often triturated coral with such débris as mollusc shells and echinoderm spines, and gives evidence that the inner portions of the reef before elevation were either dead or but feebly growing. The last class of rock is the 'champignon'; it is a highly metamorphosed coral rock with scattered inclusions of calcium phosphate derived from guano. Metamorphosed coral limestones from all the islands in this region contain inclusions of calcium phosphate, and it is necessary to

credit the latter substance with some power of inducing metamorphosis though its nature is not understood. In all the three classes of rock pits and subterranean caverns are numerous and usually contain salt-water which fluctuates tidally. They all show signs of erosion and are increasing in size. Evidence is thus given that the reef as a whole before elevation was but badly consolidated and that even now water can percolate completely through the land-rim.

A few exceptional rocks remain to be dealt with. Ile Esprit, in the lagoon, consists of a ridge of rock half a mile long and some 30 ft. high, and is thus the highest rocky point on the atoll. The body of the ridge consists largely of calcium phosphate: on its sides are pinnacles and walls of a much denuded rock composed almost entirely of mollusc shells. Plain Cabris on Picard island again shows somewhat similar structures but a crystalline mineral resembling apatite should be noted as also fossil bones of the giant land tortoise. It may be mentioned that Esprit island is of special importance if an attempt is made to reconstruct the early stages of Aldabra after elevation.

(*b*) The second section of the atoll consists of the Fringing Reef. Evidence obtained from the amount of wave erosion and from rocks of elevated coral limestone still in situ proves that the fringing reef of Aldabra is a ledge of elevated coral rock cut down to low water mark and preserved at that point by growing lithothamnia and by the piling of sand and coral on the flat. The fringing reef of Aldabra therefore is not a sign of the increase of the atoll seaward, but on the contrary is evidence of the loss of land owing to wave erosion.

(*c*) The last section of the atoll is the *lagoon*, some of the features of which have already been mentioned. Those to be further treated are the extent of lagoon erosion and the formation of passes. Judging by the maze of small islands and rocks, once part of the land-rim, at least two-thirds of the lagoon were at one time land. It is still increasing for the rock is reduced to mud; the mud is swept to sea and the land continues to diminish. Contrasting lagoon erosion with sea erosion it will be noticed that while the latter is uniform over any stretch of coast the former is just the reverse, and is extremely irregular. Passes therefore must be formed from the lagoon side of the land-rim and not from the sea. All stages of pass formation are shown on Aldabra. At Camp Frigate the mangrove swamp passes completely through the land-rim to the sea, though but little water flows through yet. The western channels show complete passes through the land-rim but the fringing reef has not yet been divided. Grande Passe finally is a pass with a complete channel, and it further shows that the channels of the passes

do not increase indefinitely in size though the land continues to be cut down as far as low water mark. This point will be further illustrated by the atoll of Cosmoledo, which will be described later. The structure of Aldabra has now been described, and before proceeding further it may be mentioned that any land connection with Madagascar is shown to be obviously impossible.

The Flora and Fauna of Aldabra must be treated very briefly for the collections are not completely worked out.

Flora.

The Flora contains, oecologically, the following sections: (1) *Pemphis jungle* which grows on metamorphosed rock and contains little but Pemphis acidula. (2) *Open country jungle* which contains a varied vegetation of Madagascar origin. (3) *Mangrove swamp* containing three genera of Rhizophoraceae and also the pseudo-mangroves Avicennia, Sonneratia and Carappa. (4) *Shore zone* containing typical coral-sand plants such as Tournefortia argentea and Scaevola koenigii. The zone owes its origin to the fact that sand is blown on to the land from the reef and forms a little soil near the shore.

Fauna.

The Mammalia, as regards indigenous species, are represented by two bats only, though one Pteropus aldabranus is peculiar. Reptiles comprise the Giant Land Tortoise (Testudo daudinii), the Green Turtle (Chelone mydas), the Hawksbill (Chelone imbricata), two species of Gecko and a Skink. The distribution of the Giant Land Tortoises affords an interesting problem. They have been found in recent times on the Seychelles, the Mascarenes, Madagascar, Aldabra, Assumption and Cosmoledo. Their presence on continental islands such as the Seychelles is easily accounted for by the previous land connection, but this explanation will not apply to islands of purely oceanic origin such as Aldabra. The fossil bones on the latter atoll appear to prove that they do not owe their distribution to transport by man: they are at present too large to drift on wreckage or logs, or if drifted before their large size was evolved, then they present a wonderful case of parallel evolution. The matter still awaits a satisfactory solution.

Amphibia are absent from Aldabra and the fish though specifically numerous call for no special mention.

The land invertebrata are at present being worked out but it may be mentioned that the majority seem to be derived from Madagascar: indeed it may be said that the whole land fauna with the exception of the Giant Land Tortoise is such that it may have been obtained from neighbouring lands without necessitating any former continental connection.

Next passing to the other islands visited, a brief summary of the chief structural characters of each may be given.

Assumption is not an atoll but is an elevated reef composed of the same classes of rock as constitute the land-rim of Aldabra. The rock is extremely cavernous; water can penetrate completely through the land and there is no doubt that, as the caverns inter-connect, the island will be broken up into a number of rocks.

Astove is an elevated atoll with a very perfect land-rim which is divided by only one pass. It is composed of the same three classes of rock as were found on the land-rim of Aldabra, though coral in position of growth is even more common. The lagoon is very shallow and probably existed almost entire at the time of elevation. It is getting larger; fresh passes are forming and the atoll will be speedily broken up.

Cosmoledo is also an elevated atoll but differs from Aldabra and Astove in having but half its circumference capped with land. The remainder, the reef, is elevated coral rock cut down to low tide level and piled with coral débris while the edge is protected by growing lithothamnia. There is no doubt however that once the whole circumference of the rim was land. Sand derived from disintegrated rock is abundant and hides the rock of the islands so that there is a tendency as the rock land is removed to replace it by sand cays.

The state of Cosmoledo must be considered as forecasting in part the future condition of Aldabra, and in conclusion an hypothetical biography of the latter atoll may be given, basing the account on the facts obtained from the various coral islands in the region.

The geology of Aldabra shows that it is entirely of oceanic origin and has been built up by active coral growth, and therefore our history of the atoll must begin with its formation as a reef beneath the sea. There must first have been, however, some base or mountain-top which reached to within 40 fathoms of the surface, for it is well known that reef-building corals cannot live at greater depths. Of the nature of this base we have no evidence, though it may be taken as probable that it is of the nature of a volcanic mound. We know of volcanoes in the Comoros to the S.W. and the Sealark dredged volcanic mud near Providence to the E., and there is thus no inherent improbability in the existence of volcanic action in between. However this may be, it is necessary to assume the presence of some base, colonised by corals, forming a reef which gradually grew up until it nearly reached the surface. This may be deduced from the fact that the fossil corals are all of a shallow-water facies. As to the form of the reef we suppose both from the geology and from theoretical

considerations that the outsides were flourishing, while the inner portions were but feebly growing or dead. This means that the shape of the reef was probably that of a shallow basin.

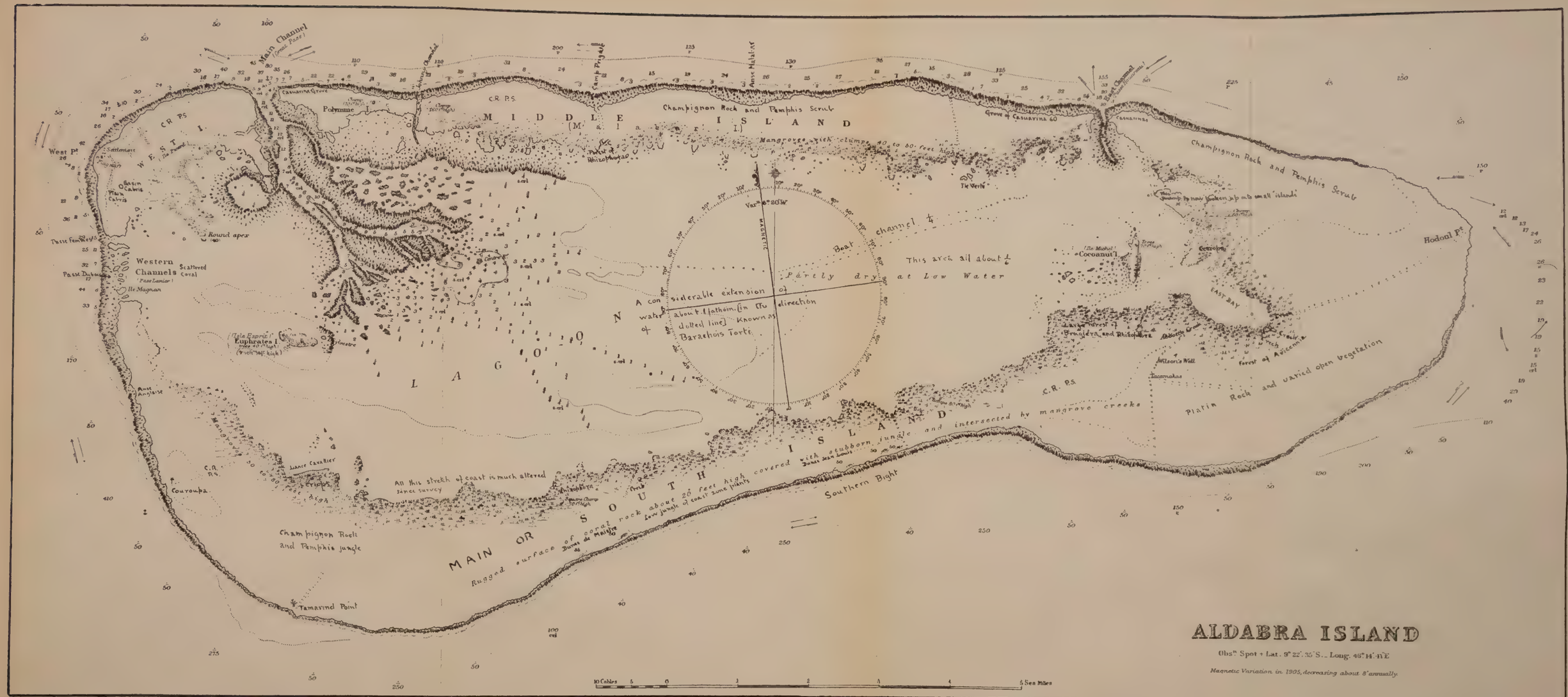
The next point in its history was its elevation above the surface of the sea, though how this occurred is somewhat doubtful. It would naturally be supposed that it was accomplished by a slight local elevation of the earth's crust, but against this is the objection that there is evidence of an elevation of equal amount throughout the western Indian ocean, and it is difficult to conceive of an alteration in the level of the earth's crust which would be so uniform over such a large area. The only alternative is an alteration in the level of the sea, and it may be mentioned that there are points in favour of this hypothesis though they are of too indefinite a nature to bring up for discussion in this paper.

The extent of the elevation may be placed at about 60 ft., for Esprit island is 30 ft. high, and it is probable that the reef hardly reached the surface before elevation, while since then an enormous amount of denudation has occurred. After elevation the whole reef was placed out of water and the depression in the centre was dry. In Astove it will be remembered there was evidence that the present lagoon was largely existing before elevation, and may therefore be called a primary lagoon, while in Aldabra we now see that the present lagoon must be a secondary lagoon developed after elevation. The question of its formation will be dealt with shortly, but first certain deductions must be made from Esprit island which consists almost entirely of calcium phosphate. The phosphoric acid was undoubtedly derived from guano, and to have formed the phosphate rock on Esprit it is necessary to suppose that a solution obtained entrance to a large underground cavern which may have contained a certain amount of sand or mud. The whole space was gradually filled with the deposit, and to have obtained such a large quantity it is necessary to suppose that the top of Esprit was the sink or drain from a large area covered with guano. This area was probably the primary lagoon which after becoming dry was filled with guano by oceanic birds. For the present, therefore, it is supposed that all the peculiar phosphate rocks of Aldabra were formed by a solution of guano obtaining entrance to a subterranean pocket.

Turning next to the formation of the secondary lagoon, mention was made of the fact that the rock as a whole is still very cavernous so that water can penetrate completely through the land-rim. At the time of elevation this feature must have been more marked, and therefore the sea could at once begin dissolving away rock and enlarging any spaces. Simultaneously rain-water denudation was acting on the surface and the natural

result was a gradual decrease in level. In time the subterranean caverns, becoming larger, joined each other and finally a series, probably in the neighbourhood of Grande Passe, obtained open connection with the sea. After this a tidal current swept in and out continually dissolving away rock and forming a lagoon. The transition from this stage to the present one is easy: a further pass, Passe Houareau, next formed and the lagoon gradually obtained its present dimensions, though Ile Esprit, on account of its being composed of an insoluble rock, was left standing as a clue to former conditions. The level of the land gradually decreased by rain-water denudation and fresh passes continued to form until the present state of things obtained. As to the future we may safely prophesy the formation of more passes: lagoon erosion will proceed with increased speed though it is noticeable that land is only cut down to low tide mark, for below this lithothamnia, corals and the piling of sand form a protection against speedy erosion. Looking further into the future and bearing in mind the present state of Cosmoledo we see that the rock islands will gradually be divided up and be separated from one another by long stretches of reef. The islands may be eaten away until no land remains, though it is more probable that it will continue to exist in the form of sand cays so characteristic of many atolls. This stage may be called the Farquhar* stage. Beyond this it is impossible to foretell; and in conclusion it may be suggested that the life-history of Aldabra will be found typical of most coral rock islands, and finally may give a clue as to the former history of many of the coral sand islands so common in the tropics.

* Farquhar atoll, 150 miles east of Astove, is in this condition. Vide *Trans. Linn. Soc.*, Vol. XII. p. 144.



ALDABRA ISLAND

Obsn Spot + Lat. 9° 22′ 35″ S. Long. 46° 14′ 0″ E

Magnetic Variation in 1905, decreasing about 8′ annually.

All soundings in fathoms.

Figures on the land show the heights in feet above H. W. Springs.
Double dotted line = a main track, other tracks not indicated.

crl. coral, *r.* rock, *s.* sand, *sh.* shells, *w.* white.
Single dotted line defines an area of land or lagoon.

C.R. P.S. Champignon Rock and Pemphis Scrub.

Notes on the Larger Cetacea. By D. G. LILLIE, B.A., Hutchinson Research Student of St John's College. (Communicated by Mr A. E. Shipley.)

[*Read* 22 November 1909.]

The establishment of whaling stations within recent years in three localities off the shores of the British Isles should give a new impetus to the study of Cetology, and stimulate additions to our knowledge of the larger Cetacea before these much-hunted animals become too scarce.

Hitherto our studies of these enormous creatures have been chiefly derived from isolated specimens stranded from time to time in various localities around the coasts of civilised countries, which coming by chance into the hands of zoologists, often in an advanced state of decay, have enabled them to add a few observations to the large though scattered Cetacean literature. This would indeed seem the only method possible since, during the last three centuries, the whaling industry has been confined to the wildest regions of the earth and carried on under conditions of physical privation which were beyond the endurance of all but a few.

Since the whaling industry of to-day now supplies fresh material fairly near at hand, and is destined to play a large part in any new work which may be done on the histology and general biology of whales in the near future, it will not be inappropriate here to briefly trace the history of this industry from early times until its establishment near our shores at the present day, and to give a short account of modern methods of whaling before proceeding to record an observation of more strictly scientific interest, which was made during a preliminary visit to the Irish whaling station in the past summer.

In very early times whales occasionally became stranded just as they do now, through venturing too near the shore and being left high and dry by the tide. It was, no doubt, soon discovered that the oil of these stranded individuals could be utilised; but the practice of pursuing and killing large whales, as far as is known, only dates from about the year 875 A.D., the Basques then hunted *Balaena biscayensis*, and from their word "*arpoi*," which means "to take quickly," we get the word "*harpoon*." At first the Basques attacked this whale from the shores of the Bay of Biscay; but they later put out to sea on long voyages as the quarry became scarcer.

This fishery was practically exhausted by 1607 when Henry Hudson made his first voyage to Greenland and Spitzbergen and discovered the Greenland whale (*Balaena mysticetus*) in its home

among the northern icefields. The English and Dutch immediately opened up the Arctic whale fishery, taking with them the hardy Basque seamen to act as their instructors. This fishery reached its height at the beginning of the 18th century, English, Dutch, Germans, Spaniards and Danes all taking part. In 1749 the first vessels set out from Scotland and took over the fishing in Davis Strait, while the Americans worked that of the Behring Strait, the fishery on the east coast of Greenland being then exhausted. The last great names associated with the Greenland fisheries are those of Scoresby and David Gray in the Atlantic and Scammon in the Pacific. The fishery is now almost extinct.

In 1712 the Sperm whale (*Physeter macrocephalus*) was first hunted. This fishery was started by Americans on the island of Nantucket in the Atlantic. British ships took part in 1775 and extended the fishery to the Pacific and Indian oceans, but they abandoned it in 1853. This whale is now chiefly hunted by Americans and Norwegians.

The shore-loving Californian Gray whale (*Rhachianectes glaucus*) was first pursued on the Pacific coast of North America in 1846.

In 1866 a new era began in the whaling industry. Up to this date attention had necessarily been confined to the two species of *Balaena*, the Sperm whale and the Californian Gray whale. All these slow-moving creatures could be hunted with the hand harpoon in small boats each manned by six men, which put out from the shore or from a whale ship in mid-ocean as soon as a whale was sighted.

The fast-swimming Balaenopteridae or Rorquals remained unmolested by man until 1866, when Captain Svend Foyn, a Norwegian seaman, invented a harpoon which made their capture possible. This deadly projectile is fitted with an explosive shell and is fired, together with the attached harpoon-rope, from a gun in the bow of the whaling steamer. Captain Foyn took out a patent for his invention, established a station at Vadsö and opened up the Rorqual whale fishery off the coast of Finmarken.

In 1882 his patent expired and numerous stations sprang into existence along the northern coasts of Norway and Lapland, afterwards spreading to Iceland and the Faroe Islands.

In 1903 this industry reached the shores of Scotland. There are now four stations on the mainland of Shetland and one in North Harris in the Hebrides. In 1908 a station was established on the Island of South Innishkea off the coast of Co. Mayo in the west of Ireland and a second Irish station is expected to open next summer. All the stations on the east side of the Atlantic are in the hands of Norwegians who, like the old Basque whalers before them, are the masters of the industry they invented and have been engaged by other nations to teach them their craft. The

result has been that modern whaling centres have sprung up in Newfoundland, Japan and elsewhere.

The staple quarry of the northern whale fishery is the widely distributed family of the Balaenopteridae, the last of the larger Cetacea, the Sperm whale and *Balaena biscayensis* being also taken when they can be found. The Balaenopteridae are still fairly plentiful but, at the present rate of slaughter and with the rapid spread in the use of the deadly Svend Foyn harpoon, the day of their extinction cannot be far distant, as Mr Shipley pointed out in his presidential address to Section D at the Winnipeg meeting of the British Association.

The Norwegian method of whaling is briefly as follows. A station is established on shore or afloat to which three or four strongly built steamers from 15 to 90 tons burden are attached. Each has a harpoon gun mounted in her bows. These go to sea in search of whales and stay out from two to ten days at a time. When a steamer gets within 40 yards of a whale the harpoon is fired, and the shell at the point of the harpoon is so arranged that when the harpoon enters the body of the animal it bursts and the animal is generally killed at once; but this is not always the case.

Mr Southwell records an instance of a whale, *Balaenoptera sibbaldii*, towing a Newfoundland whaling steamer for a distance of 122 miles, the screw being reversed at full speed the whole time. After a pursuit which lasted 26 hours the animal was exhausted and killed.

The Balaenidae and *Physeter* float when dead; but the Balaenopteridae always sink, probably on account of their having less oil. The Norwegians have overcome this difficulty by an ingenious device. The dead whale is brought to the side of the steamer and inflated with air. An iron pipe is thrust into the body cavity and is connected by india-rubber tubing to a pump in the engine-room. When sufficient air has been pumped into the body cavity to render the animal buoyant, the pipe is withdrawn and the wound is plugged with a piece of tarred wood. In this condition the dead whale is kept afloat, and several can be towed to the factory at one time by a whaling steamer. The air thus pumped in keeps the carcase distended until it is landed and ready to be cut up, its volume being sometimes added to by the gaseous products of decomposition. As soon, however, as the distended body is pierced by the flensing knife the imprisoned air escapes with great violence and portions of the viscera are torn away and shot out of the body cavity. The two foetal whales which I obtained this year at the Irish whaling station were extracted in a mutilated condition from among the scattered remains of organs thus expelled from the body cavities of their mothers.

Directly a whale is hauled up the slip or landing stage the whalebone and the blubber are rapidly stripped off and the latter boiled to extract the oil. The remainder of the carcase is dragged asunder by means of wire ropes and steam winches and cut up by many hands. The bones and soft parts are roughly separated, dried and ground in mills to form bone manure, flesh manure, cattle food and, at some stations, various forms of preserved meat for human consumption. In this way a whole animal 70 feet long and as many tons in weight will often disappear completely in the course of a morning.

To the scientific man a whaling station does not pretend to offer the advantages of unlimited time and comfort which are to be found in a laboratory. It is so essential, in the whaling trade, to dispose of the animals as rapidly as possible while the oil is fresh that the whalers can hardly be expected to wait for the deliberations of the anatomist. Yet there can be little doubt that to visit a whaling station such as the one in the west of Ireland is a more satisfactory method of increasing our knowledge of the larger Cetacea, than to depend upon the occasional stranding of an isolated specimen on some part of the coast. For at a station all the largest whales, with the exception of *Balaena mysticetus* and *Rhachianectes glaucus*, can usually be seen within the space of three months. Very frequently several individuals of different species can be examined and compared as regards their external characters and their internal characters also, according to the skill of the investigator in overcoming the obvious difficulties arising from the manipulation of such large creatures. The material is often sufficiently fresh for histological study, which, on account of the gigantic size of the animals, should prove of considerable interest. Moreover exceptional opportunities occur for observing the animals in the living state.

With regard to the smaller Cetaceans, since they are seldom killed by man, material is difficult to obtain; but the study of these animals is not at the moment so pressing as that of the rapidly decreasing larger forms.

It now remains to briefly record one of the several observations which were made at the Irish whaling station, during a visit of seven weeks, in the past summer.

The distribution and significance of the scanty hairs of the Cetacea do not appear to have been hitherto studied in the detail they deserve. They have been vaguely referred to as occurring on the lower jaw of some adult forms. Sometimes they have been found on the foetus only.

In two adult Sperm whales (*Physeter macrocephalus*) seen at Innishkea this summer, no trace of hairs could be found on any part of the animals even after careful searching.

In the case, however, of the Rorquals *Balaenoptera sibbaldii* and *Balaenoptera musculus* some ten individuals of each species were examined and it was found that a definite distribution of hairs could be made out in each case. Four rows of white, bristle-like hairs from half an inch to an inch in length occur on the dorsal surface of the beak or facial region of the head. These consist of two inner rows on either side of the median ridge which bears the blowholes or external nares and two outer rows following the edges of the snout, from points just behind the blowholes to its anterior extremity. The average number of hairs in each row is about eight.

On each side of the lower jaw there was a row of some five or more hairs running from the tip of the mandible along the middle line of the outer edge of each ramus to a point just in front of the eye. At the extreme anterior end of the mandible, over the ligamentous junction of the rami, there were also two rows of hairs set at right angles to those above mentioned. These occur close together and run parallel to each other from the upper to the under surface of each ramus. The hairs in these two rows are placed closer together than in the others, there being generally about fourteen hairs in each row.

On looking through the Cetacean literature there appears to be no record of the occurrence of hairs in the odontocetes, except in some foeti. It would seem that the presence of hairs in the adult is restricted to the whalebone whales and their retention and distribution over the beak and mandible in these forms may be due to their possessing a tactile function and thus serve to indicate to the animal the presence of its food.

The small size of the organisms, which generally form the food of the Mystacoceti, making them difficult to see, and the olfactory organs of the Cetacea being very reduced, it seems reasonable to suppose that the occurrence of tactile hairs over the oral region would be a distinct advantage, as the small food animals would brush against them and thus inform the whale when to open its mouth. The top of the snout of a whale is, after all, only the prolonged upper lip, where one would naturally look for vibrissae. The food of the odontocetes being of a larger size the presence of tactile hairs is not obviously required, and so the hairy covering has entirely disappeared in the adult forms of these whales. If the hairs of the whalebone whales do not function as suggested above, an interesting problem remains open to solution to explain how these animals become aware of the presence of their food.

Experimental Investigation as to Dependence of the Weight of a Body on its state of Electrification. By L. SOUTHERNS, B.Sc. (London), Whitworth Scholar, 1851 Exhibition Scholar. (Communicated by Professor Sir J. J. Thomson.)

[*Received* 9 December 1909.]

In a previous paper* the writer described an experiment designed to detect any alteration which might occur in the weight of a body when its temperature was changed. The present paper deals with an experiment which seemed naturally to follow the former one, viz. an attempt to detect change of weight due to electrification. This experiment, unlike the other, appears to give a positive result. The object of the experiment is not to actually weigh electricity, but rather to find whether the presence of electricity on a body modifies in any measurable degree the force of gravity acting upon it. While such a modification of gravity seems to provide the simplest way of explaining the results, such explanation is by no means insisted upon. Further reference to this matter will be made in the last section of the paper.

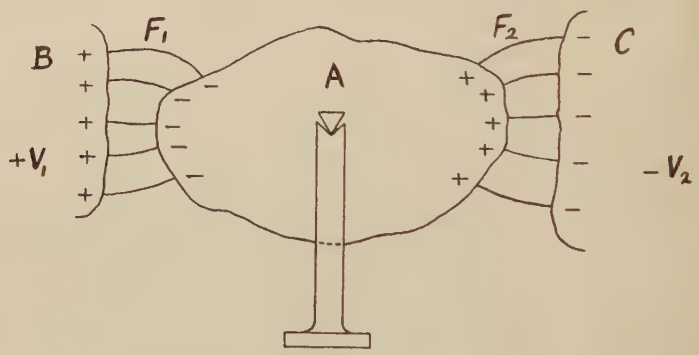


Fig. 1.

The paper is divided into the following sections:

(1) General theory and method of experiment.
(2) Description of apparatus.
(3) Detailed description of experiments and results.
(4) Conclusion.

(1) *General theory and method of experiment.*

A general idea of the method employed may be gained from consideration of fig. 1.

* *Proc. Roy. Soc.* A, Vol. LXXVIII. December 20, 1906.

Suppose a delicately but stably-balanced earthed conductor A to be placed between two fixed charged conductors B, C, at potentials $+V_1$, $-V_2$ respectively. Two electrostatic fields F_1, F_2 will be set up, and these will be quite independent of each other, provided all lines of force from B to C are intercepted by the conductor A, which is at zero potential, or by some other earthed conductor. Charges will be induced on the ends of A, which will now be subject to electrostatic forces which will in general deflect it to a new position inclined at an angle, say θ, from its original equilibrium position. If now the fields F_1, F_2 be reversed each independently of the other, so that we have B at potential $-V_1$ and C at potential $+V_2$, there will be no change in this deflection θ caused by electrostatic forces, for these are independent of the direction of the fields. But now let us suppose that a gravitation effect exists such as to cause the end of A, which is positively charged, to be a little heavier than the negatively charged end. This will give a deflection $\pm\delta$ which must be added to θ, thus with one direction of field we shall have a total deflection $\theta+\delta$, and with the direction of field reversed we shall have deflection $\theta-\delta$. The difference between the two observed deflections will thus be 2δ, the electrostatic part of the deflection being eliminated. In observing deflections practically two methods may be adopted. We may observe the zero position under no field, then put on the field in a given direction and observe $\theta+\delta$, discharge and re-determine zero, then charge in the opposite direction, and observe $\theta-\delta$ and take the difference. Or we may disregard the zero and merely, having the field in a given direction, observe position of A, then reverse the field and note the deflection 2δ which results. Both these methods have been used.

It will be seen that there is no theoretical necessity for making the fields or the conductors symmetrical or the potentials V_1, V_2 numerically equal to each other. In practice, however, it is desirable to make the apparatus and fields as symmetrical as possible, though it is quite possible to obtain results with wide differences between the potentials V_1, V_2. With symmetrical apparatus the deflection θ, due to electrostatic forces, will be small and any deflection due to a gravitation effect will be more readily observed.

The form of apparatus indicated in fig. 2 is therefore adopted. The beam AA with its curved end pieces is supported by a steel knife edge and grooved plate by means of which it is electrically connected to earth. The pierced guard cylinder D, and metallic case E, are also earthed. The field plates B, C are insulated and connected through a double reversing key to the sources of potential, which are separate for the two plates. Some of the

lines of force from these plates fall on the guard cylinder and on the case. The apparatus can be adjusted so that the forces acting on AA are very nearly horizontal, so that they produce only a very small deflection from the uncharged position. This deflection is usually made small for practical convenience, but the resulting value of δ is the same for large as for small deflections. The stability of the arrangement and method of determining sensitiveness while under the electric field will be referred to later. It may be mentioned here that variations of weight corresponding to $\frac{1}{5000}$ or $\frac{1}{10000}$ mgm. are easily detected in practice.

This method of arranging the apparatus and of applying and reversing the fields eliminates several errors which might be expected to cause difficulty in an experiment of this nature. As shewn above, the effect of electrostatic forces (which do not depend

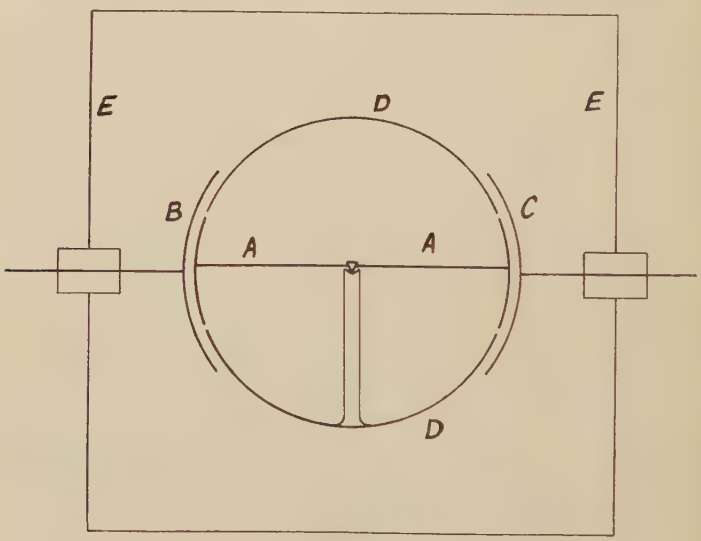


Fig. 2.

on the direction of the field) is eliminated on reversal. The same also applies to any heating effect due to currents which pass while charging and reversing, or to leakage currents, and also to electromagnetic effects, none of which depend on the direction of the currents or fields. A point which has often been mentioned to the writer is the difficulty of eliminating stray lines of force. These are really unsymmetrical portions of the field which go to make up the deflection θ as described above and are included in the preceding discussion. Experiment shews that great departures

from symmetry do not vitiate the result. Thus the field plates may be moved, or we may use either of the plates alone, the other being earthed, or we may adjust the beam in such a way that θ is large or small and in either direction from the zero. In all these cases the result is unaffected*. In most of the experiments the balance was screened from any possible external electrostatic effects, this however made no difference to the result—stray lines coming on to the beam from the external electrical apparatus gave rise to no discrepancy, these will be dealt with more fully when the experiments themselves are described. The effects do not depend on strict accuracy being attained in the magnitude of the potentials employed. A variation of eight volts out of a thousand purposely introduced produced no effect on the results. This shews that the results are not due to some variation of potentials, such as might be caused by contact difference.

Reference will be made later to some effects observed with electrometers which seem to have a bearing on the subject, but it should be borne in mind that the present instrument is not an electrometer, indeed it is the exact opposite, that is to say it is designed so that electric charges produce the least possible deflection instead of the greatest as with the electrometer.

(2) *Description of apparatus.*

The apparatus was for the most part† constructed by the writer (who was aided by a Royal Society grant) at the Technical College, Huddersfield, and afterwards set up in the Cavendish Laboratory, Cambridge. The general plan of the arrangements is shewn in fig. 3.

In the figure, A represents a table standing on rubber blocks and supporting the main instrument or balance B which is enclosed in a wooden box. C is a table on which are placed an observing telescope D, a scale E, with its sliding support and lamp F, a double pendulum electroscope G, a multi-cellular voltmeter H, keys J, K, L, M, Daniell cell N, and resistance box P, the uses of which will be described later. Another table R carries a cabinet of 1000 small secondary cells in two separate trays of 500 each, which are connected to the key L by the leads T. The diagram is approximately to scale, the distance from telescope to balance being about two and a half metres. The dotted lines represent connecting wires which are insulated on paraffin wax columns.

* Irregular fluctuations occur in the *magnitude* of the result, but they are extremely small for an experiment of this kind.

† Knife edges by Oertling.

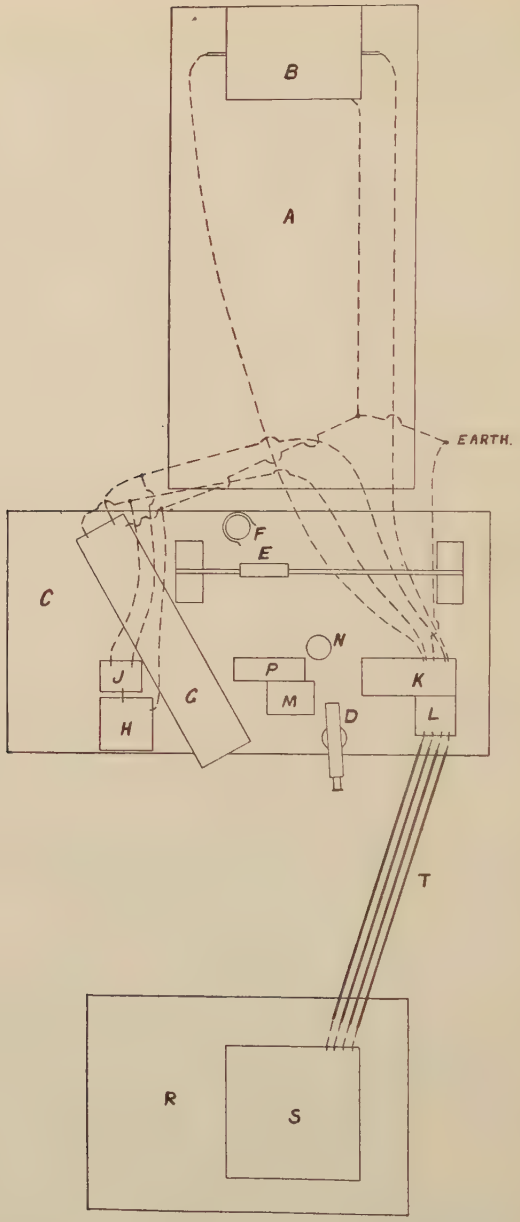


Fig. 3.

The balance itself is shewn in fig. 4. It is rigidly made so that the applied electric forces shall not produce distortion. The case, guard cylinder, field plates and beam are constructed of magnalium. Referring to the figure, *AA* represents the beam with its arresting arrangements *aaa*, *B* one of the end pieces, *CC* the guard cylinder and *DE* the insulated field plates. The beam is electrically connected to the guard cylinder by means of its steel knife edge which bears on a grooved steel plate attached to

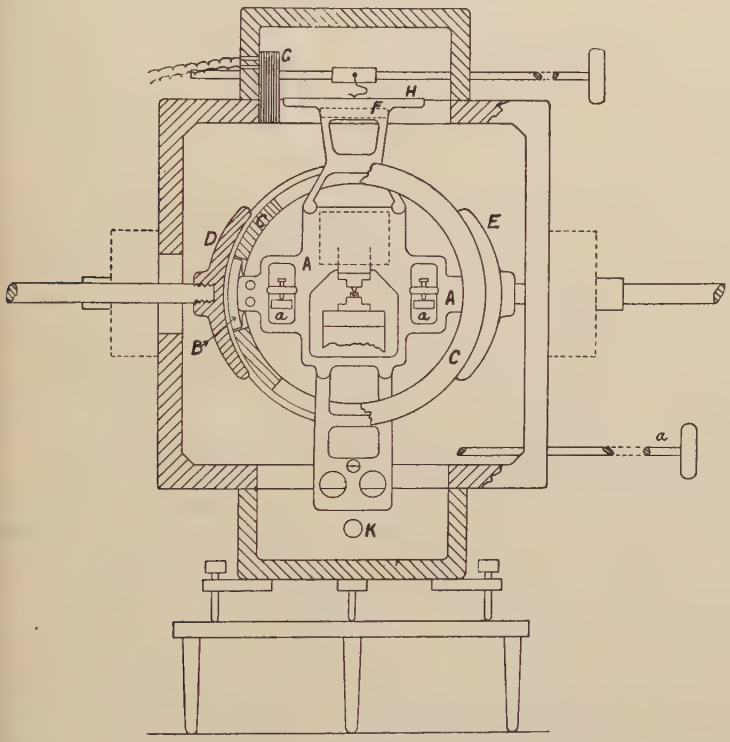


Fig. 4.

a pillar screwed to the guard cylinder. The object of the groove in the bearing plate is to prevent side slipping of the knife edge under the action of the electric forces. The guard cylinder itself is rigidly fixed to the back of the case, and the whole earthed by means of the earth wire shewn in fig. 3. (It should be noted that all the earth connections are made with one wire as shewn in that figure.)

It will be seen from the figure that a small amount of play is allowed between the piece *B* and the guard cylinder. This allows a

few lines of force from D to pass between the two, and some of these will terminate on the edges of B. Those falling on the upper and lower edges in the figure give rise to forces tending to deflect the beam. If everything is symmetrical these forces will be equal above and below, and therefore the beam will not actually be moved. But if B is slightly higher than its central position, the lower gap will be wider than the upper one and more lines of force will therefore fall on the lower edge of B than on the upper edge. These will tend to bring B back again to its central position. The beam is always in stable equilibrium under the action of the field. The application of the field, however, affects to a slight degree the sensitiveness of the apparatus as a balance, and it is therefore necessary to have some means of determining the sensitiveness while the field is on. This is provided by the arrangement shewn in the upper part of the figure. It is an adaptation of a method employed in a balance designed by Dr Hicks and used by the writer in the research previously alluded to. A small and feeble magnet is attached to the beam as shewn by the dotted lines at F. This can be attracted or repelled by a small current passing round the coil G. The deflection produced by a given current can be compared once for all with that produced by moving a rider a certain distance along the bar H. By observing the deflection produced by this current during an actual experiment with the plates charged, the sensitiveness can be determined, and the value of any deflection produced by the charges expressed in mgms. of weight. The current is not of course allowed to pass during actual observations. The current required to produce a measurable deflection of the beam, though small, is vastly in excess of any possible leakage current between the field plates in the balance, and therefore it is impossible that these should act directly on the magnet and give rise to any difficulty by causing deflections. The cell, resistance and key, N, P and M in fig. 3, are used for supplying the current. The rider is not moved during a series of observations. It is used before observations are commenced in order to level the beam. By suitably levelling the beam the electrostatic deflection θ may be varied at will, both in magnitude and direction. Many values of this deflection have been used in the experiments.

The deflections are observed by telescope and scale, and a double suspension mirror, the position of which is indicated by the circle K. The field plates D, E are insulated by sulphur plugs and have various adjustments not shewn on the figure. Other details, such as knife edge adjustments, etc., are also left out of the figure to avoid complication. The front and back of the case are of thick magnalium with plate glass windows. The latter have been covered inside with tinfoil in the later experiments,

with the exception of a small space for observing the mirror K which has been closed in with a grid of fine wire. The results, however, are exactly the same whether the windows are covered or left bare. The whole apparatus is enclosed in a box of wood 2 inches thick and provided with double windows of plate glass.

The diagram is approximately to scale. The diameter of guard cylinder is 14·3 cms., distance between this and field plates 3 mms., the end pieces B, which are square with rounded corners, 4·9 cms. square. Weight of end pieces about 28 grms. each, total weight of beam about 260 grms.

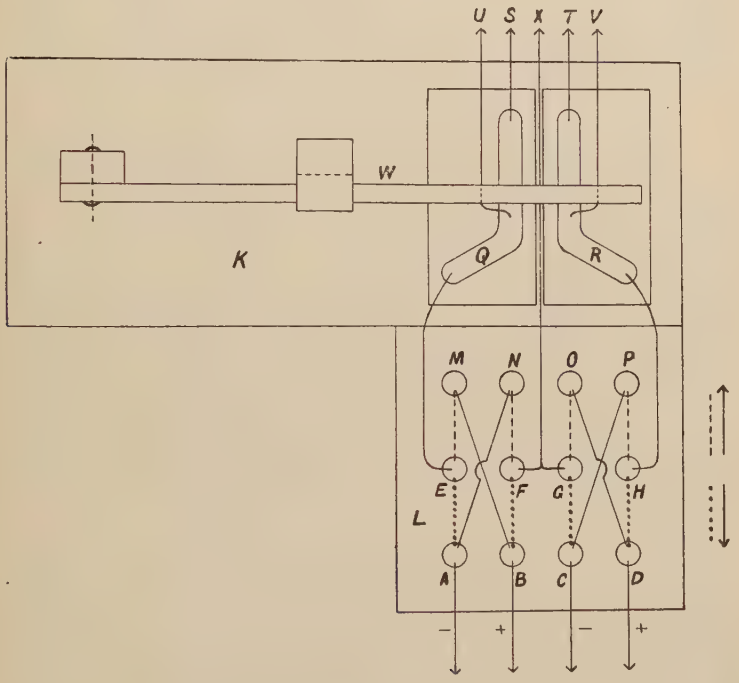


Fig. 5.

Fig. 5 is a diagram of the keys K and L of fig. 3. L is a paraffin block with mercury cups and connections as shewn. The cups A, B are connected to the negative and positive terminals of the top tray of cells and C, D to the negative and positive terminals of the bottom tray. E and H are connected to two other cups Q, R, which are permanently connected to the electroscopes and to the key J in fig. 3 by wires S, T. The wires U, V to the balance can be dipped into the cups Q, R by means of the lever W to

which they are attached. The cups F, G are permanently earthed by the wire X. By means of a rocker not shewn in L connections can be made as shewn by the dotted lines, two kinds of dots being used to shew the connections corresponding to the two positions of the rocker. The mercury in the cups B, C, N, O is higher than that in the other cups in order that earth contacts shall always be made before and broken after high potential contacts. It will be seen that the key simply consists of two ordinary six cup reversing keys fastened together. The lever W is used to throw the balance connections in or out as desired. It will easily be seen that the arrangement here described satisfies the condition that the two fields F_1, F_2 of fig. 1 shall be quite distinct from each other. By means of the key J in fig. 3 the multi-cellular voltmeter H can be connected to either of the wires S, T in fig. 5 at pleasure.

(3) *Detailed description of experiments and results.*

The several experiments will now be described in order and their results afterwards discussed.

1. *June 9th*, 1909. (Used 1000 volts on each side of balance in all cases except when otherwise stated.) There was very large electrostatic deflection θ, no attempt being made to reduce this. The zero of balance was altering rapidly. In the following tables the observations are numbered, and the arrows shew the position of the rocker of reversing key (refer to fig. 5). The column marked 2δ is calculated from three successive readings, it is the difference between the mean of the first and last of the three and the intermediate reading.

A. *Front cover of balance off.*

Scale Readings ↓		Scale Readings ↑		Readings used for calculation of 2δ	2δ
1	48·2	2	47·4	1, 2, 3	1·25
3	49·1	4	48·9	2, 3, 4	0·95
5	51·1	6	50·8	3, 4, 5	1·20
				4, 5, 6	1·25
					Mean 1·16*

* See Table on p. 369 for values of δ corrected for sensitiveness.

B. *Front cover of balance off.*

Scale Readings ↓	Scale Readings ↑	Readings used for calculation of 2δ	2δ
	1 55·5	1, 2, 3	·85
2 57 0	3 56·8	2, 3, 4	1·10
4 58·8	5 58·0	3, 4, 5	1·40
6 59·6		4, 5, 6	1·20
			Mean 1·14

C. *Cover on.* (The larger windows are coated with tinfoil.)

Scale Readings ↓	Scale Readings ↑	Readings used for calculation of 2δ	2δ
1 80·2	2 80·0	1, 2, 3	1·35
3 82·5	4 82·2	2, 3, 4	1·40
5 84·2	6 83·9	3, 4, 5	1·15
7 86·0	8 85·2	4, 5, 6	1·15
		5, 6, 7	1·20
		6, 7, 8	1·45
			Mean 1·28

2. *June 10th.*

A. *Electrostatic deflection large, but not so large as in last experiment.*

Scale Readings ↓	Scale Readings ↑	Readings used for calculation of 2δ	2δ
1 67·0	2 66·0	1, 2, 3	0·75
3 66·5	4 65·6	2, 3, 4	0·70
5 66·2	6 65·3	3, 4, 5	0·75
		4, 5, 6	0·75
			Mean 0·74

B. *Taken at* 1 *minute intervals. Reversed at* $\frac{1}{2}$ *minutes.*

Scale Readings ↓	Scale Readings ↑	Readings used for calculation of 2δ	2δ
1 65·9	2 64·9	1, 2, 3	0·80
3 65·5	4 64·9	2, 3, 4	0·60
5 65·6	6 64·8	3, 4, 5	0·65
7 65·7	8 64·9	4, 5, 6	0·75
		5, 6, 7	0·85
		6, 7, 8	0·85
			Mean 0·75

C. *Smaller electrostatic deflection, zero for no field* 120·0 *about.*

Scale Readings ↓	Scale Readings ↑	Readings used for calculation of 2δ	2δ
1 17·4	2 14·9	1, 2, 3	1·35
3 15·1	4 13·3	2, 3, 4	1·00
5 13·9	6 12·2	3, 4, 5	1·20
		4, 5, 6	1·15
			Mean 1·17

D. Cut out and earthed balance, and moved over reversing key several times. No change of zero.

3. *June* 12*th.*

A. *Taken at* 1 *minute intervals. Zero about* 63·0.

Scale Readings ↓	Scale Readings ↑	Readings used for calculation of 2δ	2δ
	1 26·9	1, 2, 3	1·05
2 27·0	3 25·0	2, 3, 4	1·25
4 25·5	5 24·1	3, 4, 5	0·95
			Mean 1·08

B. Put 4 volts in and out of one battery of cells. This made no appreciable difference to the deflections.

C. *Zero about* 52·5.

Scale Readings ↓	Scale Readings ↑	Readings used for calculation of 2δ	2δ
1 22·8	2 21·2	1, 2, 3	1·25
3 22·1	4 20·8	2, 3, 4	1·10
5 21·7		3, 4, 5	1·10
			Mean 1·15

D. *Zero taken between reversals. Observations at* 1 *minute intervals.*

Scale Readings ↓	Zero	Scale Readings ↑	Zero	Readings used for calculation of 2δ	2δ
1 21·4	2 52·4	3 20·3	4 52·2	1, 3, 5	1·15
5 21·5	6 52·3	7 20·2		3, 5, 7	1·25
					Mean 1·20

E. Put 4 volts in and out of one battery. The only effect was to alter the electrostatic deflection θ in the ratio 4 : 1000. This alteration was about 0·1 scale divisions.

4. *June* 14*th.*

A. *Zero about* 60·5. *Electrostatic deflection in opposite direction to last.*

↓ gave 72·3 ↑ gave 71·3 (about).

B. Put 8 volts in and out of one battery, no perceptible difference in deflections.

C. *Very small electrostatic deflection and zero steady.*

Zero	Scale Readings	Zero	Scale Readings	Readings used for calculation of 2δ	2δ
	↑		↓		
1 99·0	2 100·1	3 98·8	4 101·0	2, 4, 6	0·95
5 98·8	6 100·0	7 98·7	8 101·0	4, 6, 8	1·00
9 98·7	10 100·0	11 98·5	12 100·9	6, 8, 10	1·00
13 98·5	14 99·8	15 98·3	16 100·7	8, 10, 12	0·95
				10, 12, 14	1·00
				12, 14, 16	1·00
					Mean 0·98

D. *Reversed without taking zero.*

Scale Readings	Scale Readings	Readings used for calculation of 2δ	2δ
↓	↑		
1 100·7	2 99·8	1, 2, 3	0·90
3 100·7	4 99·7	2, 3, 4	0·95
			Mean 0·92

5. *June 17th.* Covered all the remaining windows with tinfoil, except for a small space for observation of mirror, in this and following experiments.

A.

Scale Readings	Scale Readings	Readings used for calculation of 2δ	2δ
↑	↓		
1 127·3	2 128·3	1, 2, 3	0·65
3 128·0	4 128·9	2, 3, 4	0·60
5 128·7	6 129·5	3, 4, 5	0·55
		4, 5, 6	0·50
			Mean 0·57

B. *Zero about* 71·0. *Reversed as rapidly as possible.*

Scale Readings		Scale Readings		Readings used for calculation of 2δ	2δ
	↓		↑		
1	125·0	2	124·9	1, 2, 3	0·55
3	125·9	4	125·8	2, 3, 4	0·55
5	126·6	6	126·3	3, 4, 5	0·45
7	127·3	8	127·2	4, 5, 6	0·55
				5, 6, 7	0·65
				6, 7, 8	0·55
					Mean 0·55

6. *June* 18*th.* Reversed connections of balance on the lever *M*. Kept all other connections as before. Effect should be in opposite direction to previous effects, which is the case.

A.

Scale Readings		Scale Readings		Readings used for calculation of 2δ	2δ
	↑		↓		
1	82·5	2	81·9	1, 2, 3	0·60
3	82·5	4	81·7	2, 3, 4	0·70
5	82·1			3, 4, 5	0·60
					Mean 0·63

7. *June* 19*th.* Balance connection reversed as yesterday but only one side of balance used, the other being earthed. Should give half previous effects.

A. *Left side of balance used. Observations at* 1 *minute intervals.*

Scale Readings		Scale Readings		Readings used for calculation of 2δ	2δ
	↑		↓		
1	107·0	2	106·3	1, 2, 3	0·30
3	106·2	4	105·7	2, 3, 4	0·20
5	105·6	6	105·0	3, 4, 5	0·20
				4, 5, 6	0·25
					Mean 0·24

B. *Right side of balance used.*

Scale Readings ↑		Scale Readings ↓		Readings used for calculation of 2δ	2δ
1	111·3	2	110·8	1, 2, 3	0·45
3	111·2	4	110·5	2, 3, 4	0·55
5	111·0	6	110·2	3, 4, 5	0·60
				4, 5, 6	0·65
					Mean 0·56

8. *July 10th.* Balance connections in normal position.

A. 1000 *volts each side as usual. Observations at* 1 *minute intervals.*

Scale Readings ↑		Scale Readings ↓		Readings used for calculation of 2δ	2δ
1	103·2	2	104·1	1, 2, 3	1·40
3	102·2	4	102·9	2, 3, 4	1·30
5	101·2	6	102·0	3, 4, 5	1·20
				4, 5, 6	1·25
					Mean 1·29

B. 660 *volts each side.*

Scale Readings ↑		Scale Readings ↓		Readings used for calculation of 2δ	2δ
1	103·3	2	103·8	1, 2, 3	1·10
3	102·1	4	102·7	2, 3, 4	1·15
5	101·1	6	101·7	3, 4, 5	1·10
				4, 5, 6	1·10
					Mean 1·11

C. 330 *volts each side.*

Scale Readings		Scale Readings		Readings used for calculation of 2δ	2δ
	↑		↓		
1	103·3	2	103·3	1, 2, 3	0·65
3	102·0	4	102·0	2, 3, 4	0·65
5	100·7	6	100·7	3, 4, 5	0·65
				4, 5, 6	0·65
					Mean 0·65

9. *July 16th.* 800 volts. Right plate of beam covered (except near edges) with shellac varnish. That side of balance only used, the left side being earthed.

Scale Readings		Scale Readings		Readings used for calculation of 2δ	2δ
	↓		↑		
1	14·5	2	14·0	1, 2, 3	0·40
3	14·3	4	13·8	2, 3, 4	0·40
5	14·0	6	13·3	3, 4, 5	0·35
				4, 5, 6	0·45
					Mean 0·40
				Corrected to 1000 volts	0·50

10. *July 19th.* Used left side only. A wire was passed through case and bent inside into proximity with the field plate and end piece of beam, but without touching them. This could be connected to one terminal of a Daniell cell, the other terminal being connected to balance case.

A. *Wire not connected to cell.*

Scale Readings ↓		Scale Readings ↑		Readings used for calculation of 2δ	2δ
1	7·2	2	6·8	1, 2, 3	0·40
3	7·2	4	7·0	2, 3, 4	0·30
5	7·3	6	7·0	3, 4, 5	0·25
7	7·2	8	6·9	4, 5, 6	0·30
9	7·3			5, 6, 7	0·25
				6, 7, 8	0·25
				7, 8, 9	0·35
					Mean 0·30

B. *Wire connected.*

Scale Readings ↓		Scale Readings ↑		Readings used for calculation of 2δ	2δ
1	7·2	2	6·9	1, 2, 3	0·30
3	7·2	4	6·8	2, 3, 4	0·35
5	7·2	6	6·8	3, 4, 5	0·40
				4, 5, 6	0·40
					Mean 0·36

Summary of Results, corrected for Sensitiveness.

Number of Experiment	Deflection 2δ observed (mean)	Sensitiveness 1·0 div. $= x$ mgms.	Electrostatic deflection θ	Reduced value of δ
	scale divs.	x		mgms.
1. A.	1·16	·00125		·0007
B.	1·14	,,	very large	·0007
C.	1·28	,,		·0008
2. A.	0·74	·00167		·0006
B.	0·75	,,	large	·0006
C.	1·17	·00125	− 105	·0007
3. A.	1·08	·00125	− 38	·0007
C.	1·15	,,	− 30	·0007
D.	1·20	,,	− 30	·00075
4. C.	0·98	·00167	+ 1	·0008
D.	0·92	,,	about	·0008
5. A.	0·57	·00200	+ 54	·0006
B.	0·55	,,	+ 54	·00055
6. A.	0·63	·00250	+ 42	·0008
7. A.	0·24	·00222		·0003*
B.	0·56	·00182	+ 22	·0005
8. A.	1·29	·00200	− 20	·0013
B.	1·11	·00154		·00085
C.	0·65	·00125		·0004
9. A.	0·50	·00200	very large	·0005
10. A.	0·30	·00333		·0005
B.	0·36	,,		·0006

* The values of δ in experiments 7—10 are not directly comparable with the previous values, as various charges were used, as described above, in those experiments.

It will be seen from the above results that a positive effect is constantly exhibited, constant in direction and as uniform in magnitude as can be expected when regard is had to the nature of the experiment, and the fact that the effect only represents a difference of a few ten thousandths of a milligramme. Indeed it is surprising that the fluctuations are not much larger. Some preliminary experiments, in which the sources of potential were charged Leyden jars, gave similar results, though they were more irregular on account of the leakage of the jars. They shewed, however, that changes in the adjustment of the apparatus which were constantly made, had no influence on the average values of the results.

From Experiment 8 *A*, *B*, *C* it will be seen that the effect—so far as measurements of the actual magnitude can be relied on—is proportional to the induced charges or to the potentials of the field plates.

A few possible sources of error may be briefly alluded to:

1. *External electrostatic action* due to lines of force entering the balance case from outside. The fact that the front of the balance case may be on or off (Experiment 1) or the windows coated with tinfoil, or left bare without sensibly altering the result, seems to shew that this source of error is inoperative. It is hardly possible that it could be otherwise, for electrostatic effects depending on the external conductors from key to balance would be eliminated on reversal, and any permanent field which might exist which could send stray lines on to the beam, would merely give a permanent deflection which would not alter the value of 2δ on reversal of the fields F_1, F_2, *unless* these stray lines penetrated to the surfaces of the end pieces where the induced charges resided. An examination of the drawing will shew how unlikely this is to be the case. Moreover the shape of the surfaces is such that very few of the lines so reaching them would tend to deflect the beam. If the result had been due to direct action of the cells, voltmeters, or other parts of the electrical apparatus, other than the conductors mentioned above, it would have changed sign in Experiment 6 when the balance connections were reversed, but this was not the case.

2. *Mechanical operation of turning the reversing key.* In Experiment 6 the result would have changed sign had it been caused by this operation. Moving the key produced no disturbance of the balance.

3. *Accidental variations in the potentials applied.* Actual variations of four and eight volts made no perceptible difference to the results. See Experiments 3 *B*, *E*, 4 *B*. These variations were much greater than any which could have occurred during the majority of the experiments.

4. *Small permanent fields inside the balance case.* The remarks under 1. above apply to this case also. In order to test directly Experiment 10 was tried, in which a small field was purposely introduced. The small effect recorded is within the limits of experimental error, as an examination of the actual observations will shew. As a matter of fact, no movement whatever of the scale could be detected when the field was put in or out.

5. *Surface effects of the pieces carrying the induced charges.* In Experiment 9 one of the end pieces was coated (except near the edges) with shellac, and this end of the apparatus only was charged. It was feared that the shellac might give rise to electrostatic difficulties, but none were experienced, thanks no doubt to the approximate symmetry of the apparatus. It is probable that the field plate was a little further from the beam than usual in this experiment. The result was the same as in Experiment 7 *B*, when the same side of the apparatus was used. The value in Experiment 9 is probably a little low (perhaps 1 in 5 or 6) due to the suspected increase in the distance between the plates, but the similarity of the results shews that these are not caused by any peculiar action of the metal surfaces.

Some other sources of error, such as heating and electromagnetic effects, have already been dealt with.

(4) *Conclusion.*

Whether the effect be due to a modification of gravity or not, the experiments appear to shew that a body, positively charged with 20 electrostatic units of electricity, behaves as though it were heavier than the same body negatively charged, by an amount of the order of ·0007 mgms. If the effect be due to a change in gravity, there will exist in an electrostatic field a small amount of energy due to this cause, and this will involve a very slight alteration in the configuration of the lines of force from that given by ordinary electrostatic laws. This would be so small as to pass unnoticed in ordinary cases. It would be such that with given conductors charged in a given way and then reversed as to sign of charge, we should obtain two slightly different configurations of field. Delicate instruments might detect this, and it is possible that the different zeros of a quadrant electrometer, with the needle positively and negatively charged, and quadrants earthed, may be due to this cause, though the writer has not investigated these effects, for the explanation of which the above suggestion may be quite inadequate.

It was suggested to the writer that the effects might possibly be caused by an accumulation of air condensed on the surfaces of the metal, the quantity being greater when the surface was positively than when negatively charged. Experiment 9 was

really made in view of this suggestion, but from the point of view of energy it would appear difficult to explain the result in any such way as this. For we may liken the metal surface to a series of buckets which are filled with condensed air, when the surface is charged positively, but empty when it is charged negatively. Now if we have a number of such surfaces, fixed to spokes, radiating from a central axis so as to form a kind of wheel (like a water-wheel) and place this between charged field plates, we should have all the buckets on one side full and those on the other side empty, and the wheel would revolve. It is difficult to see where the energy would be drawn from to keep up this rotation.

It may be of interest, as a mere speculation, to see whether the theory of gravitation which supposes that the attraction between unlike charges is somewhat greater than the repulsion between like charges, could be modified to suit the present results. Let us suppose that the repulsion between two positive units at a given distance apart is P, and that between two negative units N, and that the attraction between a positive unit and a negative unit is M. Then the attraction between a neutral body (containing a unit of positive and a unit of negative electricity) and a positive unit will be $M-P$, and that between the neutral body and a negative unit $M-N$. The difference between these will be $N-P$, which is the effect found in the experiments. Again the attraction between two neutral bodies (each containing one positive and one negative unit) will be $2M-(P+N)$ and this will be ordinary gravity. The last expression is much smaller than $N-P$ and thus we should have the result that while N and P both differ considerably from M, their mean is very nearly equal to M. Since in the attraction M one charge of each kind is employed, it is perhaps not unnatural to suppose that this might be the case. The actual differences between M, N and P would of course be exceedingly small in comparison with their absolute magnitudes. Assuming M to be the ordinary electrostatic attraction, we have two equations for finding N and P. It should be remembered that in the experiment, as in this theory, no lines of force pass between the attracting masses. In the experiment the lines do not pass to the earth, but sideways to the field plates. Turning aside from such hasty speculations, the writer would be most grateful for suggestions as to possible sources of error in the experiments, as he hopes to have an opportunity of repeating them later with improved apparatus, the design of which might be modified in view of any suggestions thus made.

In conclusion the writer desires to express his thanks to Sir J. J. Thomson and to Professor W. M. Hicks and Dr S. R. Milner for valuable criticisms and for the kindly interest which they have taken in the experiments.

Note on an Attempt to Detect a Difference in the Magnetic Properties of the Two Kinds of Ions of Oxygen. By Miss D. B. Pearson. (Communicated by Prof. Sir J. J. Thomson.)

[*Received* 11 November 1909.]

During the past year attempts have been made to discover whether a magnetic field effects any separation of ionised oxygen into its positive and negative constituents. Were such a separation detected it would point to a difference in the magnetic properties of the two kinds of ions.

The method of experiment was briefly as follows. Oxygen, ionised by passage through a long tube lined with uranium oxide, was passed between the poles of a strong electro-magnet and then into a Faraday Cylinder connected to a Dolezalek Electrometer. At first the gas was driven steadily through the apparatus with the electro-magnet not active. The charge going to produce the electrometer deflection was then merely that due to the excess of one ion over the other, produced by the difference in the rates of diffusion of the two kinds of ions. Readings of the rate of deflection of the electrometer needle in these circumstances were taken. Then the magnetic field was put on across the tube, through which the oxygen was passing, and observations of the movement of the needle once more made. If one ion were more magnetic than the other it would have had a greater tendency than the other to get into the region between the pole-pieces of the electro-magnet, where the field was strongest. Consequently the gas beyond this region would have been poorer in the more magnetic ion, and this dearth of one of the charged constituents of the gas reaching the Faraday Cylinder would have affected the rate of deflection of the electrometer needle.

As a matter of fact, numerous observations gave no certain indication of any effect produced by the magnetic field. All the readings were small and variable, both with and without the electro-magnet active. The means of the two series of readings were, however, so closely concordant that the results may be fairly taken to shew that the difference between the magnetic properties of the two kinds of ions of oxygen, if it exists, is beyond the limits of accuracy of the method used.

It would be out of place here to discuss in full quantitative results, but it will perhaps be well to refer to them shortly. The electrometer deflections were of the order of 3·5 scale divisions

per minute whether the magnetic field was off or on. Such a deflection was found to be produced if, in the conditions of the experiment, 5·5 °/₀ more of one ion than the other reached the Faraday Cylinder. Although an exact limit cannot be set, it is certain that a readjustment of ions sufficient to change the rate of deflection of the needle by ·5 scale divisions could not have escaped detection. The magnetic field used cannot therefore have prevented more than ·8 $\left[=\frac{\cdot 5}{3\cdot 5}\text{ of } 5\cdot 5\right]$ °/₀ ions of either sign from escaping beyond it to the cylinder. The magnetic field employed was found to be about 6000 gauss at its maximum, and its greatest rate of variation was 5000 gauss per centimetre.

On the theory of the motion of charged Ions through a Gas. By Sir J. J. Thomson, Cavendish Professor of Experimental Physics.

[*Read* 8 November 1909.]

In the usual method of calculating the velocity of a charged ion through a gas, the expression $\frac{Xe}{m}\frac{\lambda}{u}$ for the velocity is obtained on the assumption that the ion after each encounter with a molecule of the gas starts afresh and is as likely to move in any one direction as the opposite. The momentum communicated to the ion by the electric field in the interval before a collision is assumed to be transferred to the molecule of the gas during that collision, from which it follows that the maximum velocity communicated to an ion by the electric field is that imparted to it in the short interval between two collisions. It seems clear however that the time during which the velocity is acquired by the ion is the average time during which the ion continues to move in one direction and not the time between two collisions, and the time taken for an ion to have its motion reversed by its collision with the other molecules may be much greater than the time between two collisions. This will certainly be the case if the mass of the ion is considerably greater than that of the molecules against which it strikes. For let us suppose that the average kinetic energy of the ion is equal to that of the molecules. Let m be the mass of the ion, u its velocity, M and v the corresponding quantities for the molecules, then since

$$mu^2 = Mv^2,$$

$$\frac{mu}{Mv} = \frac{v}{u} = \sqrt{\frac{m}{M}},$$

thus the momentum of the ion will be $\sqrt{\frac{m}{M}}$ times the momentum of the molecule against which it strikes and so the ion will have to collide with at least $\frac{1}{2}\sqrt{\frac{m}{M}}$ molecules before its motion is reversed. From this it would appear that the quantity λ which appears in the expression for the mobility of the ion should be, not the ordinary free path, which depends only on the size of the ions and molecules and not upon their masses, but that free path multiplied by some fraction of $\sqrt{\frac{m}{M}}$; if we call λ_0 the ordinary free path, the expression for the mobility would be

$$p\,\frac{e}{m}\frac{\lambda_0}{u}\sqrt{\frac{m}{M}},$$

where p is a numerical coefficient which we have not determined.

If the absolute temperature is θ

$$\tfrac{1}{2}mu^2 = \alpha\theta,$$

where α does not depend upon the nature of the ion or of the gas. Thus the mobility will be

$$p\,\frac{e\lambda_0}{\sqrt{2M\alpha\theta}},$$

i.e. it will not depend upon the mass of the ion except in so far as mass is an indication of size, and since λ_0 diminishes on this view as the size of the ion increases, λ_0 would depend to some extent on the value of m.

It must be confessed however that in this as in many other problems the methods founded on the mean free path leave much to be desired, and are far less satisfactory than the method introduced by Maxwell when he replaced the idea of collisions between hard elastic spheres by that of the effects produced by forces exerted by one molecule on another. Maxwell gave a complete solution when these forces are repulsions varying inversely as the fifth power of the distance. It happens that on the simplest view we can take of the forces between a charged ion and a neutral molecule, i.e. that these forces are due to the attraction between the electric charge on the ion and the distribution of electricity induced by this charge on the molecules regarded as conducting spheres, these forces will vary inversely as the fifth power of the distance unless the ion gets close to the molecule. For the attraction between an electric charge e and an unelectrified conducting sphere of radius a is equal to

$$\frac{e^2a^3}{f^3}\,\frac{(2f^2-a^2)}{(f^2-a^2)^2}$$

(see Thomson's *Electricity and Magnetism*, 4th edition, p. 154), where f is the distance between the charge and the centre of the molecule; when f is a considerable multiple of a, this expression is approximately

$$\frac{2e^2a^3}{f^5},$$

and thus varies inversely as the fifth power of the distance between the ion and the molecule. In this case the force is an attraction while Maxwell considers the case of a repulsion. Maxwell's investigation can be applied to the case when the force is attractive with hardly any modification, the only change that is required is in the numerical constants which Maxwell denotes by A_1, A_2; these have not the same values for repulsion as for attractive forces, since they depend on the magnitude of the apsidal distance; with this exception Maxwell's results can be applied

without modification to the case when the forces are attractive. Maxwell gives an expression for the coefficient of diffusion of one gas A into another B of the form

$$D_{12} = \frac{1}{2h}\sqrt{\frac{m_1+m_2}{m_1 m_2 K}}\,\frac{1}{A_1(\nu_1+\nu_2)},$$

where m_1, m_2 are the masses of the molecules of A and B, ν_1, ν_2 the number of these molecules in unit volume, K the force at unit distance and $h = N/2p$, where p is the pressure exerted by a gas in which there are N molecules per cubic centimetre, $h = \frac{k}{\theta}$, where k is a constant and θ the absolute temperature of the gas. A_1 is a numerical constant, having when the forces are repulsive the value 2·659. Suppose now m_1 is the mass of a charged ion the preceding equation will give us the coefficient of diffusion of the ion through the gas, if we change the value of A_1 to allow for the force being attractive, and put $K = 2e^2a^3$, where a is the radius of a molecule of the gas B. We can eliminate a by means of the relation

$$\mu_2 - 1 = N\frac{\pi}{4}a^3,$$

where μ_2 is the index of refraction of the gas B when there are N of its molecules per cubic centimetre. In the case of an ion diffusing through a gas, ν_1 may be neglected in comparison with ν_2 so that

$$D_{12} = \frac{1}{2h}\sqrt{\frac{m_1+m_2}{m_1}}\sqrt{\frac{8\pi N}{e^2(\mu_2-1)}}\,\frac{1}{A_1\nu_2}.$$

The mass of the charged particle only enters this expression through the term $\sqrt{\frac{m_1+m_2}{m_1}}$, thus when the mass of the charged particle is small compared with the mass of a molecule of a gas through which it is diffusing, the coefficient of diffusion varies inversely as the square root of the mass of the ion; if however the mass of the ion is large compared with that of the molecule the coefficient of diffusion varies exceedingly slowly with the mass of the charged ion. Hence it seems to me that we can attach but little value to the determinations of the atomic weight of the emanations made by measuring their rate of diffusion through air or hydrogen, for if these were positively charged heavy particles the rate would be practically the same whether the atomic weight of the emanation were 200 or 2000, though this objection would not apply to methods based on the diffusion through porous plugs.

The mobility of an ion, i.e. the speed with which it moves through the gas under unit electric force, is connected with the rate of diffusion of the ion by the equation

$$k = D_{12}\frac{Ne}{\pi},$$

where π is the pressure due to N molecules per cubic centimetre. Hence

$$k = \sqrt{\frac{m_1+m_2}{m_1 m_2}}\sqrt{\frac{8\pi N}{\mu_2 - 1}}\frac{1}{A_1 \nu_2}.$$

We see from this that the mass of a heavy ion has not much effect on its mobility; the mobility of an ion consisting of a charged molecule of the gas through which it is moving is $\sqrt{2}$ times the mobility of one whose mass is very much greater than that of a molecule of the gas; the mobility does not however depend on the charge on the ion. The velocity, through hydrogen, of an ion made of a molecule of hydrogen charged with electricity, would be about half as much again as that of a charged molecule of methyl iodide through hydrogen and so would easily be distinguished from it.

Mobility of an ion through a mixture of gases.

If we have a small number of charged molecules A diffusing through a mixture of gases B and C we can readily prove by Maxwell's method that D, the coefficient of diffusion of A through the mixed gases, is given by the equation

$$D = \frac{1}{2hA_1}\frac{1}{\nu_2\sqrt{\frac{m_1 m_2}{m_1+m_2}}\sqrt{\frac{\mu_2 - 1 \cdot e^2}{8\pi N}} + \nu_3\sqrt{\frac{m_1 m_3}{m_1+m_3}}\sqrt{\frac{\mu_3 - 1 \cdot e^2}{\pi N}}},$$

where ν_2 and ν_3 are respectively the number of molecules of B and C present per unit volume, m_2, m_3 the masses of the molecules of these gases, μ_2 and μ_3 the indices of refraction of these gases when there are N of their molecules per unit volume.

The mobility k of the ion through these gases is given by

$$k = \frac{1}{A_1}\frac{1}{\nu_2\sqrt{\frac{m_1 m_2}{m_1+m_2}}\sqrt{\frac{\mu_2 - 1}{8\pi N}} + \nu_3\sqrt{\frac{m_1 m_3}{m_1+m_3}}\sqrt{\frac{\mu_3 - 1}{\pi N}}}.$$

Let us consider the application of this equation to the experiments lately made by Mr Wellisch on the mobilities of the ions through mixed gases and calculate the difference between the mobilities through the mixture of charged molecules of B and

C. Mr Wellisch has proved to a high degree of accuracy that in these mixed gases there is only one mobility. Let us take the case of O_2 and SO_2 investigated by Mr Wellisch and calculate the velocity (1) of an oxygen ion consisting of a charged molecule of oxygen, and (2) that of an ion of SO_2 consisting of a charged molecule of SO_2; let us suppose that the partial pressures due to these gases are the same so that $\nu_1 = \nu_2$. If m_2 is the mass of an oxygen molecule, m_3 that of an SO_2 molecule, $m_3 = 2m_2$, and $\mu_2 - 1 = \cdot0003$, $\mu_3 - 1 = \cdot00066$; substituting these numbers we find that the velocity of a charged oxygen molecule through the mixture would be about 15 per cent. greater than that of a charged molecule of sulphur dioxide, a difference which could easily have been detected in Mr Wellisch's experiments. If the ions from oxygen and sulphur dioxide had been much more complex than the single molecule of these gases their mobilities through the mixed gases would have been much more nearly equal than for the simple ions, and the difference in this case might have escaped detection.

We shall consider in the light of the expression given above for the mobility the various views that have been taken of the nature of the ion. There are two main points to be considered with regard to the ion, (1) is the ion more complex in structure than a molecule, i.e. does it consist of an aggregation of molecules, and (2) can the electric charge on the ion leave the ion and find another home, thus producing a new ion, or is the charge bound by an indissoluble bond to the molecules forming the ion. It would seem clear from the great increase in the mobility of the negative ion which takes place in flames that the negative charge must be able to leave one ion, exist for a time as a corpuscle and then form a fresh ion: the question then is confined to the positive ion, and we have to consider whether or not a positive charge, carried by something much more massive than a corpuscle, can leave one positive ion, unite with a molecule of the gas, and form a new positive ion. If there were no transference of charge from the positive ion, then if the positive ion was a single molecule or even two or three molecules, there would in mixtures of gases be two sets of positive ions moving with speeds sufficiently different to have been detected in experiments like those made by Mr Wellisch. Wellisch made some other experiments where a small quantity of methyl iodide was mixed with a large quantity of hydrogen, and it was found that the velocity of the positive ions (which had originated from the methyl iodide) through the mixture was the same as that of the positive ion through pure hydrogen when the ion had originated from the hydrogen. If the positive ion in hydrogen had been a single molecule of hydrogen the velocity of the positive ion in pure hydrogen would be $\sqrt{2}$ times

the velocity in the mixture instead of being equal to it. We may therefore conclude that if there is no transference of the charge the mass of the ion must be a considerable multiple of that of the molecule.

We must therefore consider whether the hypothesis of complex positive ions is sufficient to explain the results which have been obtained as to the mobility of the positive ion. The expressions given for the mobility shew that when the mass of the ion is large compared with the mass of a molecule of the gas through which it is moving the mobility of the ion will be independent of the electric charge carried by it, practically independent of the mass of the ion, and at constant pressure will vary directly as the absolute temperature, and will be inversely proportional to the $\sqrt{m(\mu-1)}$, where m is the mass of a molecule of the gas through which the ion is moving and μ its refractive index. The result that the velocity is independent of the charge and mass of the ion and depends only on the nature of the gas through which the ions are moving is not in accordance with the results obtained by Professor H. A. Wilson for the mobility of ions in salted flames, for he found that the velocity of the positive ions when salts of the alkaline earths were placed in the flame was but little more than half the velocity of the positive ions when salts of the alkali metals were placed in the flame. Again though, as Mr Phillips' results shew, the law that the mobility at constant pressure is directly proportional to the absolute temperature is very approximately obeyed at moderate temperatures, it breaks down altogether at temperatures as high as those which occur in flames. Thus the experiments of H. A. Wilson and Moreau shew that at a temperature of about 2000° C. the velocity of the positive ions in flames impregnated with salts of the alkali metals is about 60 cm./sec. for a potential gradient of 1 volt per centimetre; the velocity of positive ions through air at 0° C. is about 1·4 cm./sec., so that the velocity at 2000° C. would if it were proportional to the absolute temperature be 12 cm./sec., which is only one-fifth of the actual value. The rapid increase in mobility with temperature is what we should expect if the positive charge could, like the negative, pass from one molecule of the gas to another.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On the relative velocities of diffusion in aqueous solution of rubidium and caesium chlorides. By G. R. Mines, B.A., Fellow of Sidney Sussex College. (From the Physiological Laboratory, Cambridge.)

[*Read* 24 January 1910.]

In the course of some physiological work with the chlorides of the alkali metals certain results led me to inquire as to the relative rates of diffusion of these salts in aqueous solution. While many data are available regarding the salts of lithium, sodium, and potassium, I have been unable to find any figures relating the two rarer members, rubidium and caesium, to the rest of the group.

The method I have used offers certain advantages in the simplicity of the manipulations involved and the small quantity of material needed for a determination. I therefore publish this short account in the hope that it may be of use to others. Graham* showed nearly fifty years ago that sodium chloride diffuses in a gelatine jelly as quickly as in distilled water. Voigtländer† found that the rate of diffusion of salts in agar-agar was unaffected by changes in the strength of the jelly from 1%—5%.

My plan has been to keep a solution of salt of known concentration in contact with a free surface of jelly, and to ascertain the rate of progress of the salt by measurement of the electrical conductivity at a fixed distance below the surface of the jelly.

* Thomas Graham, *Phil. Trans.* 1861, p. 183.

† Voigtländer, *Zeitschr. Physik. Chem.* III. 1889, p. 316.

The apparatus employed for this purpose is depicted in fig. 1. It consists of a resistance cell of special shape. The platinum electrodes are fused into opposite points in the wall of a glass tube fifteen centimeters long and thirteen millimeters in internal diameter, about twenty-two millimeters from the lower end, which is sealed. During an experiment the mouth of the tube is

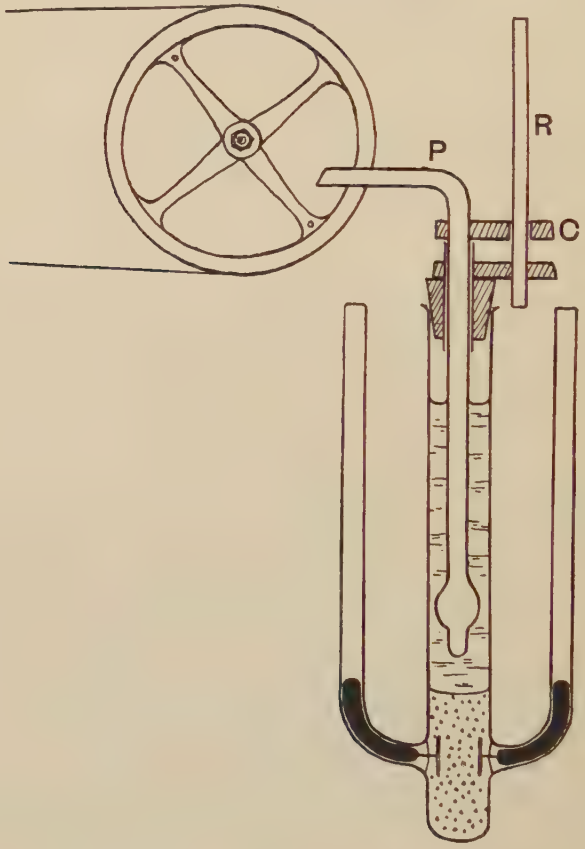


Fig. 1.

closed by a cork pierced by a short tube in which slides the glass plunger *P*. This is prevented from turning by the piece of cork *C*, which, gripping the plunger and sliding loosely on the rod *R*, acts as a guide. The plunger is raised and allowed to drop every two minutes by a wheel moved by clockwork and having two pins projecting from opposite spokes in the same direction as the axis.

Five cubic centimeters of a warm 4% solution of well washed gelatine* were introduced by a pipette and allowed to solidify. The cell was immersed in a water bath kept at 18° C. The resistance of the jelly was determined by the telephone method, using a post-office resistance box; the alternating current was led off from the primary of a small induction coil, the secondary being connected in series with a 10 c.p. lamp to the 50 volt, 90 ~, house supply.

The resistance of the jelly was not affected by standing in contact with distilled water for over three hours: it is therefore certain that the progressive changes in conductivity observed when salt solutions are substituted for the water are not complicated by the diffusion of impurities away from the gelatine. The salt solutions used were deci-normal. 10 c.c. of the solution were poured on the jelly, filling the cell to the level indicated in the figure, and the resistance of the gelatine between the electrodes was measured, usually at intervals of half an hour, for the next eight or ten hours.

The specific conductivity was obtained for each reading by dividing the constant for the cell (·643) by the resistance in ohms. From this the specific conductivity of the gelatine at the beginning of the experiment was deducted, the result indicating the specific conductivity due to the salt. The molecular concentration of the salt corresponding to each of these values was read off from curves constructed from the figures given by Kohlrausch† and his co-workers. At the dilutions with which we are concerned the specific conductivity is very nearly in linear proportion to the concentration.

On plotting the concentrations against the times from the beginning of the experiment the curves were found to be at first convex to the base line and towards the end of the experiment slightly concave, but in every case over a period of at least two hours their progress was rectilinear. The inclination of this part of the curve differed for each salt. I assumed that during the rectilinear period the rate of rise in concentration would be directly proportional to the velocity of diffusion of the salt. In order to test the validity of this assumption I made experiments with lithium, sodium and potassium chlorides, since the relative rates of diffusion of these substances have been determined already by physicists, using other methods‡.

* A gelatine jelly of this strength has been shown by Hardy to have the structure of an open network.

† Quoted in *Handbuch der anorg. Chemie*, II. 1, Abegg u. Auerbach, 1908. Also, Landolt-Börnstein's *Tabellen*, 1903.

‡ I have unfortunately been unable to apply the method of calculation, since worked out by Mr Hill, to these observations owing to the neglect of a simple precaution in my first experiments. The solution was introduced at a temperature

Time	Temperature of Bath	R Resistance in ohms	$K\left(=\frac{\cdot 643}{R}\right)$	$K - K_{gel.}$	Molecular concentration	
11.45 a.m.	18°	—	—	—	—	Cell with 5 c.c. gelatine and 10 c.c. distilled water placed in bath.
12.15 p.m.	18°	2335	·0002755	—	—	
1.25 „	18°	2335	·0002755	—	—	
1.30 „	—	—	—	—	—	Removed water and introduced 10 c.c. M/10 NaCl warmed to 18° C.
2.0 „	18°	2310	·0002782	·0000027	·00003	
2.30 „	18°	2210	·0002912	·0000157	·00015	
3.0 „	18·1°	2207	·0003150	·0000395	·00035	
3.30 „	18°	2040	·0003551	·0000795	·00074	
4.0 „	18°	1620	·0003986	·0001213	·00113	
4.30 „	18°	1406	·0004575	·0001820	·00171	
5.0 „	18·1°	1225	·0005250	·0002495	·00226	
5.30 „	18°	1085	·0005925	·0003170	·00303	
6.0 „	18°	970	·000663	·0003875	·00371	
6.30 „	18°	877	·000735	·0004595	·0044	
7.0 „	18°	794	·000810	·0005345	·00514	
7.30 „	18°	730	·000881	·0006055	·00587	Straight part of curve.
8.0 „	17·9°	676	·000951	·0006755	·00656	
8.30 „	18°	628	·001024	·0007485	·0073	
9.0 „	18°	585	·001094	·0008185	·00801	
9.30 „	18°	551	·001165	·0008895	·0087	
10.0 „	18°	521	·001232	·0009565	·00937	
10.30 „	18°	497	·001294	·0010185	·00999	
11.0 „	17·5°	476	·00135	·0010745	01052	

Specific conductivity of NaCl solution at beginning of experiment ·00920 = Concentration ·1 N.

„ „ „ „ end „ ·00835 = „ ·086 N.

One experiment may be quoted at length.

In this experiment there was a period of four hours, during which the increase in concentration proceeded at a practically uniform rate.

At 5.30 the concentration was ·00303 Mol. NaCl.
At 9.30 „ „ „ ·0087 „ „

Thus in four hours the increase was ·00567 or ·001417 per hour.

Experiments made in the same way with the other salts yielded the following numbers. The observations are plotted in fig. 2.

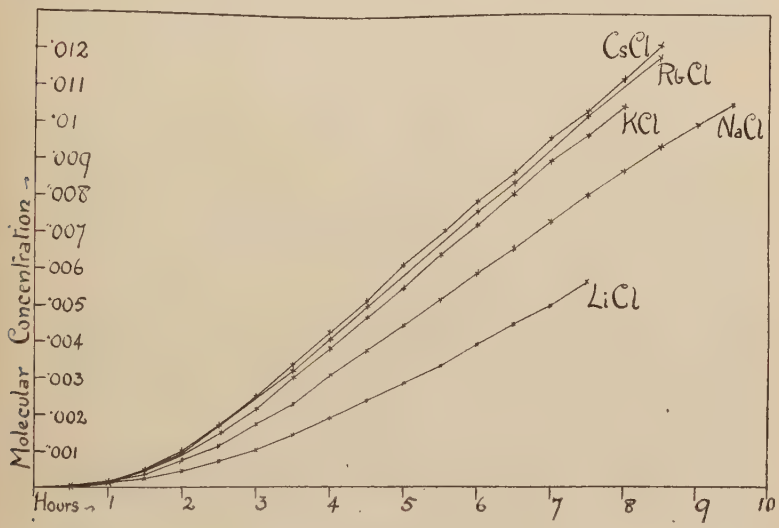


Fig. 2.

Deci-normal	Caesium chloride.....	·0018	Mol. per hour
„	Rubidium chloride ...	·001748	„ „ „
„	Potassium chloride ...	·001707	„ „ „
„	Sodium chloride	·001417	„ „ „
„	Lithium chloride	·001165	„ „ „

Or taking the rate of diffusion of potassium chloride as unity, and disregarding the last decimal place, we have

CsCl 1·05(5), RbCl 1·02(3), KCl 1·000, NaCl 0·830, LiCl 0·68(4).

below that of the water bath and some few minutes must have elapsed before 18° was reached. The effect of this delay is to slightly displace the curve to the right without affecting its form except at the very beginning. My calculations are based entirely on the *inclination* of the curve during a period beginning three or four hours after the experiment was started.

In the following table these results are compared with some figures given by previous observers for the relative diffusivities of potassium, sodium and lithium chlorides.

CsCl	RbCl	KCl	NaCl	LiCl	Observer
—	—	1·0	·8337	—	Beilstein*, 1856
—	—	1·0	·763	·55	Schuhmeister†, 1879
—	—	1·0	·77	·674	Long‡, 1879
—	—	1·0	·805	·685	Öholm§, 1905
—	—	1·0	·835	—	J. C. Graham‖, 1905
—	—	1·0	·76	·717	J. C. Graham¶, 1907
1·05(5)	1·02(3)	1·0	·830	·68(4)	Mines, 1910

It will be noticed that in the last two columns my numbers fall within the limits of variation of those quoted above. This fact I consider to justify the extension of the method to the study of rubidium and caesium chlorides. In making these experiments I aimed only at getting a rough comparison of the diffusion rates of these substances, but I believe that the method might be used to obtain values of considerable accuracy.

It would be easy to simplify the conditions by keeping one end of the cylinder of jelly in contact with a dilute salt solution renewed at frequent intervals and the other end in contact with distilled water. It is obvious that the method is limited to those electrolytes which are not hydrolysed in solution.

Conclusion. In dilute aqueous solution at 18° C. rubidium chloride diffuses slightly faster than potassium chloride and caesium chloride slightly faster than rubidium chloride.

The chlorides of the alkali metals show a rise in their rate of diffusion which follows the same order as the increase in their molecular weight and in the velocity of their kations.

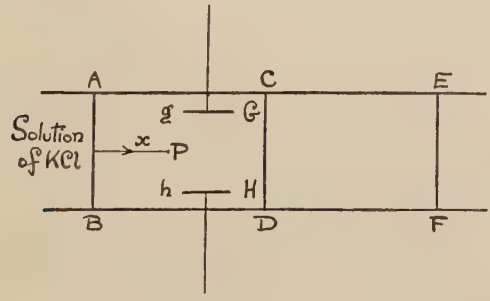
My friend, Mr A. V. Hill, of Trinity College, has kindly indicated a mathematical treatment of the subject in the following note.

* Beilstein, *Liebig's Annalen*, XCIX. 1856, p. 165.
† Schuhmeister, Landolt-Börnstein's *Tabellen*, 1905.
‡ Long, *Annalen der Physik u. Chemie*, IX. 1880, p. 613.
§ Öholm, *Zeit. Physik. Chem.* L. 1905, p. 309.
‖ J. C. Graham, *ibid.* p. 257.
¶ *Ibid.* LIX. 1907, p. 691.

Note on the use of the experimental method described in the preceding paper. By A. V. Hill, Scholar of Trinity College, and George Henry Lewes Student.

ACDB is the tube containing the gelatine, which stretches from *AB* to the bottom *CD* of the tube. Then the conditions throughout the gelatine *ACDB* must be the same as if we had double as much gelatine *AEFB*, and KCl (e.g.) solution at each end. For in the first case the bottom of the tube hinders diffusion across the plane *AB*: in the second case from considerations of symmetry there can be no diffusion across the plane *CD*.

Gg and *Hh* are the platinum electrodes. $AC = \frac{a}{2} = CE$.



Let y be the concentration at any point P measured a distance x along the tube.

Then we have the equation for the diffusion,

$$\frac{\partial y}{\partial t} = k \frac{\partial^2 y}{\partial x^2}$$

if t is the time.

A solution of this equation which satisfies all conditions is

$$y = y_0 + \sum_{r=1}^{r=\infty} A_r e^{-k\frac{r^2\pi^2}{a^2}t} \sin \frac{r\pi x}{a}.$$

For (1) at $x = 0$, $y = y_0$, the constant concentration of KCl.

(2) at $x = a$, $y = y_0$.

(3) y can be made equal to 0 at $t = 0$.

We have at $t = 0$

$$0 = y_0 + \sum_{1}^{\infty} A_r \sin \frac{r\pi x}{a}.$$

Multiplying by $\sin\frac{r\pi x}{a}$, and integrating between the limits 0 and a, we have

$$0 = y_0 \int_0^a \sin\frac{r\pi x}{a} + A_r \int_0^a \sin^2\frac{r\pi x}{a}$$

$$= -y_0 \frac{a}{r\pi}\left[\cos\frac{r\pi a}{a} - \cos 0\right] + A_r \frac{a}{2},$$

when r is even $\quad A_r = 0,$

when r is odd $\quad A_r = -\frac{2}{a}\cdot\frac{a}{r\pi} y_0 \cdot 2 = -\frac{4y_0}{r\pi},$

$$y = y_0\left\{1 - 4\sum_1^\infty \frac{1}{(2n-1)\pi} e^{-k\frac{(2n-1)^2\pi^2}{a^2}t} \sin\frac{(2n-1)\pi x}{a}\right\}.$$

Hence we may draw two conclusions.

(1) To make results with the instrument easily and directly comparable it is advisable to keep $\frac{a}{2}$, the amount of gelatine used, the same in different experiments.

(2) With this condition if kt in one experiment $= k't'$ in another, the concentration at any point P is the same in either experiment. Hence if we compare the times at which equal concentrations are reached these must be inversely proportional to the diffusion constants of the two salts. With a series of observations therefore at equal intervals of time the concentrations can be plotted and the curve compared with the standard curve obtained for one salt whose diffusion constant is known. The ratio of the times to equal concentrations can be compared at several points of the curve: this ratio should be found nearly constant, and its mean value for several concentrations will give the ratio of the diffusion constants very accurately.

If we suppose the conductivity to be directly proportional to the amount of salt between the two electrodes we shall have the conductivity

$$\Lambda = \mu y_0 \int_p^q \left\{1 - 4\sum_1^\infty \frac{1}{(2n-1)\pi} e^{-k\frac{(2n-1)^2\pi^2}{a^2}t} \sin\frac{(2n-1)\pi x}{a}\right\} dx,$$

where p and q are the values of x for the ends g, h, and G, H, of the electrodes, and μ is some constant, depending on the conductivity and the size of the plates.

$$\Lambda = \mu y_0\left[(q-p) - 4a\sum_1^\infty \frac{1}{(2n-1)^2\pi^2} e^{-k\frac{(2n-1)^2\pi^2}{a^2}t}\left\{\cos\frac{(2n-1)\pi p}{a} - \cos\frac{(2n-1)\pi q}{a}\right\}\right].$$

From a measurement of the constants a *direct* estimation might be made of the diffusion constant of any one salt.

By plotting the *logarithms* of the time and the *logarithms* of the conductivities the curves for two different salts will be found to be the same, though situated at a different position. The amount that one must be displaced along the axis of time in order to coincide with the other gives the ratio of the diffusion constants: the amount of displacement along the axis of conductivity gives the ratio of the conductivities.

A note on some fossil plants from Newfoundland. By E. A. Newell Arber, M.A., F.L.S., F.G.S., Trinity College.

[*Read* 7 February 1910.]

The present preliminary note is concerned with two fossil plants, not hitherto recorded from Newfoundland. Our knowledge of the plant-remains from this region is at present very slight, and is confined to a few Upper Carboniferous species described by Dawson*. The impressions, discussed here, occur in a sandy shale exposed in the bed of the Shenanditti River, on the west side of Red Indian Lake in the interior of Newfoundland.

Text-fig. 1. *Sphenophyllum tenerrimum*, Stur.
Two leaf whorls, each composed of six or more repeatedly dichotomous, narrow segments. No. 269 Sedgwick Museum, Camb. ×2.

The specimens, which are well preserved, were presented to the Sedgwick Museum by the generosity of Mr Rowland Feilding in 1908.

There are only two species to be recorded and these were associated. One of them is a *Sphenophyllum*, which appears to

* Dawson, *Rep. Foss. Plants, Lower Carbon. and Millstone Grit, Canada* (Geol. Surv. Canada, 1873), pp. 29, 32, 34.

be very closely similar to, if not identical with, *Sphenophyllum tenerrimum*, Stur*, known from the Lower Carboniferous rocks of Silesia. Both leaf whorls and stem impressions are represented. Two of the former are seen in text-fig. 1, magnified twice.

The whorl consists of six or more dichotomously branched narrow segments. In some cases the leaves dichotomise twice, in others, three times. The first dichotomy may take place at the base of the leaf, or a little above the base.

Sphenophylls possessing this reduced and repeatedly forked type of foliage are known both from the Lower and Upper Carboniferous. In some Coal measure species, both normal wedge-shaped leaves and leaves with reduced forked segments may occur on the same stem. In the case of *S. tenerrimum*, Stur, however, as in the present specimen, there are no indications that would lead one to suppose that the plant was heterophyllous.

With the Newfoundland specimens we may also compare the *Sphenophyllum subtenerrimum* of Nathorst† from the Upper Devonian rocks of Bear Island in the Arctic regions. Here however the leaf whorls appear to be smaller in size‡, and the leaf segments even more delicate.

Impressions of stems of *Sphenophyllum*, possibly belonging to the same species as the leaves described above, are also associated. One of these is shown in text-fig. 2, where two somewhat tumid nodes are seen, with which fragments of leaves, which lie at right angles to the plane of the stem as seen in this specimen, appear to be still in continuity. The internodes exhibit two rather sharp longitudinal ridges, which are fairly distant from one another.

This stem appears to be somewhat stouter, and the internodes more elongated, than the stem specimens of *S. subtenerrimum* figured by Nathorst.

The other plant collected from the Shenanditti River is of special interest for it appears to be new to science, at least specifically, if not generically. In Mr Feilding's collection, several examples of a large fan-shaped leaf are more or less completely preserved. These leaves often exceed 14 cm. in length and breadth. The nerves radiate from the base and are numerous, dichotomising here and there in the broader portion of the leaf. All the examples are detached. At the base the leaf narrows somewhat rapidly and appears to have been sessile.

These leaves recall some species of the rare and little known genus *Psygmophyllum*, Schimper, more closely than any of the

* Stur (Culm Flora, Part II.), *Abhandl. k. k. Geol. Reichsanst*, Wien, Vol. VIII. 1877, p. 108, pl. VII. (XXIV.), figs. 1—14.

† Nathorst, *K. Svenska Vetenskaps-Akad. Handl.*, Vol. XXXVI. No. 3, 1902, p. 23, pl. 2 (figs. 14—17), pl. 3 (figs. 7, 8), pl. 4 (figs. 14—23), pl. 5 (fig. 5).

‡ *Ibid.*, pl. 4 (figs. 22, 23).

other fossils recorded from the Palaeozoic rocks. *Psygmophyllum* as at present constituted is probably a composite genus. At a future opportunity it is proposed, when describing these fossils more fully, to review the present position of our knowledge of the plants which have at one time or another been assigned to it. The present preliminary note may serve to record the discovery of these interesting fossils in a region which hitherto has remained almost a *terra incognita* in a palaeobotanical sense.

Text-fig. 2. *Sphenophyllum*, sp.
A stem showing two nodes, to which fragments of leaves are attached. No. 339 Sedgwick Museum, Camb. $\times 1\frac{1}{2}$.

The only evidence of the age of the rocks from which these specimens were obtained is that presented by the fossils themselves. From the occurrence of *Sphenophyllum tenerrimum*, Stur, one would suppose that the period is either Lower Carboniferous or Upper Devonian, the former being more probable.

A note on Cardiocarpon compressum, Will. By Mrs E. A. Newell Arber, D.Sc., Newnham College. (Communicated by Mr E. A. Newell Arber.)

[*Read* 7 February 1910.]

In 1877 Williamson* described and figured a small unattached winged seed showing binary symmetry, from the Lower Coal Measure nodules of Oldham, which he referred to Brongniart's† genus *Cardiocarpon.* On account of its flattened form he gave it the specific name *compressum.* Other Cardiocarpons had previously been described from the British Coal Measures, but only their external appearance was known, whereas in the new species the internal structure was preserved. Williamson figured sections cut from three specimens. The most characteristic feature of the seed was that the testa was divisible into two very distinct layers,—an exotesta of coarse parenchyma, and an endotesta of much smaller cells. No further description of the internal structure of *Cardiocarpon compressum,* or of that of any other seed showing bilateral symmetry from the British palaeozoic rocks, has been published since Williamson's time. The main interest of the genus *Cardiocarpon* lies in the great probability that it represents the seeds of *Cordaites*‡. The possibility is not however excluded, that seeds of this type were borne by Pteridosperms, for seeds of binary symmetry have been found attached to fronds of *Aneimites*§ and *Pecopteris Pluckeneti*‖. The following note is based on material collected by Prof. F. W. Oliver, Dr D. H. Scott, and Prof. Weiss, and generously made over to the present writer for examination. The material consists entirely of sections of unattached seeds occurring in the "coal balls." The same seed is seldom met with in more than one section.

The contrast between exotesta and endotesta already mentioned affords the readiest means of identifying *Cardiocarpon compressum* in microscopic section. There is sufficient variation among the specimens, both in dimensions and structure, to suggest that *Cardiocarpon compressum,* instead of being a single species, may possibly represent an assemblage of seeds belonging to closely

* W. C. Williamson, "On the Organisation of the Fossil Plants of the Coal-measures," Pt. viii., *Phil. Trans. Roy. Soc.* Vol. clxvii. Pt. i. p. 213.

† A. Brongniart, *Prodrome d'une histoire des végétaux fossiles,* Paris, 1828.

‡ F. C. Grand'-Eury, *Mémoire sur la Flore Carbonifère du Département de la Loire,* 1877, p. 233.

§ David White, "The Seeds of Aneimites," *Smithsonian Miscellaneous Collection,* Vol. xlvii. 1905, p. 322.

‖ F. C. Grand'-Eury, "Sur les graines trouvées attachées au Pecopteris Pluckeneti, Schlot," *Comptes Rendus,* t. cxl. 1905, p. 920.

allied plants. The sclerotesta or shell consisted of two flattened valves, pointed above and broadest near the base. In both length and breadth these valves slightly exceeded half a centimetre. In well preserved specimens the shell is enclosed in a delicate sarcotesta forming a thin layer over the surface of the valves, and at their lateral edges extending beyond them as a wing lying in the plane of flattening. The wing appears to have terminated in a short downward prolongation on each side of the hilum, giving the seed a remote resemblance to a mitre. It should be mentioned that Williamson took a different view of the structure, and figures the sarcotesta as continuous round the base of the seed.

Williamson's preparations did not admit of his observing the vascular system, which is of some interest. The main supply bundle enters the hilum and expands into a funnel-shaped mass below the nucellus. Here it gives off two opposite branches in the plane of flattening of the seed. These branches are at first horizontal, but they soon bend downwards and outwards, traversing the shell in an oblique direction. On emerging into the sarcotesta they curve upwards and run to the micropyle following the slight groove at the junction of the valves. The base of the nucellus is supplied by tracheides given off from the expanded end of the central bundle above its two main branches. I have not been able definitely to satisfy myself as to whether a nucellar vascular system arises from this basal tracheal plate.

Inside the shell there are indications of an "inner flesh," which is however seldom preserved. The nucellus seems to have been free from the integument from the base upwards. The megaspore is usually empty and contracted, but in one case a tissue occurs within it which may be a prothallus.

I have provisionally adopted Williamson's name for this seed, *Cardiocarpon compressum*, but it is doubtful whether it should be permanently retained. The question of nomenclature, which presents some difficulties in this case, will be discussed at length in a detailed paper which I hope to publish shortly.

On a new species of Physostoma from the Lower Carboniferous Rocks of Pettycur (Fife). By W. T. Gordon, M.A., B.Sc., Falconer Fellow of Edinburgh University, and Advanced Student Exhibitioner of Emmanuel College. (Communicated by Mr E. A. Newell Arber.)

[*Read* 7 February 1910.]

Among the rarer petrifactions of plant-remains occurring in the Calciferous Sandstone Rocks at Pettycur are seed-like bodies of various kinds. Perhaps the commonest example is *Lepidocarpon wildianum*, Scott*, while occasionally the pteridospermous fructification—*Conostoma ovale*, Williamson†, is also met with. Under the name *Conostoma intermedia*‡ Williamson also placed what are probably elongated specimens of *C. ovale*. These two seed bodies are very distinct, for *Conostoma* has a funnel-like micropyle, whereas *Lepidocarpon* has no true micropyle. The megaspore in the latter was exposed to the exterior by means of a slit-like aperture between two enveloping outgrowths from the sporophyll.

Last year Prof. Oliver§ published his work on *Physostoma elegans*, and showed that in that genus the apex of the seed consisted of closely adpressed, free, tentacular processes; the micropyle was thus capable of great enlargement.

In the same paper he refers to *C. ovale* as being probably the seed of *Heterangium grievii*, Williamson. The only evidence however is association. Recently, while examining a new species of Heterangium from Pettycur, I discovered several specimens of *C. ovale*, and among them what appears to be a new and distinct seed. The specimen had been cut through in an oblique tangential direction, but by careful preparation I was able to get one of the sections more nearly radial. One section is through the micropylar orifice and the other passes through the chalaza.

General Structure.

The seed is radially symmetrical and is considerably larger than *C. ovale*. The following table gives the dimensions of the new seed and those of *C. ovale*.

* Scott, *Phil. Trans. Roy. Soc.* B. Vol. cxciv. 1901, p. 291.
† Williamson, *Ibid.* Vol. clxvii. 1877, p. 243. ‡ *Ibid.* p. 246.
§ Oliver, *Ann. Bot.* Vol. xxiii. 1909, p. 73.

	New Seed (*Physostoma*)	*Conostoma ovale*	
Length	3·8 mm.	3·2 mm.	Average of 5 radial sections
Breadth	3·3 mm.	2·1 mm.	

The length of the new seed is probably greater than indicated, since both sections are oblique, and 4 mm. would probably not be an excessive estimate of the true length. The breadth is also considerable, and indeed the almost globular shape of the seed is striking. The base is rather flat and the micropyle is not prominent.

The seed at the apex shows a number of tentacular processes and the outer surface is studded with small peg-like hairs. The occurrence of these characters warrants its inclusion in the genus Physostoma as lately defined by Oliver.

All the Pettycur seeds I have examined are poorly petrified, and this new one is no exception. The description must therefore be meagre.

The nucellus is capped by the pollen chamber but there is no dome-like projection of the nucellar apex into the cavity of the pollen chamber. Indeed, in that region the seed is quite like Conostoma and not Physostoma. Outside the membrane of the nucellus the tissues are all decayed; only the vascular bundles remain. Near the chalaza however and in one of the apical processes a slight amount of delicate parenchymatous tissue can be observed, but it is too fragmentary to warrant detailed description.

The outer layers of the integument have all decayed into a structureless dark zone about ·2 mm. thick. There are no ribs on the outside of the seed, but the whole periphery is covered with small dark dots. These lie in a zone concentric with the seed and about ·1 mm. outside. Near the micropyle these dots are seen to occupy the ends of short finger-like cells which clothe the seed externally. They are quite similar to the peg-cells of Lagenostoma and may be equivalent to the hairs on *Physostoma elegans*. In this new specimen the hairs are equal in length all over the surface. In outline the hairs are short and have rounded ends.

The most interesting part of the seed, however, is the micropylar end. There we find a pollen chamber in the usual position at the apex of the nucellus. The orifice of this chamber is not visible owing to the obliquity of the section and it is uncertain whether there was a funnel-shaped aperture as in *Physostoma*

elegans or a depressed aperture as in *Conostoma ovale*. The micropyle which lies immediately over the opening of the pollen chamber is also cut tangentially and in the most radial section is clearly bounded by three separate tentacle-like processes.

The chalazal end is also interesting from the fact that the bundles enter the seed independently as in Physostoma.

This new species of Physostoma, which is more ancient than *P. elegans* recently described by Professor Oliver, combines with its Physostoma characters, others which are typical of the genus Lagenostoma. The tentacular processes at the apex suggest affinities with Lagenostoma in that they are less divergent than in Physostoma (i.e. they clasp the pollen chamber more tightly), and thus the seed has the whole apex flattened as in Lagenostoma. On the other hand the structure of the pollen chamber, so far as is known, is quite unlike that of Lagenostoma. The seed may however eventually require to be placed in a new genus.

In conclusion I have to thank my supervisor Mr Arber for his kindly criticism and advice.

On the relation between the fossil Osmundaceae and the Zygopterideae. By W. T. Gordon, M.A., B.Sc. (Edin.), Falconer Fellow of Edinburgh University, and Advanced Student Exhibitioner of Emmanuel College. (Communicated by Mr E. A. Newell Arber.)

[*Read* 7 February 1910.]

The structure of the axis in the living Osmundaceae has given rise to two main contentions as to the evolutionary position of the group. The axis is regarded by Jeffrey* and Faull† as reduced from a more complex dictyostelic ancestor, and by Boodle‡, Seward and Ford§, Chandler‖, Tansley¶, and Kidston and Gwynne-Vaughan** as the latest development in an ascending series from a protostelic ancestor.

This latter view is supported by the ontogenetic development of living species and, now that Kidston and Gwynne-Vaughan have completed their studies on the fossil representatives, by the phyllogeny of the group as well.

The present living genera may be placed in two categories. *Osmunda regalis* may be taken as one extreme type where the leaf gaps are long and a cross-section of the stem consequently showing a ring of several xylem strands. In cases where the leaf gap is short as in *Todea barbara* and *T. superba* a cross-section shows the xylem as a much more continuous ring. This may be looked upon as the other extreme.

The stems of the most recent fossil representatives of the Osmundaceae conform to these two extremes. Even *Osmundites Skidegatensis*, Penhallow, is of the Osmunda regalis type, though it may be regarded as an even more extreme example. (This species seems to lend colour to Jeffrey's reduction theory, but the weight of the other evidence completely overrules this single exception.) The tertiary species *Osmundites schemnicensis* (Pettko) and *O. Dowkeri*, Carruthers, resemble *Osmunda regalis*, and so does the Jurassic form *O. Gibbiana*, Kidston and Gwynne-Vaughan. *O. Dunlopi*, Kidston and Gwynne-Vaughan, another species from the Jurassic, has a xylem ring which is almost later-

* Jeffrey, *Phil. Trans. Roy. Soc.* Vol. cxcv. 1902, p. 127.
† Faull, *Bot. Gaz.* Vol. xxxii. 1901, p. 418.
‡ Boodle, *Ann. Bot.* Vol. xvii. 1903, p. 515.
§ Seward and Ford, *Trans. Linn. Soc. London*, Series 2, Vol. vi. 1902, p. 254.
‖ Chandler, *Ibid.* Vol. xix. 1905, p. 406.
¶ Tansley, *New Phytologist*, Reprint No. 2, 1908.
** Kidston and Gwynne-Vaughan, *Trans. Roy. Soc. Edinburgh*, Vol. xlv. Pt. 3, 1907, p. 759; *ibid.* Vol. xlvi. Pt. 2, 1908, p. 213; *ibid.* Vol. xlvi. Pt. 3, 1909, p. 651.

ally continuous, i.e. the leaf trace departs typically in a protostelic manner. This also occurs occasionally in *T. superba*, so that *O. Dunlopi* may be considered as pointing to *Todea* rather than to *Osmunda regalis*.

In the Jurassic period therefore forms existed which were practically as divergent as the living ones are. The only difference is shown in *O. Dunlopi*, where the leaf trace is typically protostelic in its departure. The type of this last species is still further accentuated in Upper Permian times in the case of *Zalesskya*, Eichwald, and *Thamnopteris*, Eichwald. Both of the last-named genera have not only the xylem ring continuous laterally but they have no central pith. The central tissue of the stem consists of short, reticulated tracheides.

In the Permo-Carboniferous genus *Grammatopteris* Kidston and Gwynne-Vaughan see the earliest example of a plant with Osmundaceous affinities, and here we find a perfect protostele with only one type of xylem tissue. This genus however is still imperfectly known.

The leaf trace in the fossil Osmundaceae also shows a very distinct passage from exarchy to endarchy, and from protostelic departure to cases where leaf gaps occur. The most interesting example is probably *Thamnopteris schlechtendalii*, since in it we occasionally find leaf traces with double mesarch protoxylem groups. Even where such double protoxylems are found the trace develops normally afterwards.

Turning now to Zygopterid alliance, we find that the youngest known representative is Asterochloena Corda ranging to the Devonian. The stem xylem in this genus is stellate with exceedingly long rays and a pith is always present. The trace has two mesarch protoxylem groups. In the Permo-Carboniferous of Autun *Z. Brongniarti*, Renault, occurs. The stele in this case has almost circular xylem with a true pith and a system of internal tracheides in that pith. The outer xylem ring is interrupted by the departing leaf traces. *Z. Grayi*, Williamson, from the Coal Measures, has a stellate xylem but the rays are very short. A central pith and system of internal tracheides are present.

Corresponding to *Z. Brongniarti* in the Permo-Carboniferous we have *Z. Corrugata*, Williamson, in the Coal Measures. The organisation of this stem is quite similar to *Z. Grayi*, but in outline it has become more circular. The differences between the outlines in these two species seems to be correlated with the greater length of the internodes. The lately discovered stem of Diplolabio Römeri Solms from Lower Carboniferous rocks exhibits the most primitive type of zygopterid stem so far known. The stem as seen in transverse section, in cases where no petioles are departing would be classed as a Botryopterid. The wood is solid and of two kinds,

forming an inner and an outer zone. The thickenings on all the elements are reticulate, except the protoxylem groups, where they are scalariform. The leaf trace departs in a protostelic manner, and has two immersed protoxylem groups.

Among the Botryopterideae, *B. cylindrica* (Williamson), from the Coal Measures, represents probably the highest known type of development. Here the inner elements of the stem xylem are smaller than those of the outer xylem. In *B. antiqua*, Kidston, there is only one type of xylem element. In the last species I have been able to show that the leaf trace is at first mesarch.

The thickenings on the xylem elements in these three groups are interesting. In the Osmundaceae we get a series from multiseriate pits through scalariform to reticulate thickenings. Reticulate tracheides only occur in the archaic forms *Zalesskya* and *Thamnopteris* and then only in the inner wood. The last-mentioned type of thickening must therefore be considered the most primitive. Among the Zygopterideae we find a similar series from scalariform to reticulate (in *D. Römeri*); in the Botryopterideae from scalariform to reticulate.

Except for the multiseriate pits on the external xylem elements *Thamnopteris* could not be distinguished from the zygopterid *D. Römeri* on stem structure. The leaf trace has of course typically only one protoxylem group, but occasionally examples with two are found. This feature may be considered as being probably primitive since the double group ultimately functions as if it were a single one. In *D. Römeri* the leaf trace has typically two protoxylems and I have never observed either more or fewer than that number. It is clear then that at first sight in their most primitive representatives the two groups of the Osmundaceae and the Zygopterideae cannot be distinguished in their anatomical features. The two groups show exactly parallel development from the two primitive types just cited, and it thus seems probable that they have diverged from a common stock. The geological position of the various representatives of these groups points to the conclusion that the Zygopterideae diverged from the parent stock before the Osmundaceae did; and the greater suitability of the latter group for present conditions of growth has allowed it to persist after the Zygopterideae had perished.

I desire to express my indebtedness to Mr E. A. Newell Arber, under whose supervision my work is being carried on, for his criticism and advice.

On the occurrence of Schizoneura paradoxa, Schimper and Mougeot, in the Bunter of Nottingham. By R. D. VERNON, B.Sc., 1851 Exhibition Research Scholar, and Advanced Student Exhibitioner of Emmanuel College. (Communicated by Mr E. A. Newell Arber.)

[*Read* 7 February 1910.]

From the English Trias recorded fossils are extremely rare and appear to be restricted to the Upper or Keuper division. The Lower Trias has hitherto yielded, so far as I am aware, only derived fossils which are contained in the constituent pebbles of the middle or "Pebble Bed" subdivision of the Bunter. The discovery of fossil plant-remains from the Bunter of Nottingham is thus of some general interest, for they appear to be the first contemporaneous fossils from the Bunter of England. Equisetaceous pith casts of doubtful attribution were the first specimens obtained; they were provisionally determined as *Equisetites* sp., and are so mentioned in the recently published Geological Survey Memoir on the Nottingham district*. Subsequent collecting has yielded a large number of fragmentary plant-remains including not only pith casts but also leaves, stems, and roots; occasionally specimens are found showing two or more organs still in organic connection. All the fossils may be referred to *Schizoneura paradoxa*, S. and M., and there is reason to believe that they originally grew on the spot where they now occur.

The fossiliferous section is a now disused sand-pit at the foot of Colwick Wood, by the west side of the Great Northern Railway bridge over Colwick Road, Nottingham. The section shows the regularly-bedded marls and thick sandstones of the Lower Keuper (Waterstones) with a thin conglomerate at the base, resting on a sharply marked and eroded surface of the Bunter Pebble Beds, which here consist of strongly current-bedded pebbly sandstones containing nodules and lenticular beds of red marl. It is from the uppermost lenticular bed of marl in the Bunter, a few feet below the basal conglomerate of the Keuper, that all the plants have been collected. The section is figured and described at length in the Geological Survey Memoir on the district†.

* "Geol. country between Newark and Nottingham" (*Mem. Geol. Surv.*), 1908, p. 42.

† *Ibid.* p. 37.

Equisetales.

Schizoneura paradoxa, Schimper and Mougeot, 1844.

1828. *Calamites arenaceus*, Brongniart, *Hist. végét. foss.* Pl. xxiii. fig. 1.
1844. *Schizoneura paradoxa*, Schimper and Mougeot, *Monog. Plant. foss. Vosges*, p. 48, Pls. xiv. xv. xvi.
1844. *Calamites arenaceus*, *ibid.* p. 57, Pl. xxviii. fig. 2.
1844. *Calamites mougeoti*, *ibid.* p. 58, Pl. xxix. figs. 1, 2, 3.
1870. *Equisetites mougeoti*, Schimper, *Traité*, p. 279, Pl. xii. fig. 4.
1907. *Equisetites arenaceus*, Arber in Wills' *Geol. Mag.* Dec. 5, Vol. iv. p. 32.
1910. *Schizoneura paradoxa*, Wills, *Proc. Geol. Assoc.* Vol. xxi. Part 5, p. 272.

Localities. Voltziensandstein (Upper Bunter) of the Vosges. Lettenkohle (top of Muschelkalk) at Neuewelt, Basle, Switzerland. Lower Keuper Sandstone at Bromsgrove and other places in Worcestershire. Bunter Pebble Beds of Nottingham.

Diagnosis. From a study of the specimens in the Strassburg Museum, including the original material of Schimper and Mougeot, and of the Worcestershire specimens, described in his recent paper, Wills* has drawn up the following diagnosis:

"Large plants, stems up to two inches in diameter, stems divided into nodes and internodes, internodes long, larger stems either ridged or smooth, pith either wholly or partially hollow; branches borne in whorls at the nodes, external surface of branches usually smooth or nearly smooth; leaves long and strap-like, borne in whorls at the nodes, leaves united either into a sheath or into sheath segments or free to their bases, border of leaves smooth, median part of leaves with fine close-set parallel nerves, cone probably with peltate areas on external surface, roots arising from the nodes, roots small and repeatedly branched."

Schizoneura must still be regarded as an imperfectly defined genus. Of the cone our knowledge is of the slightest, and of the internal anatomy of the plant we are quite ignorant. The fragmentary nature of the stem casts renders identification in many cases doubtful or even impossible. Ridged and grooved casts are of common occurrence, some of these may represent the external surface of the stem, or they may be the impressions of the external surface more or less modified by the internal structure, as pointed out by Wills; more usually they are interpreted as pith casts. Such fragmentary pith casts cannot be generically identified, they may belong to one of several members of the Equisetaceae.

* Wills, "The Fossiliferous Lower Keuper Rocks of Worcestershire," *Proc. Geol. Assoc.* Vol. xxi. 1910, Pt. 5, p. 272.

A further source of possible error depends on the fact that this diagnosis is necessarily founded on fragmentary material from various localities occurring in association with remains of other plants. This latter objection does not apply to the fossils described below since they appear to belong wholly to one species and are not associated with any other plant-remains.

Description of the Specimens.

Detached leaves. These are fragments of long, narrow, delicate, grass-like leaves, usually found detached but more rarely occurring in organic connection with a stem. They have smooth entire margins with a median group of exceedingly fine parallel nerves.

One example has four and another has six leaves, in each case the margins of the leaves are arranged parallel to one another. These specimens may be regarded as portions of sheath-segments composed of several leaves, originally united by their smooth margins along the commissural lines. By splitting along these lines the leaves have, in some cases, become either partially or wholly free.

Leafy shoot. A cast of the external surface of a small stem about ·5 cm. in diameter with a smooth, irregularly wrinkled surface, bears a node from which arise four or more leaves forming a leaf-sheath. Only the basal portions of the leaves are preserved, and the clasping nature of the leaf-sheath cannot be made out because the specimen has been flattened by crushing.

Leafless stems. The material furnishes at least four kinds of casts and impressions of leafless stems:

(*a*) The first type are characterised by a smooth, irregularly wrinkled surface and distinctly swollen nodes.

In these features they agree exactly with casts of the external surface of leaf-bearing stems.

(*b*) Occasionally a cast of the above type is folded into somewhat broad furrows and narrow ridges.

A similar structure occurring in casts of *Schizoneura meriani* was explained by Schimper* as due to an accident of preservation.

(*c*) Several casts of portions of large stems differ from type (*a*) only in the fact that their smooth surfaces are faintly but regularly ridged and grooved. They may be regarded as either the casts of the external surface of large, faintly-ribbed stems, or the ribbing may be ascribed to the action of the internal structure.

(*d*) What appears to be a more internal cast than the above has internodes strongly marked with longitudinal ridges and

* Schimper, *Traité de Palaeontologie Végétale*, 1869, p. 282.

grooves, which, in the best preserved specimens, appear to be continuous at the node. The specimens vary greatly in size, in the coarseness and fineness of the ribbing, and in the character of the ribs, which are generally broad and nearly flat-topped. Reversed casts also occur in which the ridges are represented by grooves, causing the true grooves to stand out as narrow ridges, whilst the nodal constriction is now represented by an upstanding ridge. These specimens are undoubtedly Equisetaceous pith casts, which, by themselves, cannot be generically identified.

Branching. Certain specimens of the (*d*) type appear to be casts of the pith cavity of large branches. They are gradually contracted towards the base of the branch, just as is the case with the medullary casts of Calamites. Large scars which occur on the nodes of other casts of the (*d*) type may thus be interpreted as branch scars.

Nodal Diaphragms. Some interesting examples of nodal diaphragms have been collected. One specimen of the exterior portion of a stem with a node shows a diaphragm displaced by crushing, somewhat in the manner so often found in *Equisetites lateralis.* Other specimens have a smooth, central concave portion, bearing at the node a number of long, stout, narrow, spoke-like structures which appear to be branches.

Many of the diaphragms, apparently belonging to the underground stem, are still in the position of growth and appear to have suffered hardly any distortion. Radiating from the edge of the node are four, five, sometimes six, roots which at first are confined to one plane; they bear numerous branching rootlets which ramify in all directions. One specimen of a small diaphragm, only 12 mm. in diameter, has, attached to the node, four roots which bear numerous rootlets. Another broader, thicker, more woody looking structure with a smooth surface is also borne at the node; this appendage is quite different from, and very much larger than the associated roots; it appears to be a portion of a stem.

Roots. In addition to the roots described above which are found still attached to the node, there are numerous examples of detached fragmentary roots and rootlets, which show the same characters. All possess a smooth, irregularly wrinkled, or finely reticulated, external surface. Occupying the central axis of the root in several specimens is a stout central structure which probably represents the stele, its surface bears the imprint of longitudinal and transverse lines giving an appearance strikingly suggestive of cell structure.

The rootlets differ from the roots merely in size; the angle they make with the root is sometimes 60° but usually 90°.

Conclusion.

The fossils indicate a large plant with smooth stems at least three inches in diameter. Only the smaller stems appear to have borne leaves, which were at first united by their margins to form leaf segments or leaf sheaths. Later the leaves became more or less free to their bases by splitting along the commissural lines. The upper portion of the stem bore whorls of branches at the nodes, whilst the underground stem gave off numerous slender, probably adventitious, roots ending in branching rootlets.

The vegetative organs of this plant are so similar to those of the Keuper specimens of *Schizoneura paradoxa*, S. and M., described by Wills, that they may be regarded as the same species.

At the same time a few minor differences are noticeable. The pith casts differ amongst themselves, and also from the Keuper specimens, in size, and in the relative coarseness and fineness of the ribbing; such variations may be expected to occur even in different parts of the same plant.

Whilst the Keuper specimens of the root usually make an angle of 60° with the rootlets, in the Bunter plant this angle is nearly always 90°.

From the puckering of the leaves in the Keuper fossils Wills inferred that those organs were thick structures; but the leaves of the Bunter were undoubtedly thin and delicate.

In conclusion it may be remarked that this discovery of the first evidence of contemporary life in the Bunter rocks of England is not merely an interesting fact. The occurrence of the same species of land plant in both the Lower Keuper and the Bunter is of importance as evidence that the unconformity between these two divisions of the Trias in the Midlands is probably not an indication of a long lapse of time. It further suggests that the physical conditions which prevailed during these two periods were not very diverse.

Notes on the genus Schizoneura, Schimper and Mougeot. By L. J. Wills, M.A., F.G.S., Fellow of King's College. (Communicated by Mr E. A. Newell Arber.)

[*Read* 7 February 1910.]

The genus *Schizoneura* though of widespread occurrence is still very imperfectly known. It was founded by the eminent French palaeobotanists, Schimper and Mougeot, in 1844*, on specimens from the Upper Bunter (Voltzien-sandstein) of the Vosges Mountains, which they described as *S. paradoxa*. Their definition is somewhat loose, but its essential points are that the plant is of an Equisetaceous nature, with stem divided into rather long internodes; the leaves which are borne in whorls at the nodes are at first united into a leaf-sheath. This later splits along commissural lines into sheath-segments or into individual leaves.

Though various species have since been described from different horizons and countries, little has been added to our knowledge of the structure of this plant since 1844. Recently I have had the good fortune to collect abundant material from the English Lower Keuper of Bromsgrove in Worcestershire, which in some ways supplements our knowledge of the species *S. paradoxa*. It may be of interest to point out some details of the nature and structure of *Schizoneura* as exemplified by this, the original, species. We may then be in a more advantageous position to answer the question, whether the name *Schizoneura* has been applied to plants genetically distinct or the reverse.

I have, however, described the Bromsgrove material in some detail elsewhere†. Accordingly, I propose at present to mention only the chief points of interest displayed by this plant.

There is little doubt that *S. paradoxa* and possibly all the other species, were lovers of moist spots and, in fact, probably grew actually in the water; for we find at Bromsgrove rootlets and stems still apparently in their position of growth. A similar mode of occurrence has been noticed by other observers‡. *Schizoneura paradoxa* may well have reached a considerable height, for stems up to about 2″ in diameter and several feet in length have been discovered.

Leafy branches are the best known portions of this plant. The external surface of the smaller stems was probably smooth, while the larger ones may have been slightly ribbed. The leaves usually number seven. So far they have never been observed

* Schimper and Mougeot, *Monographie des Plantes fossiles du Grès bigarré de la chaine des Vosges*, Leipzig, 1844.

† Wills, L. J., *Proc. Geol. Assoc.* Vol. xxi. 1910, p. 271.

‡ e.g. Rogers, A. W., and Seward, A. C.; *see* Seward, *Quart. Journ. Geol. Soc.* Vol. lxiv. 1908, p. 89.

entirely united into a sheath, yet sheath-segments with varying numbers of leaves are found. On the other hand, the leaves are occasionally free to their bases. They are strap-like structures, with smooth margins and a bundle of fine parallel nerves down the centre.

The stems from which the leaves had fallen previous to preservation are of various types. They are known from impressions and casts. The external surface may have been smooth or possibly finely ribbed. The internal casts or impressions are finely ribbed. There are, however, two types of these, and their interpretation must remain ambiguous until we learn something from petrifactions about the internal anatomy of these Mesozoic Equisetales. We may, nevertheless, note that the ribbing appears to be coarser in the lower parts of the stem.

On certain of the ribbed internal casts there are at the nodes two kinds of prints:—the larger lie just above the nodal line and are branch-scars, while the smaller ones probably represent leaf-scars.

The roots of *S. paradoxa* have been found, and closely resemble those of Calamites (*Pinnularia*). It is doubtful whether the fructification has ever been discovered.

The ribbed internal casts mentioned above sometimes encase an innermost cylindrical cast. This is smooth, and probably represents a cast of a partially hollow pith. Such a structure has not been noticed before, though I found exactly the same phenomenon in several examples of the so-called pith-cast of *Equisetites mougeoti*, Schimper and Mougeot, from the Bunter of the Vosges Mountains, which are preserved in the Strasburg Museum. This plant is there associated in the same beds with *S. paradoxa*. Though the smooth outer surface of *E. mougeoti* is known, examples with leaves have never been described. The pith-casts are indistinguishable from those of *Schizoneura*. Accordingly it becomes ever clearer that *E. mougeoti* is merely the name given to large leafless stems of *S. paradoxa**. One point, however, is not evident: the smooth external cast seen in *E. mougeoti* has not, so far, been found at Bromsgrove. Yet the external surface of the smaller branches is smooth, and we may probably assume that the larger examples have still to be sought for.

How far the presence of this innermost cast can be taken as a distinguishing mark of *Schizoneura* cannot be determined as yet, until it has been observed or disproved in other species.

* In this connection it may be noted that Seward (*loc. cit.* p. 88) figures as *Schizoneura*, sp. *a*, a stem with smooth external surface enclosing a ribbed pith-cast, similar to both *E. mougeoti* and *S. paradoxa*, and calls attention to the resemblance of the whole fossil to the former.

Having now examined what is known of the plant originall described as *Schizoneura,* let us turn to a consideration of th other species, with the special object of seeing whether the genu is homogenetic.

Names have been given to seven species ranging in age fro the Permo-Carboniferous to the Rhaetic, while several othe fragments of stems have been referred to the genus. The follow ing table gives the distribution in time and space and the chie characteristics of the various named species.

One or two notes are necessary relative to the table. I the first place it will be observed that the description of th nervation of both *S. africana* and *S. gondwanensis* differs fro the usually accepted one which states that stout median nerve are present. I have, however, examined undoubted specimens o *S. gondwanensis* at the South Kensington Museum, on some o which, e.g. No. V. 7099, several distinct fine parallel nerves ca be seen.

The original specimen also of *S. africana* (*Asterophyllites* o Hooker*) in the Museum of the Geological Society likewise show some three distinct nerves down the centre of some of the leave I use the term leaf here, not as Seward has done in redescribin *S. africana* †, where he applies the word to what I would call sheath-segment, but in the same sense as Schimper and Feist mantel.

In the second place, little is known of *S. wardi,* and th description given by Zeiller leaves some doubt as to its nature It seems, however, possible that the nervation is multiple.

Let us now refer to the table. It is evident at once tha Halle is well justified in classing *S. carrerei, S. hoerensis* an *S. meriani* together, and the institution of the genus *Neocalamite* appears expedient. The type of plant represented by *Schizoneur* (*Neocalamites*) *carrerei* seems to have been almost world-wide i distribution in Rhaetic times; so far so, in fact, that the Euro pean, Asiatic and African species are almost indistinguishable.

Of the remaining species, *S. wardi,* although but imperfectl known, appears to have approximated to the *Neocalamites* type yet it may have possessed multiple nervation. In this case, a it is likely that the nervation is a generic quality, it could no be easily classed with that group but would fall in more naturall with the remaining species.

These three, *S. paradoxa, S. africana* and *S. gondwanensi* agree closely with one another in the possession of a multipl nervation, in the small number of leaves, and in the usual unio

* Bain, A. G., "On the Geology of Southern Africa," *Trans. Geol. Soc.* Ser. Vol. VII. 1845–56 (1852), p. 175.

† Seward, A. C., *loc. cit.* p. 89.

Neocalamites, Halle	7. *S. carrerei*, Zeiller	Many, long, probably always free to their bases	Uninerved	Rhaetic	Tonkin and probably S. Africa
	6. *S. hoerensis*, Schimp.	ditto	ditto	ditto	Sweden and Europe generally
	5. *S. meriani*, Brongn.	Fewer than in Nos. 6 and 7	Said to be uninerved. Traces of fine, very close-set, parallel nerves in one specimen	Lettenkohle and Lower part of Keuper?	Central Europe
Schizoneura, Schimper and Mougeot (sensu stricto)	4. *S. paradoxa*, Schimp. and Moug.	Few, broad, united at first into sheaths and later into sheath-segments, sometimes free	Several fine parallel nerves aggregated into the middle of leaf	Bunter and Lower Keuper	Alsace and England
	3. *S. africana*, Feistm.	Fairly broad, about 30 in number. United into several sheath-segments	At least three fine parallel nerves down the centre of each leaf	Beaufort Series (Permian)	Roggeveld (Fish R.), S. Africa
	2. *S. wardi*, Zeiller	About 27 narrow leaves; doubtfully united into sheath	Surface of leaf is uniformly striated lengthwise by very fine and close lines which in the median region become better defined and constitute a more or less distinct median nerve	Karharbari Beds (Lower Gondwana Series)	India
	1. *S. gondwanensis*, Feistm.	Number 10–22. Broadish, usually united into ovate sheath-segments. When only two sheath-segments are present they simulate opposite leaves	Three, or possibly more, distinct fine parallel nerves down centre of leaf	Upper and Lower Gondwana Series Upper Productive Coal Measures Karoo Beds?	India New South Wales Nyasaland

of the leaves into sheath-segments. Their distribution in time is also of interest, for this type of *Schizoneura* appears to have almost died out before the advent of *Neocalamites* in the Lettenkohle and Lower Keuper. It may be that the one type gave rise to the other, but this does not seem likely. In this connection I would like to point out that Halle comments on the resemblance and possible relationship of *Neocalamites* with *Calamites*. In the same way, one may draw a comparison between *Schizoneura* proper and Grand'-Eury's *Calamodendron* type of Calamite. Thus, the whorls of leaves are separated by long internodes and the upper branches probably radiated out in all directions, while the leaves themselves had several nerves instead of a single median one, all of which facts are points of resemblance between the two genera. At the same time we must recognise that, until we know more about the internal structure and fructification of this important transitional group of the Equisetaceae the relations that they bear, both to the Calamites and to the modern Equiseta, must remain a secret.

Conclusion. The species so far described as *Schizoneura* may be divided into two groups. The one has been given the name *Neocalamites* by Halle, and includes *S. carrerei*, *S. hoerensis*, and *S. meriani*. The other may be termed *Schizoneura*, sensu stricto and embraces *S. gondwanensis*, *S. africana* and *S. paradoxa*. Until we have more satisfactory information about *S. wardi* we are unable to decide to which group it belongs.

On Petrified Plant Remains from the Upper Coal Measures of Bristol. By D. G. Lillie, B.A., Hutchinson Student of St John's College. (Communicated by Mr E. A. Newell Arber.)

[*Read* 7 February 1910.]

In working out the flora of the Bristol Coalfield, which consists entirely of the impressions of fossil plants, I was fortunate enough to obtain a small quantity of petrified material. This was entirely unexpected, because no other source of structural material of Upper Coal Measure Age is known in Britain. Our whole knowledge of the anatomy of the plants of this period is entirely founded on Lower Coal Measure material from the Yorkshire and Lancashire Coalfields.

The horizon of the petrified material from the Bristol Coalfield is Upper Coal Measures. This is shown by the associated impressions, an account of which has recently been published elsewhere*. The material was obtained from the base of the Pennant Grit at one locality, Staple Hill, three miles to the north-east of Bristol.

A sinking for coal which penetrated for about 300 feet, was made a short time ago in this locality and was abandoned in the Bristol equivalents of the New Rock Series of Radstock, a short distance below the base of the Grit. On this level a massive conglomerate was passed through, the exact thickness of which does not appear to be known; but the bed was probably of no great thickness. Mr Bolton, F.G.S., Curator of the Bristol Museum, who first examined the sinking, pointed out to me that plant remains occurred, associated with the pebbles, in this conglomerate, and suggested that I should collect the material for examination.

At the time when I visited the locality the sinking had been abandoned for some time and the supply was found to be very limited. It does not appear likely that further material will be obtainable. This peculiar conglomerate has not been recognised elsewhere in the coalfield. I understand that Mr Bolton will publish a full account of this conglomerate, so I need not enter into further details upon the subject in this note.

The plant remains, associated with the pebbles in the sandstone matrix, consist of fragmentary portions of stems and petioles, thoroughly isolated from one another and varying greatly in size and diameter. They are calcified, and the structure is preserved, the preservation being excellent in some cases.

* Lillie, *Geological Magazine*, Dec. 5, 1910, Vol. VII. p. 58.

A large number of them have been isolated from the matrix and sections cut from them. The greater number of the specimens consist of decorticated stems of *Cordaites*, showing the pith and the secondary wood. This genus appears to be particularly abundant in this locality. In addition to *Cordaites*, at least one example of a new member, either of the same or an allied genus, has been obtained. The attribution of most of the plants to the genus *Cordaites* is borne out by the fact that examples of the characteristic pith casts, *Sternbergia* (*Artesia*), of this genus also occur in the conglomerate. In addition, a specimen of a well preserved *Myeloxylon*, the petiole of a *Medullosa*, has also been obtained. A contribution on the detailed structure of these fossils is in progress.

On the assimilating tissues of some Coal Measure Plants. By H. Hamshaw Thomas, B.A., Downing College.

[*Read* 7 February 1910.]

The recent work in Fossil Botany has been concerned almost entirely with morphological and phylogenetic considerations and little attention has been given to the physiological significance of the tissues studied. It is, however, very important from all points of view to study so far as possible the biology and physiology of the fossilised plants. This can be attempted by an examination of our material from the standpoint of physiological anatomy. The process of carbon assimilation is one of the most important of the plant's vital activities and is closely connected with leaf structure. Very little research has hitherto been done on the subject of leaf structure in the Coal Measure period, and at the suggestion of Mr Arber, the author is endeavouring to obtain more information on this subject.

The leaves of some members of the *Calamocladus* section of the Calamites have been studied in detail. They are small linear structures, almost cylindrical in cross-section. The simpler types seldom exceed a length of 3 or 4 millimetres, and were about ·8—1 mm. broad. They were borne in whorls on long slender, probably pendulous, stems. In the longer leaves a considerable portion of the tissue is composed of thick-walled sclerenchymatous elements which replace the mesophyll, forming a strand on the adaxial side and increasing in proportions towards the apex of the leaf. When this strengthening tissue is absent the leaf tissues present a concentric arrangement, the central vascular bundle being surrounded first by the bundle sheath and then by the pallisade tissue.

The mesophyll of the leaf is entirely composed of isolated cylindrical cells, running perpendicularly out from the bundle sheath to the epidermis, and forming a continuous pallisade tissue. These cells are often completely separated from each other by very large air spaces, and the tissue was consequently of an extremely spongy character. Though this system of spaces would provide an ample path for the circulation of carbon dioxide, it would also probably bring about rapid transpiration. In the larger leaves it is noticeable that the air spaces are considerably reduced. There are indications that, as in the pallisade tissue of modern leaves, the cells were specially modified to absorb a maximum of the incident light.

The stomata were confined to the adaxial side of the leaf, which in the living state was probably also the lower side. Here they occur in considerable numbers, one example showing a distribution of about 440 per sq. mm. They were about ·024 mm. long and ·018 mm. broad. In shape, the guard cells resembled very closely those of modern Equisetums, and in one specimen they show also the transverse striations of thickening material, which are so characteristic of the recent plants. The guard cells are usually level with the surface of the epidermis but may be slightly sunk in it. The latter tissue forms a well-marked feature in the leaves.

Towards the centre of the leaf the pallisade cells abut directly on the bundle sheath, and are often somewhat enlarged at the point of contact. This enlargement is a feature sometimes observed in modern plants, and Haberlandt has figured a similar arrangement in the connection between the "Zuleitungsystem" and the "Ableitungsystem" in *Cycas circinalis**. There can be little doubt that the bundle sheath, termed by Hick† "melasmatic tissue," functioned as the path of conduction for the products of assimilation. It is a conspicuous feature in most specimens, because its cells are characterised by the large mass of opaque carbonaceous matter which they contain. No distinct tissue can be recognised which could be called phloem, nor can any sieve tubes be made out. Some leaves show thin-walled elongated parenchymatous cells associated with the xylem, while in others this feature is completely absent. In each leaf, however, the amount of assimilation could not have been large, for the leaves are very small; the products could easily be conducted the short distance to the stem by diffusion through the elongated cells of the bundle sheath.

The cells of the thin-walled inner zone of the cortex of the young stems often contain black contents like those of the bundle sheath, and it may be suggested that these also were connected with the formation of carbohydrates.

It would appear then that in the Calamites the leaves show a structure adapted for carbon assimilation, even though they were very small. They were probably produced in large numbers and were sufficient to provide for the nutrition of the plant.

The leaves of other Coal Measure plants are now being examined. Some Lepidodendrons possess foliar members in which we find a considerable proportion of mesophyll, which may be either of a spongy type composed of rather stellate cells, or in

* Haberlandt, "Verleichende Anatomie des assimilatorischen Gewebesystems der Pflanzen," *Pringsh. Jahrb.* 1882, Taf. VI. fig. 15.

† Hick, "On the structure of the leaves of Calamites," *Mem. and Proc. Manchester Lit. and Phil. Soc.* Vol. IX. 1895, p. 179.

other cases may bear considerable resemblance to the mesophyll of the Calamites. Tissue of the latter type has also been described by Renault* in *L. esnostense*, where it has the form of chains of cells with numerous interspaces and united by short transverse branches.

In material from Shore, Littleborough, Lancashire, leaves have been found which are probably identical with a type described by Felix† from Westphalia some time ago. There is no record of their previous discovery in this country, and as yet their affinities are uncertain. It seems probable that they belong to some species of *Lepidophloios*. They are about 6 mm. across, and fragments over 3 cms. long occur. In these the mesophyll appears differentiated into spongy tissue and pallisade cells, and thus more closely resembles that of the higher plants.

In so far as the present work has progressed there would appear to have been considerable variety in the assimilating systems of the different members of the Carboniferous flora, and these may be closely compared with those found in living plants.

* Renault, *Flore fossile d'Autun et d'Epinac*, Paris, 1896, p. 179.
† Felix, J., *Westphalischen Carbon Pflanzen*, Berlin, 1886, Taf. v. fig. 6.

The production of Cathode Particles by Homogeneous Röntgen Radiations. By R. T. Beatty, M.A., B.E., Emmanuel College. (Communicated by Prof. Sir J. J. Thomson.)

[*Read* 21 February 1910.]

Several physicists have investigated the cathode particles produced when Röntgen radiations fall upon various substances. The recent work of Cooksey* and Innes† has shown that the velocities of these cathode particles are independent of variations in the intensity of the Röntgen radiations used, and are also independent of the nature of the substance struck, but that the velocities increase or decrease with an increase or decrease in the penetrating power of the exciting Röntgen radiations.

The discovery of homogeneous Röntgen radiations, emitted by certain metals when exposed to suitable Röntgen radiations, enables one to use definite beams which differ widely in penetrating power, and it seemed that by using such beams more definite information might be gained about the cathode particles emitted from metals placed in the path of such beams.

When a very thin silver leaf was placed in the path of the homogeneous radiations described above, cathode particles shot out from its surface. It was determined to investigate the coefficients of absorption of these cathode particles in air and in hydrogen. Hydrogen was chosen on account of its anomalous behaviour with regard to ionisation phenomena.

The radiations from a Röntgen bulb, proceeding in a direction normal to the plane of the paper (fig. 1) fell upon the metal which acted as radiator. Plates of Fe, Ni, Cu, Zn, As, Sn were used as radiators.

A homogeneous radiation then proceeded from the radiator. Part of this entered the cylindrical brass chamber A through the thin parchment window. It then passed through the silver leaf, and was finally totally absorbed in the thick brass disc DD which served as electrode. DD was covered with paper to prevent the emission of cathode particles from its surface.

Another portion of the radiation entered the primary electroscope (fig. 1), which served to standardise the amount of homogeneous radiation emitted by the radiator.

As the quality of the homogeneous radiation is unaffected by variations in the bulb, and as the quantity of radiation entering the chamber A is always the same fraction of that entering P

* Cooksey, *Am. Jour. Sci.*, iv. 24, 1907, p. 285.
† Innes, *Proc. Roy. Soc.*, Ser. A, lxxix., Aug. 2, 1907, pp. 442—462.

(for a given radiator), no discordance in the results can arise from variations in the bulb.

The electrode D was parallel to L and distant 1 cm. from it. D was connected to the gold leaf of the secondary electroscope, and could be earthed by a key. The chamber A was kept at a potential of 200 volts.

When the bulb was in action the air between D and L was ionised. Thus a charge was given to D, as soon as the connection to the earth was broken. The method of experimenting was to vary the pressure of the air inside A, and to measure the ionisation between D and L corresponding to each pressure. The amount of this ionisation was in each case standardised by means of the primary electroscope.

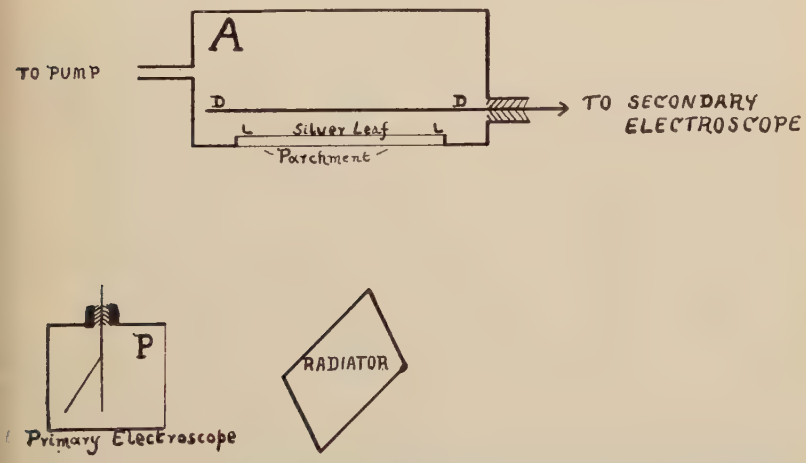


Fig. 1.

This ionisation is due to two causes: (1) Ionisation due to X-radiation alone. This has been shown by Crowther* to vary directly as the pressure of the air. (2) Ionisation due to cathode particles emerging from L. The amount of ionisation due to this source will remain constant as long as the pressure is great enough to absorb all the particles. When the pressure falls below a certain value some of the particles will reach D before being absorbed and the ionisation will decrease.

Fig. 2 shows how the ionisation due to each of these sources varies with the pressure, and how the actual curve found is the sum of these separate effects.

* Crowther, *Proc. Roy. Soc.*, Ser. A, LXXXII., March 10, 1909, pp. 103—127.

Hence given the actual curve we find the part due to cathode ionisation by drawing through the origin a line parallel to the straight portion of the curve, and subtracting its ordinates from those of the actual curve. This process gives the curve showing the increase of cathode ionisation up to a certain pressure.

The pressure at which the ordinate of the cathode curve is half the length of the maximum ordinate gives the pressure at which half the cathode particles starting from L reach D.

Knowing the distance between D and L (1 cm.), the temperature of the room, and this critical pressure, we can now calculate the thickness of the layer of air at 170 mm. pressure and 15° C. temperature, which would absorb one-half of the energy of the cathode particles starting from L (Table I, column 1).

TABLE I. *Air.*

Radiator	d in cms.	λ in cm.$^{-1}$	Cathode energy emerging ÷ absorption of X-radiation in leaf	Total cathode energy produced in leaf ÷ absorption of X-radiation
Fe	·00804	87·2	·346	30·2
Cu	·01349	51·9	·512	26·0
Zn	·0164	42·7	·575	24·5
As	·0255	27·43	1·120	30·8
Sn	·176	3·97	6·470	25·7

If we assume that $Ce^{-\lambda x}$ represents the amount of energy which gets through a thickness x (in cms.) of air at normal pressure and temperature we can now calculate λ, the coefficient of absorption of the cathode particles by air. For when

$$x = d, \quad e^{-\lambda d} = \tfrac{1}{2}; \quad \therefore \lambda = \frac{\cdot 7}{d}.$$

The values of λ are given in column 2.

Further, since the experimental curves can be split up as shown in fig. 2, we can determine the ratio of the ionisation caused by the emerging cathode particles to the ionisation due to the X-radiation in the layer of air between D and L. For the sake of uniformity we shall measure this X-ionisation when the air is at a pressure of 760 mm.

This ratio has really a very simple meaning. Barkla and Sadler* have shown that if we take any two substances which do not give a homogeneous radiation under the stimulus of a

* Barkla and Sadler, *Phil. Mag.*, May 1909, p. 751.

certain series of X-radiations, then the ratio of their coefficients of absorption for this series of X-radiations remains constant for each one of this series.

Since silver* and air† exhibit no homogeneous radiation when stimulated by the range of radiations here employed, we may assume that their coefficients of absorption have a constant ratio for each X-radiation.

We shall further assume that the ionisation in air is proportional to the coefficient of absorption of air.

Hence ionisation of air $\propto$ absorption by air of radiation absorption by air $\propto$ absorption by silver.

$\therefore$ ionisation of air $\propto$ absorption by silver.

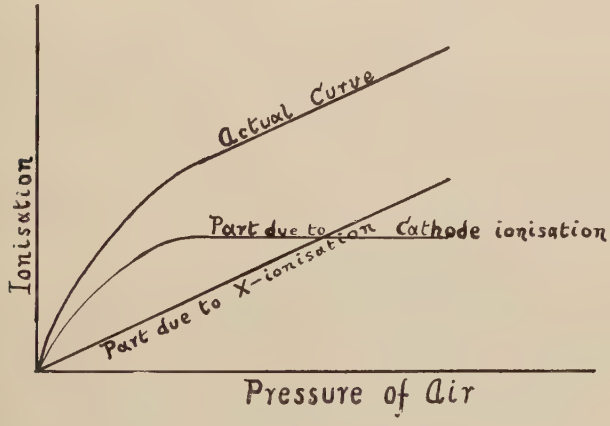


Fig. 2.

Hence the ratio (Table I, column 3) is a measure in arbitrary units of the ratio of the energy of cathode particles emerging from the silver leaf to the amount of energy absorbed by the leaf from the X-radiation.

From the numbers in Table I, column 3, we can attempt to calculate the total energy due to cathode particles produced in the whole volume of the leaf when the leaf absorbs a fixed quantity of radiation.

Two assumptions must be made:

(1) That each cathode particle moves initially in the direction of the incident X-radiation.

(2) That these particles are absorbed according to an exponential law in the silver leaf itself.

These assumptions have not been definitely proved, but there

* *Ibid., loc. cit.*

† Beatty, *Phil. Mag.*, Nov. 1907, pp. 604—614.

is evidence in favour of both of them, and they are the simplest that can be made.

Now if I be the amount of ionisation in air due to the cathode particles which emerge from the leaf, and λ the coefficient of absorption of these particles in silver, then a simple integration shows that λI is the amount of ionisation which would be caused if all the cathode particles set free in unit thickness of the silver were able to spend their whole energy in ionising air instead of being absorbed by the leaf.

Since for any set of cathode particles the absorption in air is proportional to the absorption in any other substance*, we may use the former value instead of the latter, if we are not compelled to use absolute values.

Hence if we multiply the numbers in Table I, column 3, by the corresponding values of λ in column 2, we get numbers giving the amounts of cathode energy liberated by different kinds of X-radiations when they are equally absorbed in silver. The results are given in column 4.

These numbers are seen to be of the same order of magnitude though the numbers in column 3 vary widely. No closer agreement is to be expected considering the three assumptions which have been made in getting this result.

Hence we are justified in concluding that *the amount of energy spent by the incident radiation in producing cathode particles is proportional to the energy of that radiation which is absorbed by the leaf.*

The experiments were now repeated, using pure hydrogen instead of air in the chamber A. The results are given in Table II.

TABLE II. *Hydrogen.*

Radiator	d in cms.	λ in cm.$^{-1}$	$\frac{\text{Range in } H_2}{\text{Range in air}}$	$\frac{\text{Total cathode ionisation in } H_2}{\text{Total cathode ionisation in air}}$
Fe	·0410	17·05	5·12	1·01
Cu	·0733	9·55	5·44	·99
Zn	·0909	7·71	5·54	·98
Sn	1·37	·51	7·79	1·00

The first column gives the thickness in cms. of the layer of hydrogen at 760 mm. pressure and 15° C. required to cut down the energy of the cathode particles to half its value.

* Lenard, *Wied. Ann.*, LVI., 1895, p. 255.

Column 2 gives the coefficients of absorption by hydrogen.

Column 3 is obtained by dividing the values of λ given in Table I by the corresponding values in Table II. It will be seen that *the ratio increases as the cathode particles become more penetrating.*

Column 4 gives the ratio of the total ionisation produced by a given bundle of cathode particles in hydrogen to that produced by the same bundle in air. *The ratio is unity within the limits of experimental error.*

In Table III are added some data previously found for cathode rays. It will be seen that the constants relating to the cathode

TABLE III.

Velocity of corpuscles expressed as due to a potential fall in volts	λ for air	λ for hydrogen	Range in H_2 / Range in air	Authority
4,000	645	144	4·48	Lenard
20,000	31	—	—	Seitz
40,000	3·8	·47	8·05	Lenard

particles due to the Sn radiation are very close to those found by Lenard for corpuscles possessing a velocity due to a drop of potential of 30,000 volts.

Relation between the absorption of the cathode particles by air and the absorption of the existing X-radiations by aluminium.

This relation was found to be linear, and is shown in fig. 3. The abscissae are taken from figures due to Barkla and Sadler*, with the exception of that relating to Sn, for which radiation it was found by the author that $\frac{\lambda}{\rho} = 1{\cdot}65$, the absorber being aluminium.

Relative ionisation in air and hydrogen due to homogeneous X-radiations.

The ionisation in hydrogen due to soft X-radiations is so small that the straight portion of the typical curve shown in fig. 2 was almost horizontal. A separate set of experiments was made to determine this ionisation accurately. The silver leaf was removed from the chamber A and the plate electrode replaced by an aluminium wire bent into a ring. The ionisation observed in the chamber when filled with hydrogen and air respectively was then determined. The results are given in Table IV.

* *loc. cit.*

TABLE IV.

Radiator	Ionisation in hydrogen / Ionisation in air
Fe	$\frac{1}{175}$
Cu	$\frac{1}{175}$
Zn	$\frac{1}{175}$
As	$\frac{1}{175}$
Sn	$\frac{1}{95}$

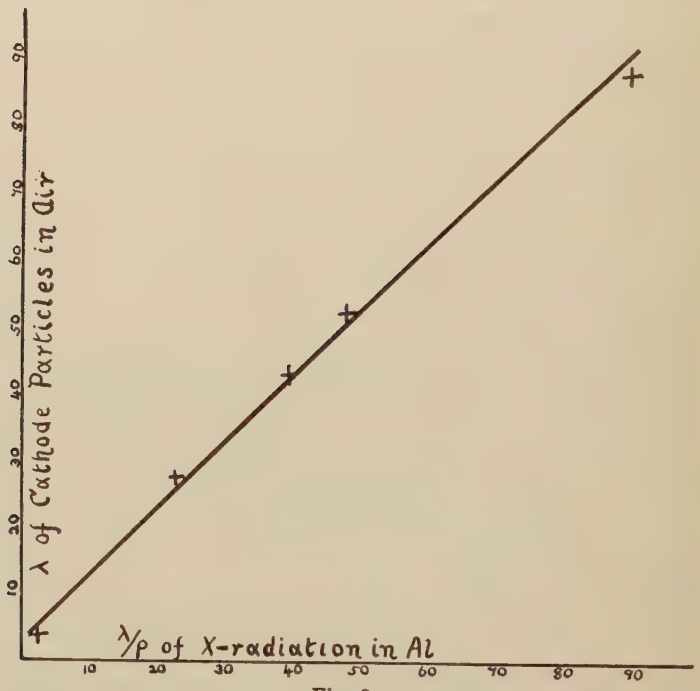


Fig. 3.

NOTE. In a recent communication to *Nature* (October 28, 1909) Sadler has determined various relations among the cathode particles excited by homogeneous Röntgen radiations. His results do not in all cases agree with the relations here described.

I have much pleasure in thanking Sir J. J. Thomson for his interest in this investigation.

The solution of a system of differential equations occurring in the theory of radio-active transformations. By H. Bateman, M.A., Trinity College.

[*Read* 21 February 1910.]

1. It has been shown by Prof. Rutherford * that the amounts of the primary substance and the different products in a given quantity of radio-active matter vary according to the system of differential equations,

$$\left.\begin{aligned} \frac{dP}{dt} &= -\lambda_1 P \\ \frac{dQ}{dt} &= \lambda_1 P - \lambda_2 Q \\ \frac{dR}{dt} &= \lambda_2 Q - \lambda_3 R \\ \frac{dS}{dt} &= \lambda_3 R - \lambda_4 T \\ &\cdots\cdots\cdots\cdots \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (1),$$

where $P, Q, R, S, T, \ldots$ denote the number of atoms of the primary substance and successive products which are present at time t.

Prof. Rutherford has worked out the various cases in which there are only two products in addition to the primary substance, and it looks at first sight as if the results may be extended to any number of products without much labour.

Unfortunately the straightforward method is unsymmetrical and laborious, and as the results of the calculations are needed in some of the researches which are being carried on in radio-activity the author has thought it worth while to publish a simple and symmetrical method of obtaining the required formulae.

2. Let us introduce a set of auxiliary quantities $p(x)$, $q(x)$, $r(x)$, ... depending on a variable x and connected with the quantities $P(t)$, $Q(t)$, $R(t)$, ... by the equations,

$$p(x) = \int_0^\infty e^{-xt} P(t)\, dt, \quad q(x) = \int_0^\infty e^{-xt} Q(t)\, dt \ldots\ldots (2).$$

It is easily seen that

$$\int_0^\infty e^{-xt} \frac{dP}{dt}\, dt = -P(0) + x \int_0^\infty e^{-xt} P(t)\, dt \cdots\cdots (3),$$
$$= -P_0 + xp,$$

* *Radio-activity*, 2nd edition, p. 331.

where p is written for $p(x)$, and P_0 for $P(0)$, the initial value of $P(t)$.

Multiplying equations (1) by e^{-xt}, and integrating from 0 to ∞ with regard to t we obtain the system of equations

$$\left.\begin{aligned} xp - P_0 &= -\lambda_1 p \\ xq - Q_0 &= \lambda_1 p - \lambda_2 q \\ xr - R_0 &= \lambda_2 q - \lambda_3 r \\ xs - S_0 &= \lambda_3 r - \lambda_4 s \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4),$$

from which the values of p, q, r may be obtained at once.

If $Q_0 = R_0 = S_0 = \ldots = 0$, i.e. if there is only one substance present initially, we have

$$p = \frac{P_0}{x+\lambda_1}, \quad q = \frac{\lambda_1 P_0}{(x+\lambda_1)(x+\lambda_2)}, \quad r = \frac{\lambda_1\lambda_2 P_0}{(x+\lambda_1)(x+\lambda_2)(x+\lambda_3)},$$

and for the nth product

$$\nu(x) = \frac{\lambda_1\lambda_2 \ldots \lambda_{n-1} P_0}{(x+\lambda_1)(x+\lambda_2)\ldots(x+\lambda_n)} \quad \ldots\ldots\ldots\ldots (5).$$

Putting this into partial fractions, we have

$$\nu(x) = \frac{c_1}{x+\lambda_1} + \frac{c_2}{x+\lambda_2} + \ldots \frac{c_n}{x+\lambda_n},$$

where

$$\left.\begin{aligned} c_1 &= \frac{\lambda_1\lambda_2 \ldots \lambda_{n-1} P_0}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)\ldots(\lambda_n-\lambda_1)} \\ c_2 &= \frac{\lambda_1\lambda_2 \ldots \lambda_{n-1} P_0}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)\ldots(\lambda_n-\lambda_2)} \\ &\ldots\ldots \text{ etc.} \end{aligned}\right\} \quad \ldots\ldots\ldots (6).$$

To obtain the corresponding function $N(t)$ we must solve the integral equation

$$\nu(x) = \int_0^\infty e^{-xt} N(t)\, dt.$$

Now it has been shown by Lerch* that there is only *one continuous function* $N(t)$ which will yield a given function $\nu(x)$; hence if we can find a function which satisfies this condition it will be the solution of our problem. It is clear, however, that

$$\frac{1}{x+\lambda} = \int_0^\infty e^{-xt} \cdot e^{-\lambda t}\, dt;$$

hence the above value of $\nu(x)$ is obtained by taking

$$N(t) = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} + \ldots c_n e^{-\lambda_n t} \quad \ldots\ldots\ldots (7),$$

where the constants have the values given by (6).

* *Acta Mathematica*, 1903, p. 339.

In the case when $Q(0)$, $R(0)$, ... are not zero, we have

$$\left.\begin{aligned} p &= \frac{P_0}{x+\lambda_1}, \quad q = \frac{\lambda_1 P_0}{(x+\lambda_1)(x+\lambda_2)} + \frac{Q_0}{x+\lambda_2} \\ r &= \frac{\lambda_1\lambda_2 P_0}{(x+\lambda_1)(x+\lambda_2)(x+\lambda_3)} + \frac{\lambda_2 Q_0}{(x+\lambda_2)(x+\lambda_3)} + \frac{R_0}{(x+\lambda_3)} \\ &\text{etc.} \end{aligned}\right\} \quad ...(8),$$

and we may obtain the values of P, Q, R by expressing these quantities in partial fractions as before.

The complete solution for the case of a primary substance P and three products Q, R, S is

$$P = P_0 e^{-\lambda_1 t}, \quad Q = \frac{\lambda_1}{\lambda_2 - \lambda_1} P_0 e^{-\lambda_1 t} + \left(\frac{\lambda_1 P_0}{\lambda_1 - \lambda_2} + Q_0\right) e^{-\lambda_2 t},$$

$$R = \frac{\lambda_1\lambda_2 P_0}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)} e^{-\lambda_1 t} + \left[\frac{\lambda_1\lambda_2 P_0}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)} + \frac{\lambda_2 Q_0}{\lambda_3-\lambda_2}\right] e^{-\lambda_2 t} + \left[\frac{\lambda_1\lambda_2 P_0}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)} + \frac{\lambda_2 Q_0}{\lambda_2-\lambda_3} + R_0\right] e^{-\lambda_3 t},$$

$$\begin{aligned} S = {} & \frac{\lambda_1\lambda_2\lambda_3 P_0}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)} e^{-\lambda_1 t} \\ & + \left[\frac{\lambda_1\lambda_2\lambda_3 P_0}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)(\lambda_4-\lambda_2)} + \frac{\lambda_2\lambda_3 Q_0}{(\lambda_3-\lambda_2)(\lambda_4-\lambda_2)}\right] e^{-\lambda_2 t} \\ & + \left[\frac{\lambda_1\lambda_2\lambda_3 P_0}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)} + \frac{\lambda_2\lambda_3 Q_0}{(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)} + \frac{\lambda_3 R_0}{\lambda_4-\lambda_3}\right] e^{-\lambda_3 t} \\ & + \left[\frac{\lambda_1\lambda_2\lambda_3 P_0}{(\lambda_1-\lambda_4)(\lambda_2-\lambda_4)(\lambda_3-\lambda_4)} + \frac{\lambda_2\lambda_3 Q_0}{(\lambda_2-\lambda_4)(\lambda_3-\lambda_4)} + \frac{\lambda_3 R_0}{\lambda_3-\lambda_4} + S_0\right] e^{-\lambda_4 t}. \end{aligned}$$

The solution may evidently be obtained by superposing the solutions of the cases in which the initial values of P, Q, R, S are given by

(1) $P(0) = P_0$, $Q(0) = 0$, $R(0) = 0$, $S(0) = 0$.
(2) $P(0) = 0$, $Q(0) = Q_0$, $R(0) = 0$, $S(0) = 0$.
(3) $P(0) = 0$, $Q(0) = 0$, $R(0) = R_0$, $S(0) = 0$.
(4) $P(0) = 0$, $Q(0) = 0$, $R(0) = 0$, $S(0) = S_0$.

The method is perfectly general, and the corresponding formulae for the case of $n-1$ products may be written down at once by using (6).

The general formula covers all the four cases (*Radio-activity*, pp. 331—337). For instance in Case 2 when initially there is radio-active equilibrium, we have

$$n_0 = \lambda_1 P_0 = \lambda_2 Q_0 = \lambda_3 R_0 = \lambda_4 S_0.$$

The solutions are then

$$P = \frac{n_0}{\lambda_1} e^{-\lambda_1 t}, \quad Q = \frac{n_0}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} + \frac{n_0 \lambda_1}{\lambda_2 (\lambda_1 - \lambda_2)} e^{-\lambda_2 t},$$

$$R = \frac{\lambda_2 n_0}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)} e^{-\lambda_1 t} + \frac{\lambda_1 n_0}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)} e^{-\lambda_2 t} + \frac{\lambda_1 \lambda_2 n_0}{\lambda_3 (\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} e^{-\lambda_3 t}$$

$$S = \frac{\lambda_2 \lambda_3 n_0}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)(\lambda_4 - \lambda_1)} e^{-\lambda_1 t} + \frac{\lambda_1 \lambda_3 n_0}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)(\lambda_4 - \lambda_2)} e^{-\lambda_2 t} + \frac{\lambda_1 \lambda_2 n_0}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)(\lambda_4 - \lambda_3)} e^{-\lambda_3 t} + \frac{\lambda_1 \lambda_2 \lambda_3 n_0}{(\lambda_1 - \lambda_4)(\lambda_2 - \lambda_4)(\lambda_3 - \lambda_4)} e^{-\lambda_4 t}.$$

The solution for the case of $n-1$ products is given by

$$N = \Sigma c_r e^{-\lambda_r t},$$

where the constants c_r are obtained by expressing

$$\frac{\lambda_1 \lambda_2 \lambda_3 \ldots \lambda_{n-1}}{x (x + \lambda_1)(x + \lambda_2) \ldots (x + \lambda_n)}$$

in partial fractions.

The method by which the solution of the system of differential equations has been obtained is really of very wide application and may be employed to solve problems depending on a partial differential equation of the form

$$\frac{\partial V}{\partial t} = F\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \ldots\right) V,$$

provided the initial value of V is known.

For if we put

$$u(s) = \int_0^\infty e^{-st} V(t)\, dt,$$

$$su(s) - V_0 = \int_0^\infty e^{-st} \frac{\partial V}{\partial t}\, dt,$$

it appears that $u(s)$ satisfies the partial differential equation

$$F\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \ldots\right) U + su + V_0 = 0 \quad \ldots\ldots\ldots(9).$$

Further, if V satisfies some linear boundary condition which is independent of t the function u will generally satisfy the same boundary condition. This function (u) must be obtained from the

differential equation (10) which is simpler than (9), inasmuch as it depends upon fewer independent variables.

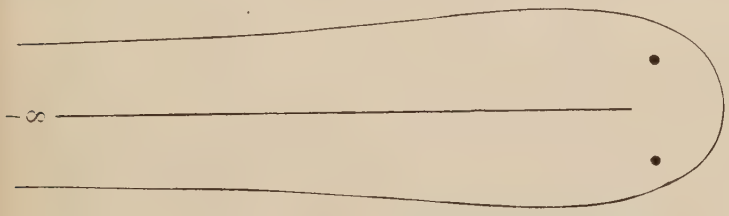
In many cases the solution of the integral equation

$$u(s) = \int_0^\infty e^{-st} V(t)\, dt$$

may be calculated by means of the inversion formula *

$$V(t) = \frac{1}{2\pi i} \int_c e^{t\zeta} u(\zeta)\, d\zeta,$$

where c is a contour which starts at $-\infty$ at a point below the real axis, surrounds all the singularities of the function $u(\zeta)$ and returns to $-\infty$ at a point above the real axis, as in the figure.



The conditions to be satisfied by $u(\zeta)$ in order that this inversion formula may be applicable have not yet been expressed in a concise form.

The formula may be used to obtain the solution of a problem in the conduction of heat when we require a solution of

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t},$$

which satisfies the boundary conditions

$$V = 0 \text{ when } x = 0 \text{ and } x = a,$$

$$V = f(x) \text{ when } t = 0.$$

The solution found in this way is identical with the one given in Carslaw's *Fourier's Series and Integrals*, p. 383.

* A particular case of this formula has been given by Pincherle, *Bologna Memoirs*, 10 (8), 1887.

On double-sixes. By W. Burnside, M.A., F.R.S.

[*Received* 7 February 1910.]

The configuration of twelve lines, known as a double-six, arises naturally in connection with the theory of cubic surfaces. Its connection with the theory of quadric surfaces has not, so far as I know, been investigated. It is the object of the present note to establish the existence of a double-six from what I believe to be a fresh point of view, connecting it with a pair of quadric surfaces that stand in a special projective relationship to each other. A word of explanation must be given of the digression in § 3. The theory of quadrato-quadratic equations has been very completely worked out; but the particular relation here required could not be quoted in a convenient form. It has therefore been established directly.

1. Let S, S' be two quadrics. On each choose one of the two sets of generators: and let λ be the parameter that distinguishes one of the chosen set on S, and μ the corresponding quantity for S'. A generator of S meets S' in two points, through each of which will pass one generator of the chosen set on S'. In other words, if λ, μ are parameters of two intersecting generators on S and S', they are connected by an equation which is quadratic in both λ and μ. Denote this equation by

$$f(\lambda, \mu) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}.$$

Let μ_{-1}, μ_0 be the roots of this equation when λ is λ_0; λ_0, λ_1 the roots when μ is μ_0, λ_{-1}, λ_0 the roots when μ is μ_{-1}; and so on. Then there arises an, in general, unending series of quantities

$$\ldots\lambda_{-2}\mu_{-2}\lambda_{-1}\mu_{-1}\lambda_0\mu_0\lambda_1\mu_1\lambda_2\ldots \quad \ldots\ldots\ldots\ldots\text{(ii)},$$

all of which are rationally determinable in terms of any two consecutive ones. Corresponding to this series there is an, in general, open polygon whose sides belong alternately to the chosen generators on S and the chosen generators on S'; and no side of the polygon meets any other except the two which immediately precede and follow it. The theory of an equation, quadratic in both variables, such as (i) is well known. Starting from an arbitrary pair of values that satisfy it, the series of values (ii) is in general unending, but if it is periodic for some chosen initial value of λ (or μ), i.e. if it is of the form

$$\ldots\lambda_n\mu_n\lambda_0\mu_0\lambda_1\mu_1\ldots\lambda_n\mu_n\ldots,$$

then it is periodic for every initial value. Suppose now that the quadrics S, S' are such that the polygon is a (gauche)

hexagon. Then the existence of a double-six implies that when, with the remaining set of generators on S and the remaining set on S' the same construction is carried out, the polygon is again in this case a hexagon. Conversely, a proof that this latter polygon is a hexagon establishes the existence of a double-six.

2. Any two arbitrary quadrics can, by a suitable projection (which may be imaginary), be brought to the forms

$$x^2 - y^2 - a^2z^2 + b^2t^2 = 0,$$
$$c^2x^2 - d^2y^2 - z^2 + t^2 = 0.$$

A generator of a chosen system from each is given by

$$x + y + \lambda(az + bt) = 0;$$
$$\lambda(x - y) + az - bt = 0;$$

and

$$cx + dy + \mu(z + t) = 0;$$
$$\mu(cx - dy) + z - t = 0.$$

The condition that these should meet is easily found to be

$$\lambda^2\mu^2 + A(\lambda^2 + \mu^2) + B\lambda\mu + 1 = 0,$$

when $$A = \frac{(a+b)(c+d)}{(a-b)(c-d)}, \quad B = -4\frac{abcd+1}{(a-b)(c-d)}.$$

The equations to the two other sets of generators, and the corresponding conditions for them are obtained by writing $-b$ and $-d$ for b and d. It is to be noticed that writing $-\lambda$ for λ merely changes the sign of B.

3. The tangent at $\left(r\frac{1-t^2}{1+t^2},\ r\frac{2t}{1+t^2}\right)$ to $x^2+y^2=r^2$ is

$$x(1-t^2) + y\,2t = r(1+t^2).$$

If this meets $(x+a)^2 + y^2 = R^2$ in the point

$$\left(-a + R\frac{1-t'^2}{1+t'^2},\ R\frac{2t'}{1+t'^2}\right),$$

then $\{(R-a) - (R+a)t'^2\}(1-t^2) + 4Rtt' = r(1+t^2)(1+t'^2)$.

Put $$t = ms, \quad t' = ns';$$

then $$m^2n^2(R-r+a)s^2s'^2 - m^2(R+r-a)s^2 - n^2(R+r+a)s'^2 + 4Rmnss' + R - r - a = 0.$$

If $$m^2n^2(R-r+a) = R - r - a,$$

and $$m^2(R+r-a) = n^2(R+r+a),$$

the equation is

$$s^2s'^2 + A(s^2 + s'^2) + Bss' + 1 = 0,$$

where $$A^2 = \frac{(R+r)^2 - a^2}{(R-r)^2 - a^2}, \quad B^2 = \frac{16R^2}{(R-r)^2 - a^2}.$$

The equation connecting λ, μ is the same as that connecting s, s'. Hence if the polygon whose sides lie on the two quadrics is a hexagon, the circles must be such that the sides of a triangle inscribed in the second circle touch the first. The conditions for this are either

$$\text{(i) } a^2 = R^2 - 2Rr, \text{ or (ii) } a^2 = R^2 + 2Rr.$$

In the first case,

$$A^2 = \frac{4Rr + r^2}{r^2}, \quad B^2 = \frac{16R^2}{r^2},$$

giving

$$A^2 - 1 = \pm B.$$

In the second case,

$$A^2 = \frac{r^2}{r^2 - 4Rr}, \quad B^2 = \frac{16R^2}{r^2 - 4Rr},$$

giving

$$A^2 - 1 = \pm AB.$$

4. If

$$\frac{(a+b)(c+d)}{(a-b)(c-d)}, \; 4\frac{abcd+1}{(a-b)(c-d)}$$

satisfy one of these relations, then obviously

$$\frac{(a-b)(c-d)}{(a+b)(c+d)}, \; 4\frac{abcd+1}{(a+b)(c+d)}$$

satisfy the other. In other words, if for a chosen pair of sets of generators of the quadrics the polygon is a hexagon, then it is also a hexagon for the other two sets. As pointed out above, this is equivalent to establishing the existence of a double-six.

5. If the two quadrics are taken in the usual canonical form

$$ax^2 + by^2 + cz^2 + dt^2 = 0,$$
$$a'x^2 + b'y^2 + c'z^2 + d't^2 = 0,$$

it will be found that the above condition

$$A^2 - 1 = \pm B,$$

which ensures that a double-six lies on them, may be expressed irrationally in the form

$$\frac{1}{\sqrt{abc'd'} + \sqrt{a'b'cd}} + \frac{1}{\sqrt{acb'd'} + \sqrt{a'c'bd}} + \frac{1}{\sqrt{adb'c'} + \sqrt{a'd'bc}} = 0.$$

On the Procession and Pupation of the Larva of Cnethocampa pinivora. By T. G. Edwards, B.A., Emmanuel College. (Communicated by Mr H. H. Brindley.)

[*Read* 21 February 1910.]

The following note is a summary of certain observations made during a fortnight spent at Arcachon, between the 18th and 31st of March, 1909, a visit which I undertook at the suggestion of Mr Brindley with a view to clearing up certain points with regard to the processional larva of *Cnethocampa pinivora*.

The habits of this peculiar larva were first studied by Réaumur in 1736, who treated the subject in considerable detail; but as his material was all conveyed by coach from Bordeaux to his house at Paris, the conditions under which his experiments were performed were necessarily somewhat artificial. Ratzeburg also described the life history, while in the last fifteen years Fabre has recorded his observations on all stages of this moth, confirming many of Réaumur's results and adding much to his account of this insect. Lastly, a paper by Mr Brindley appeared in the *Proceedings* of this Society for 1906. In this was given a detailed account of the behaviour of a certain procession which came under the writer's observation, together with certain other points which supplemented Fabre's description.

On warm days in March and April the processions of *Cnethocampa pinivora* are frequently to be observed in the neighbourhood of the Pine Woods of the Landes, where their nests form prominent objects among the branches.

The procession is one of single file, the larvae being arranged in head to tail contact. The whole moves along a silken thread which is commenced by the leader and added to by all the larvae in succession.

The number comprising a procession varies greatly, and though I saw many processions, I only encountered three which exceeded one hundred in number. The largest of these was crushed upon the pavement of a road near the woods, but it measured 26 ft. and must have contained at least 260 larvae.

Nature of the Procession.

(*a*) *Primite.* Several experiments were made with a view to determining how far the primite may be regarded as the true leader. It was found—as Fabre states—that any larva could function as primite, and that all individuals in the procession

were alike in this respect. Yet—as will be seen later—it was found that a particular larva, when once entrusted with this position, usually retained it. On one occasion, however, I saw a voluntary change of leader, the primite detaching himself and becoming inserted at the fifth place. It seemed probable, too, that the primite was capable of taking a real initiative in certain cases, the remainder following him whether influenced by the same stimuli or not. This was shown to be the case in

(1) The selection of a path.
(2) Burrowing for pupation.
(3) Forming of a "circulating mass." (This term is explained later.)

In the first case at least two external conditions seem to influence his choice, viz. light and surface. This was suggested in the case of a small procession of seven which came under my observation. I threw the whole procession into the shade and determined the path taken by the primite by causing a reflected ray of light to be cast beneath his head. The procession followed the reflected ray, although only the primite could have been influenced by this stimulus. In the case of processions which burrowed for pupation the primite was frequently observed to test the ground with his mandibles before a circulating mass—the formation always adopted before burying—was formed.

The term "circulating mass" is used in Mr Brindley's paper to denote the assemblage of larvae moving among each other; but the assemblage, as a whole, remaining on the same spot.

In the formation of the circulating mass, the primite seemed to be taking the initiative by turning his body sharply round into a position parallel to that of the second: and if a procession was reformed without pupation the same larva usually took the lead. Thus it appears that though any larva can take up the duties of the primite, yet the primite does in a real sense lead the procession, the satellites following whether influenced by identical stimuli or not. At nightfall—according to Fabre—the procession usually returns to the nest, the primite wandering round until he strikes the outward-bound thread. I never observed this proceeding, but if Fabre be correct it would seem that the primite is capable of exercising choice in this respect. In all the cases I examined the leader refused to walk along any foreign thread—whether artificial (e.g. silk thread frayed out) or that of another procession—which I placed in his path.

In addition to light and surface, the desire to pupate and temperature might be suggested as factors influencing the primite in his movements: and—as Fabre suggests—the occurrence of processions at all is dependent upon atmospheric conditions; for in bad weather the larvae remain in their nests.

(*b*) *The function of the Thread in the Procession.* In the case of a small procession, with the aid of a lens it was easy to see that each larva secreted its own thread, which was passed between the pro-legs of the terminal segment. When the path of the procession became irregular the multiple nature of the thread was occasionally observed.

In the case of a large procession of 158 the thread became very thick, in some places a band of silk some 2 or 3 mm. in thickness being formed. In this case again the multiple nature of the thread was clearly discernible. In spite of this it does not seem that it is the thread which guides a satellite in procession so much as the tail of the larva in front. The following observation supports this view.

Two sections of a procession—each consisting of seven larvae—were progressing in the same direction, separated by about 18 inches: the second eventually caught up the first and joined it, though it did not at any time follow the silk thread laid down by the latter. On the contrary that thread was crossed several times, and joining up only occurred when the primite of the second in his wanderings accidentally struck the sixth larva of the first procession; when he stopped immediately until the last larva had passed and joined on in the right place.

The evidence of artificial breaks is of interest in this connection. It appeared in all cases examined that touch rather than sight was the chief guide to the satellites. When larvae were removed from the middle of a procession, together with the thread beneath them, joining-up always occurred if the leader of the hinder portion actually came into contact with any part of a larva in front. If the thread were left, joining-up was more frequent, but if the break were small it occurred whether the thread were removed or not.

These observations tend to show that normally the chief factor in keeping a procession together is the head-to-tail contact of the larvae. The thread is of relatively small importance in this connection, and serves rather as a guide to the primite on his return journey to the nest. But the fact that joining-up occurs more readily in the case of small breaks, when the thread is not removed, would seem to indicate that the touch of the thread may under certain conditions act as a guiding influence to the satellites.

Method of Formation of the "Circulating Mass."

When on the march, processions were frequently observed to form "circulating masses." This formation is usually adopted

immediately before burrowing for pupation: but at other times it may result from a variety of causes, amongst which may be mentioned fatigue and external interference, and sometimes for no apparent reason at all. Fabre suggests cold and darkness as influences.

When I caused a small procession to circulate in a closed orbit, a circulating mass was formed after $2\frac{1}{2}$ revolutions by the gradual diminishing of the diameter of the circle, a process which tended to crowd out some of the larvae. From this mass a procession was subsequently reformed. When formed from a straight procession the method of procedure is somewhat complicated. The primite appears to start the formation by assuming a zig-zag mode of progression, which is followed for a time by the satellites. The arrangement soon becomes obscure and difficult to follow, but there are certain noteworthy features in the behaviour of the satellites.

On entering the mass the larvae do not come to rest, but continue to move slowly and in a very characteristic way, all the time moving their heads rapidly from side to side and depositing their threads. If the procession be a large one, the satellites entering into the newly-forming mass soon begin to crowd one another out in their efforts to reach it, until ranks of four or five deep are formed. The thread in these places takes a similar course. During the day the circulating mass formation may be maintained for an hour or two; but if at the end of that period the larvae have not begun to burrow, it is usually abandoned in favour of the procession.

The manner in which the procession is reformed is of considerable interest. By marking alternate larvae (by means of fine sand or flour scattered over the dorsal papillae) I found that the original order of the procession was not retained; but in every case I observed (five different processions*) *the primite of the resulting procession was the same as that which went into the circulating mass.*

Shortly before the primite sets out a rough kind of arrangement is sometimes to be observed in the mass, many of the larvae being arranged with their heads directed towards the point at which the procession is to start. The whole process of reformation is a very orderly one, though the order in which the larvae "fall in" is not in every case obvious.

In the case of a large procession which I photographed in the act of reforming the single-file formation, there was an arrangement of larvae four or five abreast in the neighbourhood of the mass, similar to that observed during its formation.

* The largest a procession numbering 57 larvae.

Purpose of the Circulating Mass.

It is possible that the circulating mass is sometimes formed for the purpose of pupation, but again abandoned for the normal procession on account of the unfavourable nature of the ground in the spot selected. But this explanation cannot apply to the majority of cases, for processions are of normal occurrence throughout the insect's larval existence.

Whether or not the circulating mass be a "rest formation" (as has been suggested) it is not impossible that it may possess other advantages as well. The facts already mentioned suggest the possibility of there being a regular arrangement within the circulating mass itself. In this case the formation might be a means of altering the order of the larvae whilst retaining the same leader. Other facts may be mentioned pointing to the same conclusion. Processions arranged artificially, by removing larvae at random from a mass and placing them in a position of head-to-tail contact, were not found to retain their original order for very long. A circulating mass was always formed and a different order arranged for the subsequent procession. The formation of a mass, in this case, however, was due probably to external interference rather than to any fixed habit of altering the order of the procession.

Method of Pupation.

Larvae which had become isolated, from any cause, were observed to bury themselves in the soft sand by means of their mandibles; but as a general rule processions seemed to burrow collectively in the following way. A circulating mass was first formed in a position which the primite apparently selected, after testing the consistency of the sand with his mandibles. On two occasions, which came under my observation, the process differed slightly, in that the original procession voluntarily broke up into several smaller processions of twenty or so each—in a manner similar to that described by Fabre—each of which formed a circulating mass and buried itself independently. As soon as the circulating mass was formed the whole commenced to rotate, thus producing a depression in the sand which gradually deepened. The sand was at the same time loosened, and some was thrown up on to the top of the mass. All the while the larvae were depositing their silk threads until a regular network was formed round the mass, in which sand became entangled. In some cases the lower larvae appeared to assist in the process by attacking the sand beneath with their mandibles. As the process continued

the whole mass gradually became engulfed, and finally, in the course of a few days, reached a depth of several inches.

When large processions became buried in this way, without first fragmenting, bundles of considerable size were formed. I discovered one numbering more than 100, buried at a depth of 3 inches. Some of these I removed with the sand containing them. These completed pupation 19 days after burrowing.

Conclusions.

The facts recorded in the previous observations all seem to support the conclusion that, though the individuals of a procession may act alike when influenced by the same stimuli, yet each larva is capable of independent action. (Cf. voluntary change of leader, action of primite under special conditions, fragmentation of processions occasionally preceding pupation, etc.)

If this be so the procession cannot be said to act in any real sense as a single individual.

Fabre states that the poisonous properties of the larval epidermis are exceedingly severe in some cases; yet, if this be true, the effect must depend largely upon the individual. I was not affected at all, though I was handling the larvae for several hours at a time. Mr Brindley also found no irritation from the larvae, though the Tachinid flies which he observed laying their eggs in a procession were noticed by him evidently to fear and avoid the hairs. I was told, however, by an inhabitant of the district, that some persons were subject to a rash at the time when the processions were abroad. This is said to be produced by the fine red hairs of the dorsal papillae which float in the air.

Literature.

Réaumur, 1736, *Mémoires pour l'histoire des Insectes*, II. pp. 149—161.
Fabre, circa 1898, *Souvenirs Entomologiques*, sér. VI. pp. 298—392.
Ratzeburg, 1840, *Forst-Insecten*, II. p. 128, and *Stettiner Entomologische Zeitung*, p. 40.
Brindley, 1906, *Proceedings of Camb. Phil. Soc.* Vol. XIV. Part I.

Secondary Röntgen Rays from Metallic Salts. By J. L. Glasson, 1851 Exhibition Scholar of Adelaide University, Gonville and Caius College. (Communicated by Prof. Sir J. J. Thomson.)

[*Read* 14 March 1910.]

I.

It has been shown by Barkla that when an element of atomic weight greater than that of Calcium is struck by a primary X-ray beam of suitable hardness, then the secondary X-rays emitted by it form a homogeneous beam characteristic of the radiating element.

The object of the present experiments was to determine whether the nature of this characteristic secondary X-radiation is affected by the state of chemical combination of the element.

Barkla made some experiments to test this point (*Phil. Mag.* June 1906), but that was before the conception of "characteristic homogeneous radiation" was introduced. The enormous simplification brought about by its introduction, makes further experiments interesting and their interpretation simple.

In particular it was desired to investigate the effect of a change in the valency of the metal upon the characteristic radiation from it.

II.

The arrangement of the apparatus is shown in figure 1. The metal or salt to be examined was placed at *A*, the salt being contained in a thin paper tray about 1 cm. deep. *B* and *C* are two thick lead screens, 20 cm. apart with apertures 2 cm. square cut in them. The rays enter the ionisation chamber *D* by a window of aluminium leaf *W*, and the ionisation produced is measured by a Wilson electroscope and a stop watch. No balance chamber was used as it was found that the coil, fitted with a hammer break, gave a very constant discharge; the variations in a series of consecutive readings amounted to about two or three per cent.

The screens for absorbing the rays were placed at *B*. They were of aluminium when an absolute determination of the absorption coefficient was required; but in some cases filter papers were used when only comparative measurements were required.

The absorption curve was determined in the same way as that used by Barkla in his work on homogeneous X-rays. The percentage absorption caused by a given screen (II) was measured, after different proportions of the beam had been previously

absorbed by the screen (I). If a curve is plotted, having as ordinates the percentage drop caused by the fixed screen (II), and as abscissae the amount previously absorbed by screen (I), then if the beam is absorbed exponentially, the curve obtained will be a straight line parallel to the x axis.

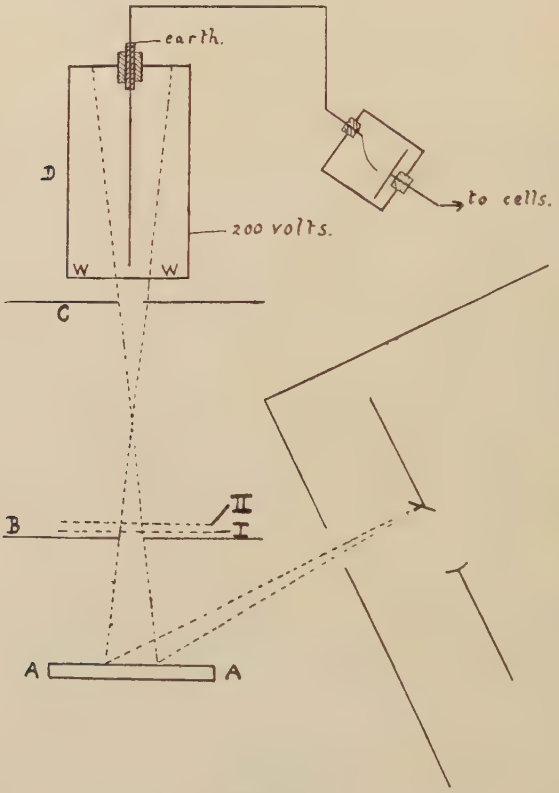


Fig. 1.

III.

Some of the curves obtained in this manner are shown in figs. 2, 3, 4, and 5.

The curves obtained when the radiator A is a pure metal are straight lines. These are inserted for comparison. The curves obtained with metallic salts are all of the same general nature. They are initially horizontal and then tend downwards. The point at which the curves begin to show this downward tendency depends on the salt used and on the hardness of the bulb.

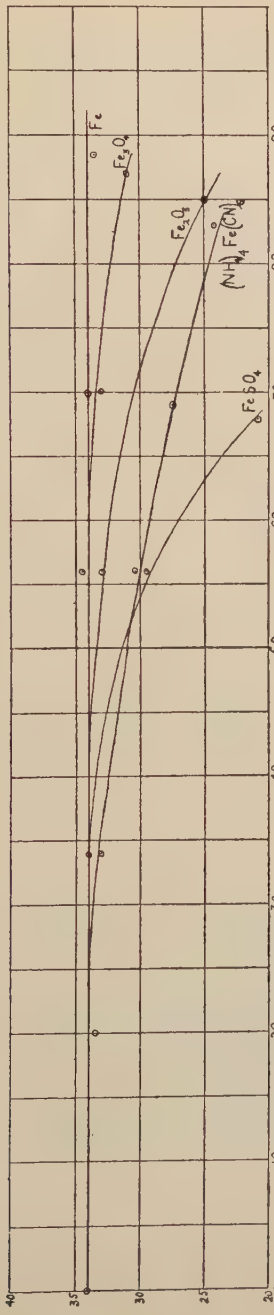


Fig. 2. Screen (II) = 3 Filter papers.

The value of the absorption coefficient calculated from the initial straight portion is always the same as that obtained for the characteristic radiation from the pure metal. The radiation from the salt thus consists of two parts:

(i) the homogeneous radiation from the metal,

and (ii) mixed with this is a "scattered" radiation due to the acid radicle considerably harder than (i).

The proportion of this scattered radiation varies with the hardness of the bulb. It is generally very small and hence does not evidence itself until the radiation has been very much reduced by screening. This has the effect of cutting off the soft "homogeneous" radiation much more than the hard "scattered," and hence the latter is shown to better advantage. The smallness of the amount of scattered radiation due to the light atoms is well shown in the case of Ammonium ferrocyanide. In this salt the weight of iron in the molecule is only about one-sixth of the whole. Yet the radiation due to the light atoms does not evidence itself until a considerable portion of the beam has been absorbed.

The information supplied by these curves may be summarized thus:

(1) The value of the absorption coefficient for the metallic radiation is unaffected by its combination with an acid radicle.

(2) It is independent of the valency of the element in the compound. This is well shown by the series of iron compounds $FeSO_4$, Fe_3O_4, and Fe_2O_3.

(3) The element may even occur in the acid radicle itself without affecting the value of λ.

IV.

In this way it is possible to determine λ for some metals which it is impossible or inconvenient to use in the uncombined state. This has been done in the case of Manganese. The salt used was the sulphate; the curve is shown in fig. 5. The value of λ obtained from the initial part of the curve is 100. This value agrees with that predicted from its atomic weight, using Barkla's curve (*Proc. Camb. Phil. Soc.* xv. 257).

I desire to thank Sir J. J. Thomson for his interest in these experiments.

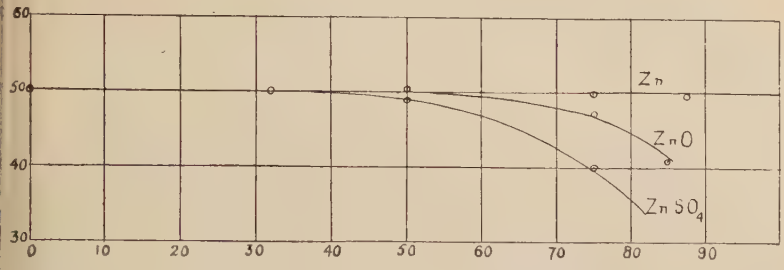


Fig. 3. Screen (II) = Aluminium ($\rho d = \cdot 017$).

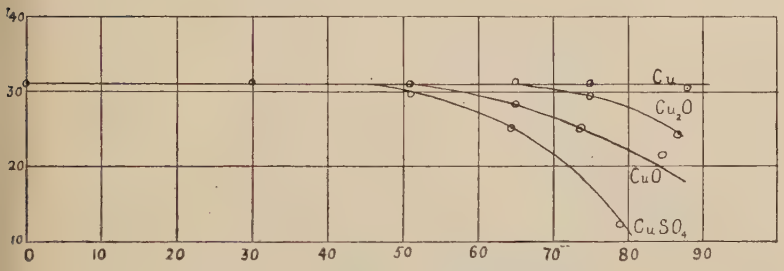


Fig. 4. Screen (II) = 5 Filter papers.

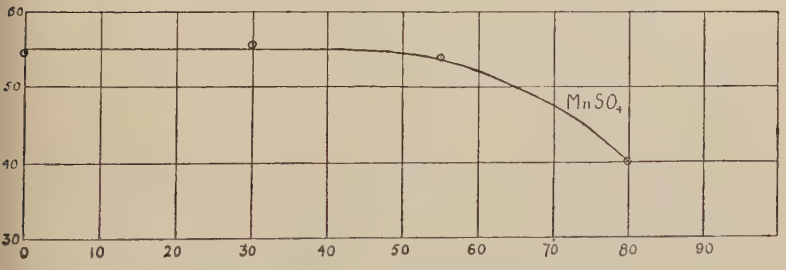


Fig. 5. Screen (II) = Aluminium ($\rho d = \cdot 008$).

On the Transmission of β-rays. By J. A. Crowther, M.A., St John's College.

[*Read* 14 March 1910.]

§ 1. The problems connected with the transmission of β-rays through matter are of considerable theoretical importance. Unfortunately they are involved at present in considerable obscurity, and a fairly extensive study by various experimenters has so far only served to demonstrate the complexity of the phenomena. The present paper deals only with a few of the many questions involved.

§ 2. The first question to be considered is the nature of the absorption itself. The absorption of the β-rays is almost invariably measured by finding the diminution in the ionization produced by them in some suitable ionization chamber, when various absorbing media are interposed in their path. This absorption may be due to two causes :—

(1) to a loss of velocity by the β-particles, such as is found to take place during the absorption of the α-rays ;

(2) to an actual decrease in the number of the β-particles themselves, due either to stoppage or deflection.

To decide this question it is necessary in the first place to determine the velocity of a homogeneous beam of β-particles before and after transmission through a suitable thickness of absorbing material.

A certain amount of indirect evidence already exists on the subject. In 1907 H. W. Schmidt* performed an experiment which at the time seemed to prove conclusively that the β-rays suffered no change of velocity even when transmitted through very considerable thicknesses of absorbing material. The essential features of his method are simple, and may be described here as they form the basis of the methods employed in the present experiments.

Three circular holes (fig. 1) are made in three lead screens, so as to define an arc of a circle of suitable radius. The system is placed between the poles of an electromagnet so that the magnetic field is perpendicular to the plane of the circle. By a suitable adjustment of the strength of the field a beam of β-rays entering the magnetic field at *A* can be made to describe the circular path *ACB*, and will produce ionization in a chamber placed at *B*. The velocity of the rays transmitted through this system is known from the value of the magnetic field, and the radius of curvature of the path.

* *Phys. Zeit.* Vol. viii. p. 371.

Schmidt placed at A a screen covered with radium E; measured the ionization produced above B, for different values of the magnetic field, and so obtained a curve connecting the intensity of the radiation passing through B with the magnetic field. The curve he obtained shows a fairly sharp maximum ionization for a certain definite field from which it falls away gradually to zero, as the field is increased or diminished.

Various thicknesses of aluminium were then interposed between the radium E and the aperture A and the experiment repeated. In every case the curves, though decreasing in height as the absorption increased, were exactly similar in shape and there was no perceptible shift of the maximum in either direction. From this result it was argued that no change in the velocity of the rays occurred during their passage through the aluminium.

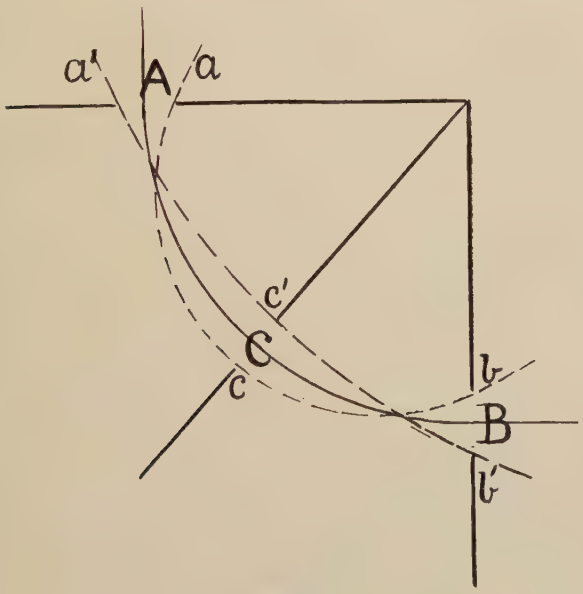


Fig. 1.

There is, however, a certain ambiguity connected with the use of the magnetic deflection method, which under the best of circumstances can only be minimised, and never completely eradicated.

If the apertures have a finite size, as must always be the case in practice, the path ACB is not the only possible path for rays to follow in order to emerge through B. A little consideration will show that rays of uniform velocity, but entering the field obliquely,

may be transmitted through the system along paths such as *acb* or *a'c'b'*, even when the field is too strong or too weak to deflect the rays, incident normally, along the path *ACB*. In this way even if the rays are of uniform velocity to begin with, there will be a definite range of field strength throughout which the rays can pass through the system. Similarly, if the rays are not homogeneous to begin with, if we are dealing with the whole of the β-rays from radium for example, there will be for every magnetic field a finite range of velocities which the β-rays may possess and still be able to pass through the system; and if the size of the apertures is considerable compared to the radius of the path, and if the incident beam is of a fairly wide angle, this difference of velocities may be very considerable. We shall return to this point later.

In addition to the broadening of the curve due to the finite width of his apertures, there was a second possible explanation of the shape of Schmidt's curves, namely that the rays from the radium *E* were not homogeneous.

It was natural to assume that the β-rays from a single homogeneous substance were homogeneous. In this case the fact that the maxima of the different curves always occurred for the same magnetic field, however much aluminium had been traversed, would prove that the rays kept their velocity unaltered during their passage through the fire.

In a recent paper W. Wilson* has given considerable indirect evidence to show that the β-rays from a single radio-active substance are not homogeneous, and in this case Schmidt's curves could only be explained on the assumption that there was a definite decrease in velocity as the rays passed through increasing thicknesses of absorbing material. In brief, if the finite width of Schmidt's curves are due to the rays from the radium *E* having a definite range of velocities, as the faster rays are proportionately less absorbed than the slower rays, for any given thickness of aluminium, the maximum should gradually move out in the direction of the stronger magnetic fields, and the higher velocities, in the absence of any compensating change of velocity in the rays.

From these considerations Wilson is able to deduce the change in velocity of β-rays for different thicknesses of aluminium. He calculates thus that the velocity of certain of his rays changed from $2{\cdot}78 \times 10^{10}$ cms. per sec. to $2{\cdot}69 \times 10^{10}$ cms. per sec. in passing through 0·489 mm. of aluminium.

It must not be forgotten, however, that the experiments of Schmidt, whichever way we regard them, do conclusively prove one important fact, namely that the β-rays from a single radio-active substance are absorbed in such a way that the distribution

* *Proc. Roy. Soc.* A, Vol. LXXXII. 1909, p. 612.

of velocities in the beam remains the same whatever the thickness of absorbing material traversed.

Under these circumstances it was felt that some more direct experimental evidence on the point was desirable.

The method employed consisted, briefly, in forming a beam of homogeneous β-rays by means of the magnetic deflection method described above, and measuring the velocity of the rays so obtained by means of a second similar system of screens. Sheets of different absorbing materials could be placed in the path of the rays between the two systems, and the velocity of the rays before and after passing through the absorbing medium could be directly determined.

As has been already explained, the magnetic deflection method only gives a truly homogeneous beam of rays when the apertures are infinitely small. On the other hand only a limited quantity of β-rays at the best can be radiated by any radio-active substance through a given area. The dimensions of the apparatus used, therefore, represent a compromise between these opposing factors.

A further point has also to be considered, namely the possible scattering of the rays during their passage through the air. Recent experiments* show that if the path is at all long, the scattering of the rays by the air may be very considerable. It is possible that this effect may not seriously affect the purity of the rays transmitted, but in cases where it is desired to work with a nearly parallel pencil of rays (as in experiments to be described later) it is certainly of very considerable importance. It was decided, therefore, that the whole operation of deflecting the rays and forming them into a pencil should take place *in vacuo*.

The form of apparatus finally decided upon is shown in section in fig. 2. The portion *EFGH* is placed between the poles of an electromagnet, so that the edges of the pole pieces lie along *EF*, and *FG*. The tube *DA* projects from the magnetic field and is carefully screened by a thick block of soft iron, so as to cut off as far as possible any strong field from this part of the path. Windows of thin aluminium foil (·002 cm.) close the apertures at *A* and *B*.

The radius of curvature of the path of the rays is 4 cms. and each of the apertures *A*, *D*, *C*, *B* is 0·5 cm. in diameter. The distance *AD* is 3·5 cms. and the depth of the box at right angles to the plane of the paper is 1·5 cms. The different apertures are bevelled, and the screens and sides of the box coated with aluminium foil to avoid as far as possible any secondary radiation. The apparatus could be exhausted by means of a water pump to a pressure of about 12 mm. of mercury, which was found to be

* J. A. Crowther, *Proc. Camb. Phil. Soc.* Vol. xv. 1909, p. 273.

sufficiently low for all practical purposes. In experiments for which a parallel emergent beam was not necessary, the radium was placed at *A*. If, however, a parallel beam was desired (as in the experiments on the absorption of the rays to be described later) the apparatus was reversed and the radium placed at *B*.

Two such chambers were made, and placed so that the window *A* of the one came directly opposite the window *B* of the other. The two magnetic fields were arranged so that rays of the proper velocity would be deflected round the two systems and emerge finally into an ionization chamber of the usual pattern.

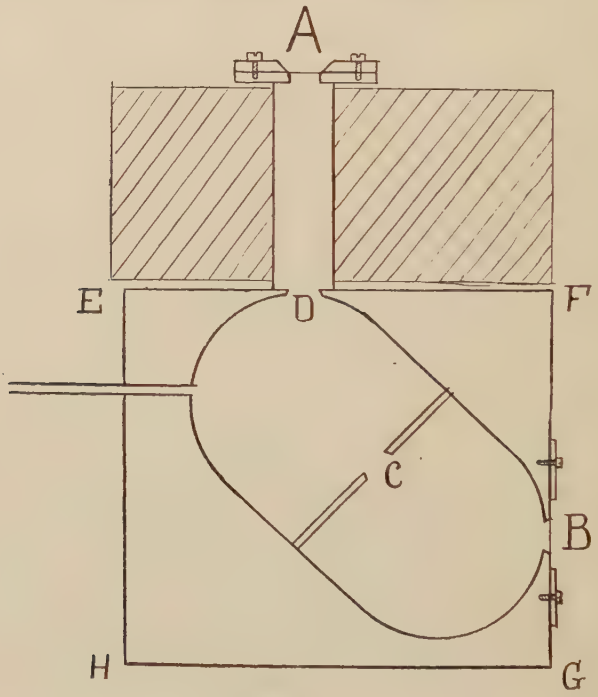


Fig. 2.

The measurements of the ionization produced were made by means of the compensating method devised for experiments on the scattering of the β-rays from uranium, and fully described in a previous paper*. In brief it consists in compensating the ionization current through the first chamber by an opposing ionization current from a second chamber connected to the same electroscope, in which the ionization can be varied, in a

* J. A. Crowther, *Proc. Roy. Soc.* A, Vol. LXXX. p. 186.

known manner, by means of a shutter sliding over a plate of uranium oxide. By placing a second small sample of radium near the compensator, the γ-ray leaks through the two chambers could be made to exactly neutralize each other, and thus the effect due to the β-rays alone could be measured directly.

Unfortunately the double system transmitted so very few β-rays that it was not found possible to measure the effect on the velocity of interposing different screens between them. Enough, however, was transmitted to test the efficacy of the systems as a means of producing a beam of homogeneous rays. Keeping the first field constant and varying the second it was found that when the mean velocity of the rays transmitted through the first system was $2{\cdot}77 \times 10^{10}$ cms. per sec., the extreme values for the velocities

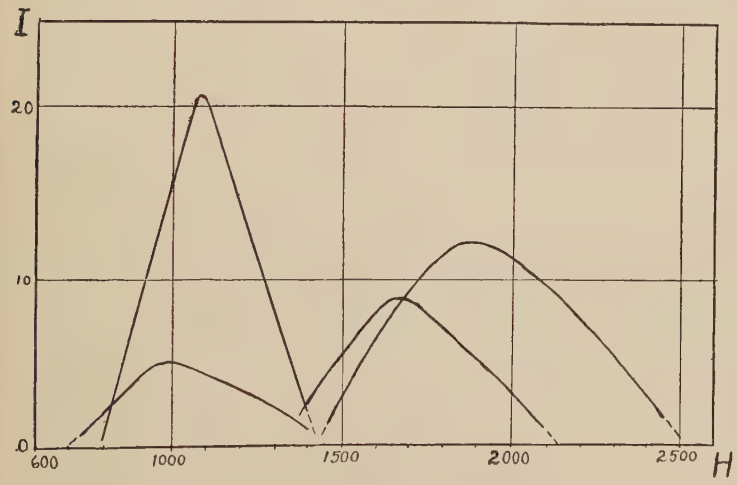


Fig. 3.

of the rays, as measured by means of the second system, were $2{\cdot}75 \times 10^{10}$ cms. per sec. and $2{\cdot}79 \times 10^{10}$ cms. per sec. The extreme velocities of the rays transmitted through one of the systems therefore do not differ by more than one per cent., plus or minus, from the mean value, and the system may be regarded therefore as giving a fairly homogeneous pencil of rays.

In order to be able to work with absorbing screens it was necessary to make a further compromise in order to obtain a measurable effect. The first system was left unaltered so that a homogeneous pencil of rays of known velocity fell upon the screen. The distance AD in the second chamber was, however, reduced to 1 cm. The field across the first system in any given experiment was kept constant while that across the second was

varied. In this way curves connecting the intensity of the β-radiation passing through the second system with the magnetic field both with and without an absorbing substance between the two systems could be obtained. The experiments were then repeated for different values of the first deflecting field, that is, for different velocities of the incident rays.

Specimen curves, obtained with aluminium as the absorbing substance, are given in fig. 3. It will be seen that there is a considerable broadening of the curves even when there is no absorbing screen in the path of the rays. As the rays emerging from the first system have been shown to be very nearly homogeneous, this broadening is due to the greater latitude of path allowed by the second system owing to the shortening of the tube AD. It will be noticed that for aluminium this broadening of the curves is no greater with than without the absorbing layers.

Fig. 3 shows the effect of interposing a sheet of aluminium 0·47 mm. in thickness between the two systems, for two different velocities of the incident beam. The upper curve in each case is the curve obtained for the incident beam in the absence of the absorbing sheet. The ordinates represent the intensity of the rays passing through the two systems, as measured by the ionization produced; the abscissae measure the magnetic field acting upon the second system.

It will be seen that in each case the introduction of the absorbing sheet produces a very definite displacement of the curve in the direction of the smaller velocities.

The actual figures deduced from these curves are given in Table I.

TABLE I.

Aluminium (0·47 mm.).

Velocity of incident rays	Velocity of emergent rays	Percentage absorption of rays by the Aluminium
$2{\cdot}735 \times 10^{10}$ cm./sec.	$2{\cdot}690 \times 10^{10}$ cm./sec.	71 °/₀
$2{\cdot}903 \times 10^{10}$ cm./sec.	$2{\cdot}881 \times 10^{10}$ cm./sec.	52 °/₀

It is evident therefore that there is a small, but perceptible decrease in the velocity of the β-rays as they pass through absorbing media.

But though this small decrease in velocity accompanies absorption, it is not sufficient to account for it. The third column in

the table gives the percentage of the incident rays absorbed in the aluminium sheet used; and it will be seen that while 71 °/₀ of the softer rays are absorbed in the aluminium screen used, the change of velocity is only of the order of about 2 °/₀. Allowing for the change in mass of the β-corpuscles with change in velocity, which is fairly rapid at these high velocities, this corresponds to a loss of energy by the β-rays of about 10 °/₀. Only a small portion of the whole absorption of the rays can therefore be due to this gradual decrease in velocity. For the main causes of absorption we must seek elsewhere. However, as we shall see later, the gradual change in velocity of the rays may have a very appreciable effect in modifying the shape of the absorption curves.

Glass screens gave very similar results to aluminium. Platinum on the other hand behaved quite differently. The curves obtained between the intensity of ionization and the strength of the second magnetic field were low, showed no decided maxima, and were very much broadened out in both directions, but particularly in the direction of the softer rays. On account of the very small amount of radiation transmitted for any given field it was not possible to determine the exact shape of the curves at all accurately. Some of the broadening may have been due to the rays on the whole emerging from the platinum more obliquely than from aluminium or glass, but it seems too great to be altogether explained on this assumption, and more probable that a homogeneous pencil of β-rays after passing through a platinum screen emerges with a very considerable range of velocities. This effect must be connected with some secondary radiation excited in the platinum. We shall return to this point later.

§ 3. *On the absorption of homogeneous β-rays.*

The absorption of the β-rays from a single radio-active substance, such as uranium *X*, or one of the radium products, may be measured by interposing screens of absorbing material of different thicknesses between the radiating layer and an ionization chamber, and measuring the change in ionization produced. If the absorbing material is of low atomic weight the absorption curve can be represented accurately by the equation

$$I = I_0 e^{-\lambda d},$$

where I is the intensity of the radiation after passing through a thickness d of absorbing material, and λ is a constant. If the absorbing substance is of high atomic weight, there is a short initial steeper portion before the curve assumes the true exponential form.

This law has been tested by many observers for practically all

substances emitting β-rays over a very wide range of absorptions. N. R. Campbell * found that for aluminium the exponential law was certainly true to within 0·5 °/₀ even when the rays had been reduced to one hundredth part of their initial intensity.

W. Wilson†, on the other hand, using a magnetic deflection method, found that the absorption of the "homogeneous" rays obtained by the magnetic deflection method certainly did not follow an exponential law, but a law which was practically linear. It is difficult to conceive any mechanism which would produce a linear law of absorption. Wilson, who worked principally with aluminium, notes that tin showed some departures from the linear law, but does not state either the nature or amount of these departures. It seems probable, therefore, that the linear law found for aluminium is only approximate and represents the balance of a variety of causes and conditions.

For example the gradual change of velocity of the β-rays in their passage through the aluminium would tend to transform an exponential law of absorption into a law which might approximate to a straight line. For consider a beam of homogeneous β-rays of velocity v, and with a coefficient of absorption λ. If the exponential law is accurately obeyed, and if the rays retain their original velocity, the curve connecting the logarithm of the intensity with the thickness of material traversed will be a straight line. As a matter of fact, however, after passing through a thickness d of material, the velocity of the rays has diminished to v', and the coefficient of absorption has increased to some value λ'. Since λ varies rapidly with v (probably as the inverse fourth power of v) the change in λ will be quite perceptible even if the change in velocity is only small. Thus a change of velocity of only 2 °/₀ such as was found to occur in 0·47 mm. aluminium might produce a change in λ of as much as 16 °/₀. Thus instead of the curve for $\log I$ being a straight line it would be some curve falling below this line, and its slope, which gives the value of λ, becomes steeper and steeper as the thickness of material is increased. In fact we should have a curve similar in appearance to that given by Wilson for the absorption of homogeneous β-rays by aluminium‡.

Prof. Sir J. J. Thomson§ has very recently published a theory of the transmission of the β-rays through matter, according to which the absorption of homogeneous rays should vary as $(1-e^{-kx})$, where x is the thickness of material traversed, and k is a constant. This would give an absorption curve decreasing at first very slowly,

* N. R. Campbell, *Phil. Mag.* Vol. xvii. 1909, p. 180.
† W. Wilson, *Proc. Roy. Soc.* A, Vol. lxxxii. 1909, p. 612.
‡ Wilson, *loc. cit.* p. 616. Fig. 4.
§ *Proc. Camb. Phil. Soc.* Vol. xv. Part v. 1910.

then much more rapidly, and finally tailing off into something not very distinguishable from the ordinary exponential law.

The experiments now to be described have been made with a view to testing the various points briefly touched upon above; and to ascertain as exactly as possible the true shape of the absorption curve for some different substances.

The apparatus used is sketched in fig. 4. The radium in a glass tube is placed at R in the centre of a large block of lead. A is one of the systems used for obtaining a homogeneous beam of β-rays, fully described in the previous section of this paper and sketched in fig. 2. D is the ionization chamber into which the β-rays pass. It is made of a copper hemisphere 12 cms. in diameter, with an inner electrode e of aluminium leaf, and a large opening dd, closed with thin aluminium leaf to admit the rays. The absorbing sheets can be introduced into the small

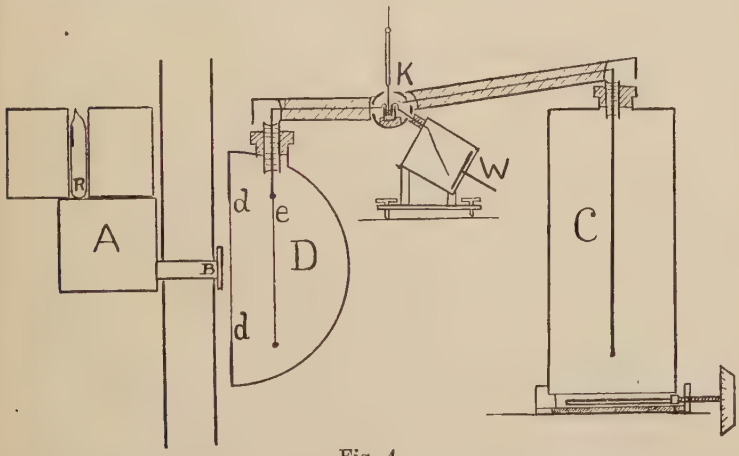


Fig. 4.

space left between the exit tube B and the window d, by means of a metal slide not shown in the figure. C is the shutter compensator described in detail in a previous paper, and W an inclined gold leaf electroscope which differs from the original design in being only 1 cm. in depth. This variation has the advantage that it allows of the use of fairly high power objective and thus of a considerable degree of magnification in the reading microscope.

C and D were charged to opposite potentials and their inner electrodes were connected through the key K to the electroscope W by wires passing through earthed metal tubes. These tubes were filled with sulphur and the key K was made small in order to reduce as much as possible the leakage from the electrodes through air ionized by γ-rays from the radium.

The chief points in the method are therefore, briefly, as follows:

(1) The homogeneous rays are produced by magnetic deflection through a suitable system of apertures in a vacuum.

(2) They fall upon the absorbing sheet normally, in a narrow and nearly parallel pencil.

(3) The intensity is measured in an ionization chamber of such a form that, at whatever angle the rays emerge from the absorbing sheet, their length of path in the chamber is practically the same.

TABLE II.

Thickness x, in cms.	I/I_0	$\log_{10} I/I_0$	$x \log_{10}(1 - I/I_0)$
Aluminium:			
·002	1·00	—	—
·004	·996	—	(– ·010)
·012	·95	$\bar{1}$·98	– ·016
·033	·77	·89	– ·021
·049	·62	·79	– ·020
·073	·29	·46	– ·010
·110	·12	·08	– ·007
Platinum:			
0	1·00	—	—
·00107	·71	$\bar{1}$·85	– ·00058
·00220	·53	·72	73
·00327	·38	·58	69
·00556	·23	·36	61

It may be permissible to note in passing the advantages possessed by the compensator method in experiments of this kind over the ordinary methods of timing the leaf over a given number of divisions. These are, in brief,

(1) The large γ-ray leak through the ionization chamber is balanced by a similar leak through the compensator. In this way the effect due to the β-rays is measured directly, and not as a difference between two other large quantities.

(2) The results are not affected by defects in the insulation, as is the case when a timing method is used. As a defect in the insulation only comes into play when the gold leaf system begins to charge up, it cannot affect the direction in which the leaf begins to move, and hence cannot affect the final balance. In a timing method, however, defective insulation will alter the

rate of leak, and may cause considerable errors in the results. This point is of special importance in the present case, as it is always difficult to secure really good insulation in the presence of the γ-radiation from any considerable amount of radium.

(3) The method is found to be very convenient in use, and capable of very considerable sensitiveness and accuracy.

Experiments have been made so far on aluminium and platinum. The results obtained are given in Table II, and are represented graphically in figs. 5 and 6. The abscissae represent

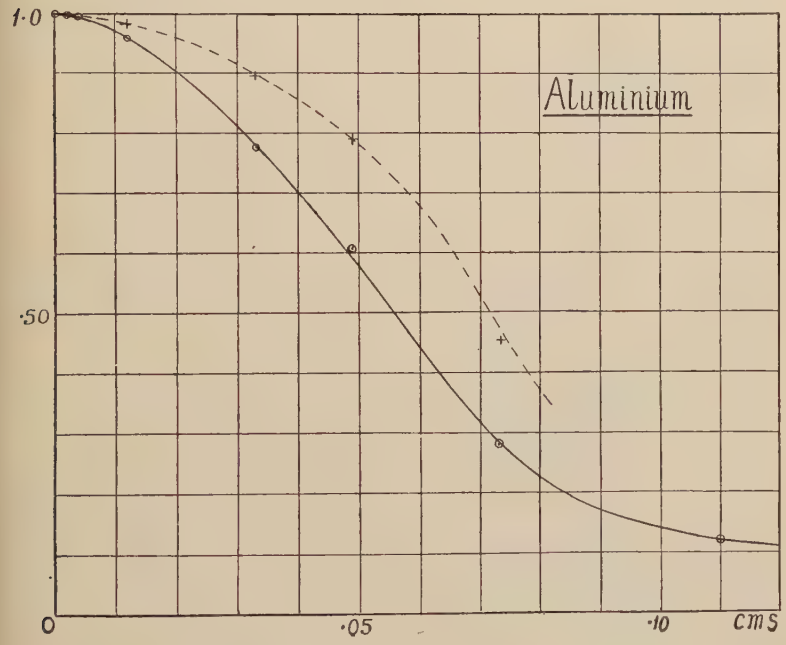


Fig. 5.

in every case the thickness of the absorbing layer; the ordinates represent in the case of the full curves the corresponding values of I/I_0 and in the case of the dotted curves the values of $\log_{10} I/I_0$. It will be seen that the two substances behave quite differently in their absorption of the β-rays, and we will consider each separately.

The curve for aluminium at first descends very gradually and the quantity of rays absorbed increases very slowly with the thickness during the initial stages. On adding further thicknesses, however, the curve begins to descend more rapidly, and for part of its course becomes practically a straight line. For greater

thicknesses still the curve has another point of inflection; the absorption becomes less rapid, and the curve appears to become asymptotic to the axis of x. The small amount of rays transmitted by these thick sheets did not allow of any very exact determination of the shape of this part of the curve. The experiments seemed to indicate, however, that it did not depart widely from an exponential form.

It will be seen that the curve thus obtained agrees well in its main outlines with the form of absorption curve predicted by Prof. Thomson's theory. It shows the same initial flat portion, the steeper middle portion and the final gradual tailing away.

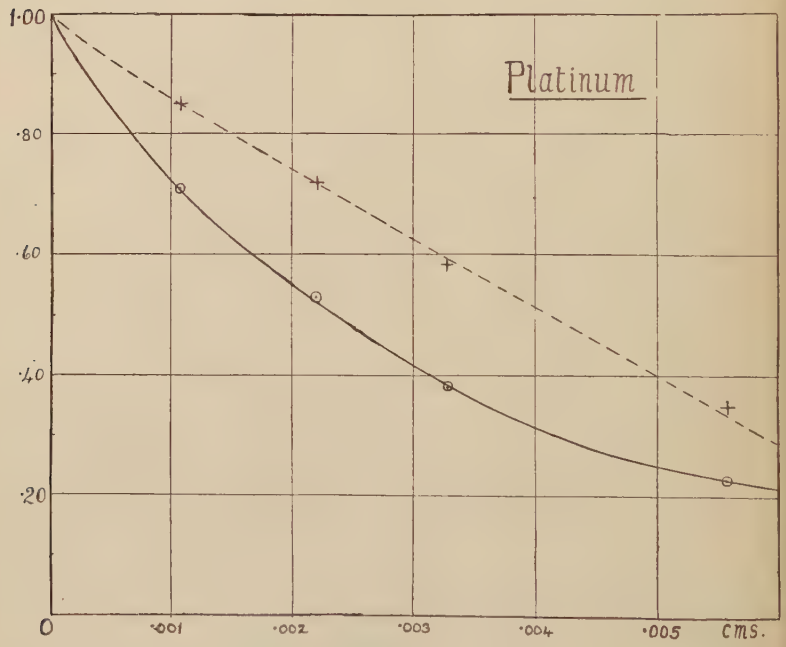


Fig. 6.

More than a general resemblance could not well be expected. Prof. Thomson's theory applies rigidly only to thin sheets of absorbing media. Again the factor k in the equation

$$I = I_0(1 - e^{-kx})$$

is a function of the energy of the rays, and decreases as the energy is diminished. As the energy of the rays is continually diminishing during their passage through the aluminium, the value of k must also diminish with the thickness, and thus cause a departure from the predicted form. The last column of Table II

gives the values of $x \log (1 - I/I_0)$ which is proportional to k, and it will be seen that this product does diminish for the thicker sheets. On the whole the agreement between the experimental results and the theory is even closer than might have been expected, but it is evident that the phenomena of absorption are too complicated to afford any rigorous test of the theory. Experiments are now being made along other lines in order to determine this point.

It will be seen that the curve differs from those given by Wilson in his paper, mainly in the initial portion. His curves show the final bending round along the x-axis, though he seems inclined to ascribe it to experimental imperfections. It seems possible that the greater width of his apertures (1·2 cms. as compared with 0·5 cm. in these experiments) and the fact that the rays fell upon the absorbing sheets over a fairly considerable angle instead of in a nearly parallel pencil, caused a kind of general levelling out, and thus obscured the initial point of inflection of the curves.

If now we turn to the absorption curves for platinum, we find an entirely different law of absorption. An examination of the dotted curve in fig. 6, representing the relation between $\log I/I_0$ and the thickness of the absorbing layer, will show that, except for a slight initial steepening, the curve may be represented by a straight line to an accuracy well within the probable errors of experiment. On the other hand the lower curve, giving the values of I/I_0, does not show the slightest approximation to a linear law. In fact the absorption of homogeneous β-rays by platinum follows exactly the same law as the absorption by platinum of the β-rays from a single radio-active substance, that is it is exponential.

The velocity of the β-rays used in these experiments was $2{\cdot}77 \times 10^{10}$ cms. per second. The mean velocity of the β-rays from uranium X, according to the measurements of H. W. Schmidt*, is $2{\cdot}79 \times 10^{10}$ cm./sec. The homogeneous rays used in these experiments had therefore practically the same velocity as the rays used in previous experiments in the absorption of the β-rays from uranium†. The value of λ/ρ for platinum (where λ is the coefficient of absorption assuming the exponential law and ρ the density) found from the present experiments is 11·9; the value obtained with the uranium rays was 9·4 Considering the very different nature of the two experiments the agreement is fairly close.

This result must undoubtedly be ascribed to some secondary radiation excited in the platinum by the incident rays. We have already seen that the rays emerging from platinum betray an

* H. W. Schmidt, *Le Radium*, Vol. VI. 1909, p. 5.
† J. A. Crowther, *Phil. Mag.* Vol. XII. 1906, p. 379.

entirely different character to those emerging from aluminium when studied in a second magnetic field. While the latter are gradually reduced in velocity without any appreciable dispersion of their velocities about the mean, the former seem to possess a very considerable range of velocities. It may be noted that McClelland* has shown that the return β-radiation from any radiator may be divided into two parts, which differ in their penetrating power and in their law of distribution. One has practically the same penetrating power as the primary β-rays, and is probably purely scattered radiation; the second type is less penetrating and is probably a true secondary radiation. The former is the predominating factor in elements of low atomic weight such as aluminium, while the latter predominates in elements of high atomic weight such as lead or platinum.

It seems, therefore, that the first stage in the absorption of a beam of homogeneous β-rays by platinum is their transformation into secondary radiation (during the initial steeper portion of the curve). The absorption of these secondary rays in the further sheets of platinum then takes place according to an exponential law. The rapidity of this conversion, which seems to be complete in 0·001 cm. of platinum, is at first somewhat startling. It is, however, quite in accordance with previous experiments† on the scattering of the β-rays from uranium, which may be regarded as showing that the distribution of the β-rays in a sheet of gold reaches its final form in a thickness of little more than 0·0002 cm.

It is interesting to notice that Sadler‡ has very recently shown that the absorption of the secondary corpuscular radiation emitted from different radiators under the action of homogeneous secondary Röntgen rays is also absorbed according to an exponential law.

We are thus led to the following result. When β-rays are emitted by any substance, whether due to its own radio-active properties, or excited by external radiation of a single definite type, the absorption of the rays emitted follows an exponential law. On the other hand the absorption of a homogeneous beam of β-rays by a substance such as aluminium which does not emit any large amount of true secondary radiation of its own, follows a law the precise nature of which remains to be determined, but which is certainly not exponential.

If this is so, it follows, as Wilson has already suggested, that the rays from a single radio-active substance are absorbed according to an exponential law by virtue of some special distribution of velocities in the emergent beam. The results obtained in the

* J. A. McClelland, *Proc. Roy. Soc.* A, Vol. LXXX. p. 501.
† J. A. Crowther, *Proc. Roy. Soc.* A, Vol. LXXX. p. 187.
‡ C. A. Sadler, *Phil. Mag.* March 1910.

present paper on the absorption by platinum, and the results of Sadler already alluded to, on the absorption of secondary corpuscular radiation, show that if this is the case a similar distribution of velocities occurs also when secondary β-rays are emitted under the influence either of homogeneous β-rays or of homogeneous Röntgen rays; while the experiments of Schmidt, described in the earlier part of this paper, show that this distribution is not disturbed during the passage of the rays through very considerable thicknesses of absorbing material.

A law so fundamental, applying equally to the emission of β-rays under radio-active disintegrations, under the action of homogeneous β-radiation, and under the action of homogeneous Röntgen rays, should be capable of some simple explanation in terms of the properties of the β-rays and of the atom. Until further experiments have been performed however it seems premature to frame any suggestion as to its cause, or even as to its precise nature. A knowledge of the true law of absorption of homogeneous rays would enable us to determine the initial distribution of velocities necessary to produce an exponential law of absorption, and Wilson, assuming that the law of absorption is linear, has calculated one such possible distribution. No theoretical evidence in favour of the distribution which he arrived at has yet been given, and the results of the present experiments seem to show that the "linear law" of absorption is only an approximation even for aluminium, and is probably the result of an interaction of the different causes producing and influencing the absorption rather than a fundamental law.

One fact alone seems perfectly clear. The absorption of the β-rays is a far more complicated phenomenon and is influenced by a far greater variety of causes than had previously been imagined. It is only by disentangling the different phenomena involved and considering each of them separately that we shall arrive at any satisfactory idea as to the nature of the processes involved.

Summary.

Experiments have been made on the velocity of a beam of homogeneous β-rays before and after their passage through an absorbing medium. It was found that aluminium caused a small but perceptible diminution in velocity of the rays, without any appreciable disturbance of the homogeneity of the beam. On the other hand a homogeneous beam of β-rays after passing through platinum emerged with a fairly wide range of velocities.

The absorption of a parallel pencil of homogeneous β-rays by aluminium and platinum has been determined. The absorption curve for aluminium is complicated, being neither "exponential" nor "linear." It approximates, however, to a form very recently suggested by Prof. Sir J. J. Thomson. The absorption curve for platinum after a steeper initial portion is exponential. This result is assigned to secondary radiation.

I take this opportunity of again expressing my best thanks to Prof. Sir J. J. Thomson for much kindly interest and many helpful suggestions.

Some Experiments on Ionisation in Dried Air. By S. G. Lusby, Emmanuel College. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 14 March 1910.]

(1) *Introduction.* In examining the physical properties of ions, one cannot help being struck by the important part which moisture plays in the determination of the coefficients which define these properties.

For example, in moist air, the coefficient of diffusion of positive ions is ·032, and that of negative ions is ·035. If, however, the gas be dried, the coefficient for positive ions drops to ·028, whilst in the case of negative ions there is a large increase to ·043. Under certain conditions one can take advantage of this difference in the rates of diffusion between the two kinds of ions and communicate a decided positive charge to the gas.

Again, the velocities of ions in any gas are affected if the gas be allowed to retain any moisture; for example, in moist air, the velocity of the positive ion under a potential gradient of 1 volt per centimetre is 1·37 centimetres per second, for the negative ion 1·51; if the air be dried, the velocity of the positive ion is changed very slightly to 1·36, whilst that of the negative ion rises to 1·87.

In both sets of measurements it is seen that the action of drying the gas is more pronounced in its effect on the negative ion, but it should be remembered that the positive ion is also affected, though in a lesser degree. Air does not show this latter modification very well—probably on account of its complex structure—but in other gases, e.g. oxygen, hydrogen, and carbon dioxide, the effect is very marked, as is seen by studying any standard tables of coefficients.

In condensation experiments this difference in the action of positive and negative ions is again noticeable; each kind of ion has its own condensation point. The experiments of C. T. R. Wilson* have shown that when the ratio of expansion of a gas is 1·25, condensation occurs round negative ions, but not till the ratio is increased to 1·31 do the positive ions produce a condensation cloud. This suggests that the negative ions act more readily than the positive ions as nuclei for the condensation of water vapour.

All these facts point to the conclusion that the negative ion is much more intimately connected with water vapour than the positive, and it was thought that if by some means an ionised gas

* C. T. R. Wilson, *Phil. Trans.* A, cxciii. 1899, p. 289.

could be thoroughly dried, then it should acquire a strong positive charge. The present paper gives an account of some experiments made to try and detect such an action in air.

(2) *Experimental conditions.* The problem was first attacked directly. A stream of undried, unfiltered air was drawn through brass tubing by means of a water pump at an approximate rate of 50 cubic centimetres per second. Near the end of the apparatus where the air entered was a roll of filter paper, 1 decimetre long, on which uranium oxide had been fixed by means of acetyl collodion: this served as the ionising agent. The ionised air then passed through a copper U-tube, which could be surrounded by liquid air, and then passed into a Faraday cylinder which was connected to a Wilson electroscope of ordinary type—a sensitiveness of 60 divisions per volt being generally employed. In order to avoid diffusion effects as far as possible, the brass piping was chosen with a fairly large diameter (4 centimetres); the U-tube was necessarily smaller, being only 1·5 centimetres in diameter, and the total length immersed in liquid air was about 15 centimetres. Liquid air was chosen as the drying agent in order to make the action as perfect as possible; this method has the further advantage that it does not cause any mechanical filtering effect such as the use of calcium chloride or sulphuric pumice might produce.

The experiment consisted in merely placing a cylinder of liquid air round the U-tube and testing the electroscope for charge. In no case could any charge be detected, even though the electroscope's sensitiveness was pushed as high as 80 divisions per volt; special care was of course taken to render all insulations perfect.

To further test the matter, the Faraday cylinder was replaced by a condenser of the type used by Zeleny and other workers with blast methods, so that the positive and negative charges could be measured separately and compared, by raising the outer wall of the condenser to an appropriate positive or negative potential whilst the inner electrode was connected to the electroscope. The diameter of the outer brass tube of the condenser was 4 centimetres, that of the inner electrode 7 millimetres, and the total length was 15 centimetres; a potential was applied sufficient to drive on the inner electrode all ions whose mobilities exceeded 1 centimetre per second. In order to avoid effects due to the charging up of the ebonite insulation, the air was made to enter and leave the condenser through earthed tubes.

On measuring the positive and negative charges in the dried air blast, a surprising phenomenon manifested itself; the two charges were exactly equal, but were two or three times the corresponding quantities in the undried air. Results were variable

from day to day, but in no case was the ratio of increase less than 2.

At first, it was thought that this effect might be due to the low temperature and dependent upon the metal of which the U-tube was composed. The copper tube was therefore replaced by a glass one of approximately the same section, and the experiment was repeated—but without any change in the result.

The uranium oxide, the ionising agent, was then suspected; if it gave off an active gas which condensed at the low temperature of liquid air, the results obtained might perhaps be explicable. Obviously this action could be eliminated by employing another ionising agent. The simplest that suggested itself was the natural ionisation of the air. It is well known that under normal conditions there is present in the atmosphere a quantity of radium emanation sufficient continually to produce small ions of both signs at the approximate rate of 30 per cubic centimetre. This gave a small but fairly constant source of ionisation to work on. Here again, the drying (or cooling) of the air almost trebled the ionisation, and the uranium was restored to its place.

The only other possible causes for the phenomenon which suggested themselves were diffusion, recombination, and conversion of large ions into small ones.

Diffusion was first tested. As was pointed out in the introductory section of this paper, the coefficient of diffusion of negative ions in air is 50 % greater than that of positive ions, and this should lead to an excess of positive ions, if conditions were such as to allow of such action. Although it seemed extremely unlikely that the large tubing used could account for the effect, the matter was tested directly. U-tubes of two different bores, one greater and one smaller than that originally used were inserted, but without any appreciable change in the result.

The question of conversion of large ions into small ones promised to give an explanation of the phenomenon. These large ions, discovered by Langevin* in 1905, have a mobility of only 3×10^{-4} centimetres per second and occur naturally in the atmosphere, there being normally about 2000 of each sign in a cubic centimetre of air. The dimensions of the testing apparatus were such that only a negligibly small portion of them were caught. In a recently published paper, M. de Broglie† has shown that these large ions are produced by the combination of ordinary small ions (whose dimensions are of molecular order) with large "neutral centres," whose linear dimensions are 100 times that of a molecule, that is about the size of ultra-microscopic particles or particles suspended in colloidal solutions, namely about 10 $\mu\mu$.

* Langevin, *C. R.* CXL. 1905, p. 232.
† de Broglie, *Journ. de Phys.* Dec. 1909,

M. de Broglie has further given a general method for detecting and controlling these nuclei. If by the action of drying, some of these large ions were broken up into a small charged portion and a large neutral system, the small portion would be caught by the testing condenser and give an effect of the nature observed. This type of analysis of the large ion, viz. into one charged and one neutral portion, is the only one admissible if we assume that the ion possesses only one electronic unit of charge and further assume that this unit is indivisible; the latter supposition seems to be supported by experiment, but the former is as yet purely gratuitous.

To test this theory, the air before entering the apparatus was deprived of all its ions—large and small—by passing through a long condenser charged to an appropriate potential. Small ions are quickly produced spontaneously, but it takes a fairly long time for these to combine with the neutral centres to form large ions. In some previous experiments carried out by the writer*, it was found that air thus deprived of large ions did not again acquire the normal number till after about 20 minutes; de Broglie (*loc. cit.*) has investigated a similar effect, and states that it requires a considerable time for the large ions to attain their maximum number, but gives no exact figures. As the time required for the air to pass from the electrical filter to the U-tube is at the most only 5 seconds, one seems justified in concluding that no appreciable reproduction of large ions occurs in this interval. On trying the above experiment, it was found that the result was quite independent of the large ions present in the air.

There now remained only the question of recombination, which however did not promise to give a solution of the problem; for recent work by Erikson† has shown that the coefficient of recombination increases as the temperature decreases, which obviously would give an effect entirely opposed to that observed. It is true that the coefficient thus found was under conditions of constant density, whereas in the present investigation conditions of constant pressure obtained. Still one could roughly suit the constant density coefficient thus found to the present case (where the pressure at any temperature would be higher) by superposing on it conditions of increased pressure. Now, according to M^{c} Clung‡, the coefficient of recombination is independent of pressure between the limits of 0·1 and 3 atmospheres; whilst Langevin§ showed that below these limits the coefficient decreases rapidly as the pressure is lowered. Thus an increase of pressure would in the

* Lusby, *Journ. Roy. Soc. N. S. W.* June 1909.
† Erikson, *Phil. Mag.* Aug. 1909.
‡ M^{c}Clung, *Phil. Mag.* March 1902.
§ Langevin, *C. R.* cxxxiv. 1902, p. 646.

former case not affect Erikson's coefficient at any temperature and in the latter case would increase it. In either case, the effect of temperature on recombination would give a result quite the opposite to that observed. Still, in order to verify this theory, the position of the ionising agent was altered, and it was placed between the U-tube and the testing condenser. Obviously, if recombination were causing the phenomenon, the current received by the electroscope would depend on the distance between the uranium and the testing condenser. The result was surprising; the ionisation in the dried air was increased to 10 times that in ordinary air, instead of only twice, as before. This seemed to point to the fact that reduced recombination occurred in the dried air, although I am not aware that any previous experimenter ever suspected or allowed for this action. To make sure of the point, the air before entering the apparatus was passed first through a tube 3 decimetres long, containing calcium chloride, and then through another tube 4 decimetres long, containing pumice, which had previously been treated with sulphuric acid in the ordinary way. On immersing the U-tube in liquid air as before the ionisation was still increased tenfold, and the negative action of water vapour on recombination confirmed.

The only other thing that remained to be tried was the effect of dust. All early experimenters on recombination found that, in order to obtain consistent results, dust must be carefully excluded from the gas experimented on—a result clearly illustrated by Owens'* experiments with tobacco smoke. Although it seemed extremely improbable that drying (or cooling) a gas could remove its dust particles, still the result of filtering the air was tried. At first, only a decimetre of cotton wool was used, with the result that, on drying the air as before, the increase in ionisation was only about 100 °/₀; when the length of cotton wool was increased to 5 decimetres the phenomenon vanished altogether, and was therefore due solely to recombination.

(3) *Discussion of Results.* The negative result obtained in the primary investigation, namely the absence of any preponderance of charge of one sign in dried air, is certainly disappointing. As was pointed out earlier, practically all the known properties of ions are modified by water vapour, such modification being more pronounced in the case of negative ions; still one must not forget that the positive ion is also affected by moisture, though to a lesser degree. It may be objected that the effect sought for is masked by the recombination effect; but that is not so, for in the first method employed the electroscope was quite sensitive enough to easily record a 5 °/₀ difference between the two charges. Further,

* Owens, *Phil. Mag.* Oct. 1899,

in the very last experiment carried out—namely, that with filtered air—the effect of drying alone was tested, and neither the positive nor the negative charge was affected. This result shows that either the ions go through the U-tube intact, or else they drop their water molecules and pass on; and their mobilities in the latter case being presumably those they would possess in dry air, they are still caught by the testing condenser.

The secondary effect, accidentally discovered, is in some ways more interesting than the main result, on account of the light which it seems to throw on the nature of the large neutral centres in the air, investigated recently by de Broglie (*loc. cit.*). In the atmosphere there are normally present three types of bodies which are of importance in an ionisation theory, viz. small ions, neutral centres, and large ions, the latter being a combination product of the two former. When a gas is in statistical equilibrium, some of the small ions are constantly combining with small ions of opposite sign, others combine with neutral nuclei to form large ions, and a proportion of these large ions combine with large ions of opposite sign. Here two main coefficients of recombination have to be considered, that of small ion with small ion, and that of small ion with large ion. Now, other things being equal, the coefficient of recombination between two ions depends on the sum of their mobilities, hence the former coefficient is approximately twice the latter. But as the large ions outnumber the small ones by 50 to 1, the chief factor in recombination is the large ion; or—to go back one step—the neutral centre. Hence anything that removes these large nuclei will tend to reduce recombination; this is the present-day method of accounting for the action of so-called "dust." As to what happens to these nuclei at the temperature of liquid air one can only speculate. They must either be deposited in the U-tube or else dissociate into smaller bodies, whose chances of combining with a small ion are rare.

de Broglie (*loc. cit.*) states that high temperatures break up the nuclei; it is reasonable to suppose that a low temperature could do the same. If we supposed them to consist partly of solid and partly of gaseous matter, then the great difference in the coefficients of expansion of the different constituents could conceivably break up the complex system at a low temperature, on the same principle that damp rocks crack in cold weather. It is important to note that water plays no part in this action; the effect occurs equally well in dry or damp air. Whatever may be the composition of the neutral centre (and therefore of the large ion) evidently water is not present to any large extent.

In conclusion, I wish to thank Prof. Sir J. J. Thomson for his valuable suggestions and kindly interest during these experiments.

On the Scattering of rapidly moving Electrified Particles. By Sir J. J. Thomson, Cavendish Professor of Experimental Physics.

[Read 21 *February*, 1910.]

When rapidly moving electrified particles pass through matter, each particle as it passes through an atom of the substance, or perhaps even when it passes close to such an atom, is deflected. The amount of the deflection will vary with the way the particle strikes the atom; there will, however, be a mean value for the deflection produced by an atom on the direction of motion of a particle passing through it, and when we are considering only the effects produced by large collections of particles we may suppose that the path of each particle suffers the mean deflection. The direction of this deflection is quite arbitrary. Let us consider now the case when a large number of particles pass through a large number of atoms and consider what would be the average deflection of the particles after they have passed through n atoms. Since the direction of the deflections are quite arbitrary, it is evident that the problem is the same as that of finding the average value of the resultant of n displacements of arbitrary phase and of constant amplitude θ; if θ is the average deflection of a particle passing through an atom. This average value is known (see Lord Rayleigh, *Theory of Sound*, 2nd Edition, Vol. I, p. 35) to be $\sqrt{n}.\theta$. Thus if the electrified particles are corpuscles moving normally through a plate of thickness t, then if there are N atoms per unit volume of the plate, and if b is the radius of an atom, the number of atoms traversed by a particle on its journey through the plate is $N\pi b^2 t$, and hence the mean value of the deflection experienced by a particle when passing through the plate is $\sqrt{N\pi b^2 t}.\theta$.

This supposes that the particle is not bent so much in passing through the plate that the length of its path is materially different from t.

We shall now proceed to calculate the value of θ.

Regarding the atom as consisting of N_0 negative corpuscles, accompanied by an equal quantity of positive electricity, the deflection a negatively electrified particle experiences when passing through the atom arises from two causes. (1) The repulsion of the corpuscles distributed through the atom, and (2) the attraction of the positive electricity in the atom.

The amount of deflection due to (2) will depend upon whether the positive electricity is uniformly distributed through the atom,

or whether it is supposed to be divided into equal units, each occupying a finite volume probably much greater than the volume occupied by a corpuscle.

We shall calculate the deflections due to the negative and positive charges separately. Let us take that due to the corpuscles first. We can show easily by the theory of forces varying inversely as the square of the distance that when the moving particle is travelling so rapidly that its deflection is small, this deflection is equal to

$$\frac{2e^2}{mV^2}\frac{1}{x},$$

when V is the velocity of the particle, e its charge, m its mass, and x the perpendicular let fall from the corpuscle on the direction of motion of the particle. Thus the mean value of the deflection produced by the corpuscles which are within a distance a of the line of motion of the particle, supposing the corpuscle uniformly distributed is

$$\frac{4e^2}{mV^2}\frac{1}{a}.$$

Now if the length of the path of the particle in the atom is l, the number of collisions between the particle and the corpuscles within a distance a from its path is, when the corpuscles are uniformly distributed, $n\pi a^2 l$, when n is the number of corpuscles per unit volume of the atom; hence by the theory of probability the average value of the total deflection of the corpuscle when passing through the atom is

$$\frac{4e^2}{mv^2}\frac{1}{a}\sqrt{n\pi a^2 l}$$

$$=\frac{4e^2}{mv^2}\sqrt{n\pi l},$$

Now if b is the radius of the atom, the mean value of $\sqrt{l}$ is $\frac{4}{5}\sqrt{2b}$. Hence θ_1 the mean deflection of the particle due to the corpuscles in the atom is given by the equation

$$\theta_1=\frac{32}{5}\frac{e^2}{mv^2}\sqrt{n\pi b}$$

$$=\frac{16}{5}\frac{e^2}{mv^2}\frac{1}{b}\sqrt{\frac{3N_0}{2}},$$

where N_0 is the number of corpuscles in the atom.

Let us now take the case of the positive electricity, let ϕ_1 be the average deflection when the positive electricity amounting

to $N_0 e$ is supposed to be uniformly distributed through the sphere of radius b, then it is easy to prove that when ϕ_1 is small it is given by the equation

$$\phi_1 = \frac{e^2}{mv^2}\frac{N_0}{b}\frac{\pi}{4}.$$

When the positive electricity is made up of definite units θ_2 the mean deflection due to these is given by the equation

$$\phi_2 = \frac{16}{5}\frac{e^2}{mv^2}\frac{1}{b}\sqrt{\frac{3N_0}{2}}\sqrt{1-\left(1-\frac{\pi}{8}\right)\sigma^{\frac{1}{3}}},$$

where σ is the ratio of the volume occupied by the positive electricity to the volume of the atom.

The mean deflection θ due to both negative and positive particles will be

$$(\theta_1^2+\phi_1^2\}^{\frac{1}{2}} \text{ or } (\theta_1^2+\phi_2^2\}^{\frac{1}{2}},$$

according as we take the first or second hypothesis.

The average deflection when passing through a thin plate whose thickness is t is $\sqrt{N\pi b^2 t}.\theta$, and substituting the values of θ, we find for this quantity

$$\left.\begin{array}{l} \dfrac{e^2}{mv^2}\left\{\dfrac{384}{25}N_0+\dfrac{\pi^2}{16}N_0^2\right\}^{\frac{1}{2}}\sqrt{N\pi t} \\ \text{or} \quad \dfrac{e^2}{mv^2}\left[\dfrac{384}{25}N_0\left\{2-\left(1-\dfrac{\pi}{8}\right)\sigma^{\frac{1}{3}}\right\}\right]^{\frac{1}{2}}\sqrt{N\pi t} \end{array}\right\} \quad \ldots\ldots\ldots(A)$$

according as we suppose the positive electricity to be uniformly distributed through the atom or done up into separate units.

As these expressions contain no unknown factors beyond N_0, the number of corpuscles in an atom, they indicate that experiments on the scattering of light by very thin plates afford a simple method of determining the number of corpuscles in the atom. In making these experiments it is necessary to remember that we have supposed the deflection small so that the length of path was equal to the thickness of the plate, and secondly we have supposed that the velocity remained unchanged; as the deflection varies rapidly with the velocity this condition is important, and it is the more so because the expressions for the change in the velocity of the corpuscle produced by collision with an atom are (see *Discharge of Electricity through Gases*, 2nd Edition, p. 378) much more complicated than those for the deflection, they involve other quantities besides the number of corpuscles in the atom and thus are not suitable for measuring that quantity. I am not aware of any experiments made quite

under the conditions contemplated in equations A, but such are now being made by Mr Crowther at the Cavendish Laboratory. The observations we have are, however, sufficient to show that N_0 is of the same order as the atomic weight of the atom.

We see from equations A that when the deflection is small the thickness of the layer of a substance required to produce a given deflection varies as

$$\left.\begin{aligned} &\frac{m^2V^4}{e^4}\frac{1}{NN_0}\left\{\frac{385}{25}+\frac{\pi^2}{16}N_0\right\} \\ \text{or}\quad &\frac{m^2V^4}{e^4}\frac{1}{NN_0}\frac{385}{25}\left\{2-\left(1-\frac{\pi}{8}\sigma\right)\right\}\end{aligned}\right\} \quad \text{.....(B)}$$

according as the positive electricity is uniformly distributed through the atom or collected into separate units. We can show that this result is true even when the deflection is not small provided the velocity of the particles remains unaltered.

For let $f(z, \phi) \times \theta$ be the fraction of the particles which at a distance z from the point of projection, measured parallel to the original direction of projection, have a deflection ϕ, where ϕ is between $m\theta$ and $(m+1)\theta$.

Let λ be the mean free path of a particle, then $f(z+\lambda\cos\phi, m\theta)$ will be got from those particles which at a distance z had deflections $(m-1)\theta$ or $(m+1)\theta$; each of these particles will have made another collision, and if they are equally likely to be deflected in one direction as the opposite, we see that

$$f(z+\lambda\cos\phi, m\theta) = \tfrac{1}{2}f\{z,(m-1)\theta\} + \tfrac{1}{2}f\{z,(m+1)\theta\},$$

(compare Lord Rayleigh, *Theory of Sound*, vol. I. p. 35).

Expanding by Taylor's Theorem we get

$$\frac{df(z)}{dz}\lambda\cos\phi = \tfrac{1}{2}\frac{d^2f(z,\phi),\theta^2}{d\phi^2},$$

or

$$\cos\phi\frac{\lambda}{\theta^2}\frac{df(z,\phi)}{dz} = \tfrac{1}{2}\frac{d^2f(z,\phi)}{d\phi^2},$$

hence if we put

$$\frac{\theta^2 z}{\lambda} = z',$$

we have

$$\cos\phi\frac{df}{dz'} = \tfrac{1}{2}\frac{d^2f}{d\phi^2},$$

an equation which determines f as a function of z' and ϕ. Since the same value of z' will give the same value of ϕ, it follows that

layers of different substances will produce the same deflection if their thicknesses are proportional to λ/θ^2; since $\lambda = \frac{1}{N\pi b^2}$ we see that this implies that the results expressed by equations B are true even when the deflections are not small.

In the preceding investigation we have supposed that the angular deflections were all in one plane, the differential equation satisfied by $f(z, \phi)$ when the deflections take place in any plane may be found as follows:

As before the particles for which $z = z + \lambda \cos\phi$, and $\phi = \phi_1$ must have come from the particles determined by z and ϕ_2 where

$$\cos\phi_1 = \cos\phi_2 \cos\theta + \sin\phi_2 \sin\theta \cos\psi, \ldots\ldots\ldots(1)$$

ψ is the angle which the plane in which the deflection takes place makes with the plane through the original direction of the particle and its direction just before it experiences the deflection θ. As all directions of ψ are equally probable the probability of ψ being between ψ and $\psi + d\psi$ is $d\psi/z\pi$. Hence,

$$f(z + \lambda\cos\phi, \phi_1) = \int \frac{d\psi}{\pi} f(z, \phi_2),$$

by Taylor's Theorem, the right-hand side is equal to

$$f(z\phi_1) + \frac{d}{d\phi_1} f(z\phi_1) \int_0^{2\pi} \frac{d\psi}{2\pi} (\phi_2 - \phi_1) + \tfrac{1}{2} \frac{d^2}{d\phi_1^2} f(z\phi_1) \int_0^{2\pi} \frac{d\psi}{2\pi} (\phi_2 - \phi_1)^2 \ldots\ldots\ldots(2).$$

From equation (1) we get

$$\phi_2 - \phi_1 = \theta\cos\psi - \tfrac{1}{2}\theta^2 \cot\phi_1 \sin^2\psi,$$

substituting this value of $\phi_2 - \phi_1$ in equation (2) we get

$$\lambda\cos\phi \frac{df}{dz} = -\tfrac{1}{4}\theta^2 \cot\phi \frac{df}{d\phi_1} + \frac{\theta^2}{4} \frac{d^2f}{d\phi^2},$$

or

$$\frac{4\lambda}{\theta^2} \frac{df}{dz} = -\frac{1}{\sin\phi} \frac{df}{d\phi} + \frac{1}{\cos\phi} \frac{d^2f}{d\phi^2},$$

or if $\cos\phi = t$, the equation may be written

$$\frac{4\lambda}{\theta^2} \frac{df}{dz} = \frac{1 - t^2}{t} \frac{d^2f}{dt^2},$$

or with the same notation as before,

$$4 \frac{df}{dz'} = \frac{1 - t^2}{t} \frac{d^2f}{dt^2}.$$

The conclusions drawn when the deflections were all in one plane may thus be extended to the more general case.

Although as yet we have no experiments in which the arrangements have been such as to admit of an accurate application of the formulae obtained in this paper, we have data by which we can calculate the order of the quantity N_0 the number of corpuscles in an atom.

Let us find the path of a particle which moves so that its deflection is equal to the average deflection for the number of collisions made by the particle, *i.e.* if ϕ is the angle through which the direction of the particle is deflected, n the number of collisions made by the particle

$$\phi = \sqrt{n}\,\theta.$$

If s is the length of path travelled by the particle, λ the mean free path $n = s/\lambda$, and $\phi^2 = s\theta^2/\lambda$.

If x is the distance, measured parallel to the direction of projection, travelled by the particle

$$\frac{dx}{ds} = \cos\phi,$$

or since

$$s = \frac{\lambda \cdot \phi^2}{\theta^2},$$

$$dx = \frac{2\lambda}{\theta^2}\cos\phi, \phi \cdot d\phi,$$

or

$$x = \frac{2\lambda}{\theta^2}\{\phi\sin\phi + \cos\phi - 1\},$$

so that when $\phi = \pi/2$, or the particle is bent at right angles to the direction of projection,

$$x = \frac{\lambda}{\theta^2}(\pi - 2),$$

when x is greater than this the particle will begin to travel back again, hence this value of x must be comparable with the distance at which the number of particles crossing a plane at right angles to the direction of projection is reduced to one half of those projected.

Substituting the value of λ/θ^2 previously found we get

$$x = (\pi - 2)\frac{m^2v^4}{e^4N\pi}\left[\frac{25}{384N_0\left\{2 - \left(1 - \frac{\pi}{8}\right)\sigma^{\frac{1}{3}}\right\}}\right]$$

if we take the second of equations (B),

putting $e/m = 5{\cdot}1 \times 10^{17}$, $e = 5 \times 10^{-10}$, $v = 10^{10}$, $N = 2 \times 3 \times 10^{19}$,

we find that for any gas at atmospheric pressure when the particles have this velocity

$$x = (\pi - 2) \times \frac{50}{N_0 \left\{ 2 - \left(1 - \frac{\pi}{8} \right) \sigma^{\frac{1}{3}} \right\}} \text{ roughly.}$$

Becker found that cathode rays of about this velocity travel through about ·5 cm. of oxygen before the number moving forwards is reduced to $\frac{1}{2}$. Putting $x = \frac{1}{2}$ we get for N_0 about 50, if the number of corpuscles were equal to the atomic weight N_0 would be equal to 16. Thus these experiments show that the number of corpuscles in the atom is of the same order as the atomic weight; we must, however, wait for experiments made on different lines before we can determine the exact relation between the number of corpuscles and the atomic weight.

Jacobi's double-residue theorem in relation to the theory of point-groups. By A. C. Dixon, Sc.D., F.R.S., Trinity College.

[Read 23 *May*, 1910.]

In this paper I have shewn how Jacobi's theorem leads directly to the chief general propositions of the theory of point-groups in a plane, and have also given a discussion of a converse theorem. No account has been taken of coincidences among the points of a group.

1. Jacobi's theorem is as follows. Let u, v, w be three polynomials in two variables x, y, of degrees m, n, $m+n-3$ respectively, and let J be the Jacobian $\dfrac{\partial(u, v)}{\partial(x, y)}$. If the mn points of intersection of the curves $u=0$, $v=0$ are all distinct and at finite distance from the origin, and are denoted by (x_r, y_r) $(r=1, 2 \ldots mn)$, then

$$\sum_{r=1}^{mn} w(x_r, y_r)/J(x_r, y_r) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

(See for instance Netto, *Vorlesungen über Algebra*, vol. II, pp. 165—173.)

The following proof is a modification of one given by Netto, after Kronecker.

If w is an arbitrary polynomial of degree $m+n-3$, it contains $\frac{1}{2}(m+n-1)(m+n-2)$ arbitrary coefficients, and if w is restricted by being supposed to vanish at (x_r, y_r) $(r=1, 2 \ldots mn)$, this number of coefficients is brought down to

$$\tfrac{1}{2}(m+n-1)(m+n-2) - mn$$

or

$$\tfrac{1}{2}(m-1)(m-2) + \tfrac{1}{2}(n-1)(n-2) - 1$$

if all the conditions $w(x_r, y_r)=0$, to be satisfied by those coefficients, are independent, that is, unless there is some relation

$$\Sigma A_r w(x_r, y_r) = 0$$

satisfied by *all* polynomials w of the degree $m+n-3$.

Now if ϕ, ψ are any polynomials of the degrees $n-3$, $m-3$,

$$u\phi + v\psi$$

is of the degree $m+n-3$ and vanishes at the mn points, and contains $\frac{1}{2}(m-1)(m-2) + \frac{1}{2}(n-1)(n-2)$ arbitrary coefficients, namely those in ϕ and ψ, these being all effective unless for some set of coefficients $u\phi + v\psi$ is identically zero, that is, unless u, v

have a common factor*; which is contrary to the supposition that the curves $u=0$, $v=0$ meet in mn isolated points.

There must then be a relation of the form

$$\sum_{r=1}^{mn} A_r\, w(x_r, y_r) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2),$$

satisfied by an arbitrary polynomial w of degree $m+n-3$.

The coefficients $\boldsymbol{A}$ can be found by constructing certain particular polynomials by Kronecker's method.

* There is a slight difference in the argument at this point when the number of variables is greater than two. Suppose for instance there to be three variables x_1, x_2, x_3 and u_1, u_2, u_3 to be polynomials of degrees m_1, m_2, m_3 and w to be of degree $m_1+m_2+m_3-4$, so that Jacobi's theorem becomes

$$\Sigma w \Big/ \frac{\partial(u_1, u_2, u_3)}{\partial(x_1, x_2, x_3)} = 0.$$

It is possible to have polynomials ϕ_1, ϕ_2, ϕ_3 of degrees m_2+m_3-4, m_3+m_1-4, m_1+m_2-4, such that

$$u_1\phi_1 + u_2\phi_2 + u_3\phi_3 \equiv 0.$$

Using a bar to distinguish the terms of highest degree we have

$$\bar{u}_1\bar{\phi}_1 + \bar{u}_2\bar{\phi}_2 + \bar{u}_3\bar{\phi}_3 = 0,$$

a homogeneous relation. It is supposed that the surfaces u_1, u_2, u_3 have no common point at infinity, and hence when $\bar{u}_2=0$ and $\bar{u}_3=0$, $\bar{u}_1$ cannot vanish, so that $\bar{\phi}_1=0$. Thus by Nöther's theorem (§ 7 below)

$$\bar{\phi}_1 \equiv \bar{u}_2\bar{\psi}_3 - \bar{u}_3\bar{\psi}_2,$$

where $\bar{\psi}_2$, $\bar{\psi}_3$ are of degrees m_2-4, m_3-4 and homogeneous. It follows that

$$\bar{u}_2(\bar{\phi}_2 + \bar{u}_1\bar{\psi}_3) + \bar{u}_3(\bar{\phi}_3 - \bar{u}_1\bar{\psi}_2) \equiv 0,$$

and that

$$\bar{\phi}_2 = \bar{\psi}_1\bar{u}_3 - \bar{\psi}_3\bar{u}_1,$$
$$\bar{\phi}_3 = \bar{\psi}_2\bar{u}_1 - \bar{\psi}_1\bar{u}_2,$$

where $\bar{\psi}_1$ is homogeneous of degree m_1-4.

Hence in the identity

$$u_1\phi_1 + u_2\phi_2 + u_3\phi_3 \equiv 0,$$

ϕ_1, ϕ_2, ϕ_3 may be replaced by the polynomials of lower degrees,

$$\phi_1 - u_2\bar{\psi}_3 + u_3\bar{\psi}_2, \quad \phi_2 - u_3\bar{\psi}_1 + u_1\bar{\psi}_3, \quad \phi_3 - u_1\bar{\psi}_2 + u_2\bar{\psi}_1,$$

and the degrees of these may be lowered similarly until we arrive at the result

$$\phi_1 = u_2\psi_3 - u_3\psi_2, \quad \phi_2 = u_3\psi_1 - u_1\psi_3, \quad \phi_3 = u_1\psi_2 - u_2\psi_1,$$

where ψ_1, ψ_2, ψ_3 are of the degrees m_1-4, m_2-4, m_3-4.

Thus if ϕ_1, ϕ_2, ϕ_3 are arbitrary polynomials of their degrees the effective number of arbitrary coefficients in

$$u_1\phi_1 + u_2\phi_2 + u_3\phi_3$$

is the number of coefficients in ϕ_1, ϕ_2, ϕ_3 diminished by the number in ψ_1, ψ_2, ψ_3, that is,

$$\tfrac{1}{6}[(m_2+m_3-1)(m_2+m_3-2)(m_2+m_3-3) + (m_3+m_1-1)(m_3+m_1-2)(m_3+m_1-3) + (m_1+m_2-1)(m_1+m_2-2)(m_1+m_2-3) - (m_1-1)(m_1-2)(m_1-3) - (m_2-1)(m_2-2)(m_2-3) - (m_3-1)(m_3-2)(m_3-3)],$$

or

$$\tfrac{1}{6}(m_1+m_2+m_3-1)(m_1+m_2+m_3-2)(m_1+m_2+m_3-3) - m_1m_2m_3 + 1.$$

Since $x^p y^q - \xi^p \eta^q = (x^p - \xi^p)\, y^q + (y^q - \eta^q)\, \xi^p$, the sum of two terms which contain the factors $x - \xi$, $y - \eta$ respectively, we have, taking the terms of u separately,

$$u(x, y) - u(\xi, \eta) = U_1 (x - \xi) + U_2 (y - \eta) \quad \ldots\ldots(3)$$

where U_1, U_2 are polynomials of degree $m - 1$ in x, y, ξ, η, and when $x = \xi$ and $y = \eta$, U_1, U_2 are equal to the derivatives u_1, u_2.

Similarly

$$v(x, y) - v(\xi, \eta) = V_1 (x - \xi) + V_2 (y - \eta) \quad \ldots\ldots\ldots(4)$$

where V_1, V_2 are of the degree $n - 1$ in x, y, ξ, η and reduce to the derivatives v_1, v_2 when $x = \xi$, $y = \eta$.

Let $U_1 V_2 - U_2 V_1 = \Delta(x, y, \xi, \eta)$, then, by substituting

$$x_r, y_r, x_s, y_s \text{ for } x, y, \xi, \eta \text{ in (3) (4)}$$

we find $\Delta(x_r, y_r, x_s, y_s) = 0$, when $r \neq s$, while it $= J(x_r, y_r)$, when $r = s$. $J(x_r, y_r)$ is not zero since the curves $u = 0$, $v = 0$ have only isolated intersections.

Now $\Delta(x, y, x_s, y_s) - \Delta(x, y, x_t, y_t)$ is of the degree $m + n - 3$ only in x, y, the terms of degree $m + n - 2$ destroying each other. Hence

$$\sum_{r=1}^{mn} A_r \{\Delta(x_r, y_r, x_s, y_s) - \Delta(x_r, y_r, x_t, y_t)\} = 0,$$

that is

$$A_s J(x_s, y_s) - A_t J(x_t, y_t) = 0.$$

This holds for all suffixes s, t and therefore the relation (2) is

$$\Sigma\, w(x_r, y_r)/J(x_r, y_r) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1),$$

which is *Jacobi's theorem.*

2. It follows directly that if w vanishes at $mn - 1$ of the intersections of u, v it vanishes at all, or that any curve of degree $(m + n - 3)$ through $mn - 1$ of the intersections of two curves of degrees m, n passes through all their intersections.

Again, let ϕ be an r^{ic} $(r < m + n - 3)$ vanishing at the first $mn - \alpha$ of the intersections, and ψ an arbitrary $(m + n - r - 3)^{\text{ic}}$. We may put $\phi\psi$ for w and thus we have

$$\sum_{r=mn-\alpha+1}^{mn} \phi(x_r, y_r)\, \psi(x_r, y_r)/J(x_r, y_r) = 0,$$

an equation which includes

$$\tfrac{1}{2}(m + n - r - 1)(m + n - r - 2) \text{ equations,}$$

linear in

$$\phi(x_r, y_r) \; (r = mn - \alpha + 1, \ldots mn),$$

and therefore gives $\phi(x_r, y_r) = 0$ for all these values of r if

$$\alpha = \tfrac{1}{2}(m + n - r - 1)(m + n - r - 2),$$

and if all these α equations are independent, that is, unless for some set of coefficients

$$\psi(x_r, y_r) = 0 \ (r = mn - \alpha + 1, \ldots mn).$$

This is the *theorem of Cayley and Bacharach*, that a curve of degree $r \ (< m + n - 2)$, which passes through all but

$$\tfrac{1}{2}(m + n - r - 1)(m + n - r - 2)$$

of the mn intersections of two curves of degrees m, n, passes through the excepted intersections also unless these excepted points lie on a curve of degree $m + n - r - 3$. There is no exception when $r = m + n - 3$.

Similarly in three dimensions, a surface of degree $m+n+p-4$, which passes through all but one of the intersections of three surfaces of degrees m, n, p, passes through all, and a surface of lower degree r must pass through all the intersections if it passes through all but

$$\tfrac{1}{6}(m + n + p - r - 1)(m + n + p - r - 2)(m + n + p - r - 3),$$

unless these excepted points lie on a surface of degree

$$m + n + p - r - 4.$$

Bacharach has further noticed that the lowest value of β, such that an r^{ic} curve ϕ can pass through $mn - \beta$ of the intersection of u, v, say A, without passing through the rest, say B, is $m + n - r - 1$, and that in such a case the points B are collinear.

For if ψ is any $(m + n - r - 3)^{\text{ic}}$ we have

$$\Sigma \phi\psi / J = 0,$$

the summation being over the β points B, and thus any ψ through all but one of these passes through the other. If

$$\beta = m + n - r - 2,$$

take ψ to consist of straight lines drawn from an arbitrary origin to all but one of the points B: this composite curve cannot *always* pass through the excepted B point, if the B points are distinct. Thus β cannot be less than $m + n - r - 1$.

If $\beta = m + n - r - 1$, suppose B_1, B_2, B_3 to be three of the points B, not in a straight line, and take ψ to consist of the line B_1B_2 and lines joining B_4, B_5, B_6 ... to an arbitrary origin: this will not *always* pass through B_3. Hence all the points B must be collinear.

Also β can have this value. For take

$$u = x f_{m-1} - f_{m+n-r-1} f_{r-n+1},$$

$$v = x f_{n-1} - f_{m+n-r-1} f_{r-m+1},$$

where f_{m-1} is an arbitrary $(m-1)^{\text{ic}}$ and so on. Then a form for ϕ is $f_{m-1}f_{r-m+1} - f_{n-1}f_{r-n+1}$: this is an r^{ic} vanishing whenever u, v both vanish, except at the $m+n-r-1$ points where $x=0, f_{m+n-r-1}=0$.

3. *Theorem of Riemann and Roch.* Take any q points (A) on a curve $u=0$, of degree m. Through these describe an n^{ic} curve $v=0\,(n>m-3)$, cutting $u=0$ in $mn-q$ other points (B).

Let λ be the number of arbitrary coefficients in an n^{ic} vanishing at all the points B, and μ the number of arbitrary coefficients in an $(m-3)^{\text{ic}}$ vanishing at all the points A. Then shall

$$\lambda = \tfrac{1}{2}(n+1)(n+2) - mn + q + \mu.$$

For if ϕ, ψ are of degrees $n, m-3$ we have

$$\sum_{A,B} \phi\psi J = 0.$$

Suppose ϕ to vanish at B; then we have here λ relations satisfied by the values of ψ at A, but among these relations

$$\tfrac{1}{2}(n-m+1)(n-m+2)+1$$

are illusory, namely those given by putting $\phi = v$ or $u\chi$ where χ is of degree $n-m$. The number of relations is thus reduced to

$$\lambda - \tfrac{1}{2}(n-m+1)(n-m+2) - 1,$$

but it is not less than this, for the number of illusory equations is the number of linearly independent $n^{\text{ics}}\,\phi$ which vanish at all the points A, B. Let ϕ be any such, then by a suitable choice of the constant a we can make $\phi - av$ vanish at a new point P on $u=0$ and therefore contain u' as a factor, u' being that factor of u which vanishes at P. Thus $\phi - av \equiv wu'$, say. If $u = u'u'' \ldots$, the factors $u', u'' \ldots$ being of course irreducible and of degrees $m', m'', \ldots$ then w is of degree $n-m'$ and vanishes at the nm'' points where $\phi=0$ and $v=0$ meet $u''=0$. Hence w must contain u'', and so on for all the other factors. Thus even when u is composite, the only n^{ics} which vanish at all the points A, B are included in the form $av + u\chi$.

The values of an arbitrary $(m-3)^{\text{ic}}$ at the points A are then connected by

$$\lambda - \tfrac{1}{2}(n-m+1)(n-m+2) - 1$$

linear relations, and μ, the number of arbitrary coefficients in an $(m-3)^{\text{ic}}$ through the q points A is therefore

$$\geqslant \tfrac{1}{2}(m-1)(m-2) - q + \lambda - \tfrac{1}{2}(n-m+1)(n-m+2) - 1,$$

or

$$\lambda + mn - \tfrac{1}{2}(n+1)(n+2) - q \ldots\ldots\ldots\ldots (5).$$

Again, in the equation $\sum_{A,B} \phi\psi/J = 0$ suppose ψ to vanish at the points A. Then ψ contains μ arbitrary coefficients and thus the values of any ϕ at the points B are connected by μ linear equations, of which none can be illusory since no ψ can vanish at all the mn points A and B. Hence λ, the number of arbitrary coefficients in an n^{ic} through the points B,

$$\geqslant \tfrac{1}{2}(n+1)(n+2) - mn + q + \mu,$$

that is, $$\mu \leqslant \lambda + mn - \tfrac{1}{2}(n+1)(n+2) - q \quad \ldots\ldots\ldots\ldots(6).$$

Comparing (5), (6), we have the *theorem of Riemann and Roch*, that

$$\lambda = \tfrac{1}{2}(n+1)(n+2) - mn + q + \mu.$$

4. It is important to prove that no other linear relation except (1) connects the values of an arbitrary $(m+n-3)^{\text{ic}}$ at the mn intersections of $u=0$, $v=0$, that is, that an $(m+n-3)^{\text{ic}}$ w can be found such that

$$w(x_r, y_r) = a_r \, (r = 2, 3 \ldots mn)$$

where the $mn-1$ quantities a_r have any values whatever. Such a polynomial is in fact given by Kronecker's method of interpolation, and is

$$\sum_{r=2}^{mn} \frac{a_r}{J(x_r, y_r)} \{(\Delta(x, y, x_r, y_r) - \Delta(x, y, x_1, y_1)\}.$$

Hence no other linear relation than (1) connects the values of w at the mn points, and from the course of the proof in § 1, any w which vanishes at the mn points must be expressible in the form $u\phi + v\psi$, where ϕ, ψ are polynomials of the degrees $n-3$, $m-3$.

Two other proofs will now be given of this result, which was first proved by Nöther (*Math. Ann.* vol. 6, p. 354).

5. In the theorem of Riemann and Roch put $m+n-3$ for n and suppose the points B to be (x_r, y_r) $(r = 2, 3 \ldots mn)$, so that q takes the value $m(m+n-3) - (mn-1)$ or $m(m-3)+1$. Thus no $(m-3)^{\text{ic}}$ can contain all the points A, and μ takes the value 0,

$$\lambda = \tfrac{1}{2}(m+n-1)(m+n-2) - (mn-1) = \tfrac{1}{2}(m-1)(m-2) + \tfrac{1}{2}(n-1)(n-2).$$

This is exactly the number of arbitrary coefficients in the expression $u\phi + v\psi$, so that the result follows.

6. For a third proof, apply Jacobi's theorem to the polynomials $(x-\xi)u$, $(y-\eta)v$, w, where ξ, η are the coordinates of an arbitrary point.

The points where $(x-\xi)u$, $(y-\eta)v$ both vanish are

(1) The points (x_r, y_r) and here

$$\frac{\partial\{(x-\xi)u, (y-\eta)v\}}{\partial(x, y)} = (x_r - \xi)(y_r - \eta) J(x_r, y_r);$$

(2) The point (ξ, η) where the same Jacobian is equal to uv;
(3) The points (ξ, Y_r) where $x = \xi$, $v = 0$: at these the Jacobian is equal to $u(y-\eta)v_2$;
(4) The points (X_r, η) where $y = \eta$, $u = 0$ and here the Jacobian is equal to $v(x-\xi)u_1$.

Thus substituting x, y for ξ, η we have

$$\frac{w}{uv} + \sum_{r=1}^{mn} \frac{w(x_r, y_r)}{(x-x_r)(y-y_r)J(x_r, y_r)} + \sum_{r=1}^{n} \frac{w(x, Y_r)}{u(x, Y_r)(Y_r - y)v_2(x, Y_r)} + \sum_{r=1}^{m} \frac{w(X_r, y)}{u_1(X_r, y)(X_r - x)v(X_r, y)} = 0, \ldots (7),$$

and the degree of w may be anything up to $m+n-1$. We have to examine the third and fourth terms on the left in (7). As to the third term, let w/uv be reduced to partial fractions as a function of y, x being treated as parametric. The denominators of these fractions are $y - Y_r (r = 1, \ldots n)$ and the factors of u.

The fraction whose denominator is $y - Y_r$ has for its numerator $w(x, Y_r)/u(x, Y_r)v_2(x, Y_r)$ and the sum of these n fractions, with sign changed, forms the third term in (7). Let their sum be brought to a common denominator, v, the numerator will then be of degree $n-1$ in y, and its coefficients being symmetrical in $Y_1, Y_2 \ldots Y_n$ will be rational functions of x, but in general fractional: let P denote this numerator. Similarly the other partial fractions will have a sum Q/u, where Q is of degree $m-1$ in y, and its coefficients are rational in x.

Thus
$$\frac{w}{uv} = \frac{P}{v} + \frac{Q}{u},$$
$$w = Pu + Qv:$$

but this identity determines P, Q uniquely if their degrees are $n-1$, $m-1$ in y, unless u, v as functions of y have a common factor, which is only true for special values of x, namely $x_1, x_2 \ldots x_{mn}$.

Similarly, if R, S are integral in x and of degrees $n-1$, $m-1$ and are such that

$$w = Ru + Sv,$$

the fourth term in (7) must be $-S/u$.

Hence (7) becomes

$$\frac{w}{uv} + \sum_{r=1}^{mn} \frac{w(x_r, y_r)}{(x-x_r)(y-y_r)J(x_r, y_r)} - \frac{P}{v} - \frac{S}{u} = 0 \ldots\ldots (8),$$

and if P is not integral its denominator is a function of x only, and that of S is a function of y only. These denominators must in fact be $\Pi(x-x_r)$ and $\Pi(y-y_r)$.

If then $w(x_r, y_r) = 0$, $(r = 1, 2 \dots mn)$, we have from (8)

$$w = Pu + Sv$$

and P, S can no longer be fractional, since w is not fractional and the denominators of P, S have no common factor.

Hence any polynomial w of degree $\leqslant m + n - 1$, which vanishes at (x_r, y_r) $(r = 1, 2 \dots mn)$, must be of the form $u\phi + v\psi$ where ϕ, ψ are polynomials of degrees $n - 1$, $m - 1$ respectively.

7. If w is of degree $m + n - 2$ only, the terms of degree $m + n - 1$ in $u\phi + v\psi$ must cancel, which can only happen if the terms of degrees $n - 1$, $m - 1$ in ϕ, ψ vanish identically, since the curves u, v have no intersection at infinity. By applying this argument repeatedly we find that the degrees of ϕ, ψ are $r - m$, $r - n$ where r is that of w.

To extend the theorem to higher values of r than those for which it has been proved, it is only necessary to note that when $r > m + n - 2$, homogeneous polynomials ϕ_1, ψ_1 of degrees

$$r - m, \; r - n$$

can be found such that the highest terms in $u\phi_1 + v\psi_1$ coincide with those in w, so that by subtraction the degree of w is reduced; when $r = m + n - 2$, the degree can be reduced by subtracting $u\phi_1 + v\psi_1 + cJ^*$, where the polynomials ϕ_1, ψ_1 and the constant c are suitably chosen. By applying Jacobi's theorem to the reduced expression $w - u\phi_1 - v\psi_1 - cJ$, whose degree is $m + n - 3$, we find $-mnc = 0$, so that c must be zero, if w vanishes at all the intersections.

8. *Converse of Jacobi's Theorem.* Suppose now that mn points (x_r, y_r) $(r = 1, 2 \dots mn)$ are such that, for *any* polynomial w of degree $m + n - 3$,

$$\Sigma \, \alpha_r \, w(x_r, y_r) = 0 \quad \dots\dots\dots\dots\dots\dots (9),$$

the coefficients α_r being independent of those in w, and let us investigate whether these mn points are necessarily the complete intersection of two curves of degrees m, n.

We may also take the conditions involved in (9) in a form in which they have been discussed by Serret, Sylvester, Clifford and others, namely

$$\Sigma \, \alpha_r (ax_r + by_r + c)^{m+n-3} = 0 \dots\dots\dots\dots\dots (10),$$

for all values of a, b, c.

Divide the mn points into two groups, A and B, containing respectively $\frac{1}{2}m(m + 3)$ and $\frac{1}{2}m(2n - m - 3)$ points. Let λ be the number of independent m^{ics} vanishing at A and l the

* See for instance *Camb. Phil. Proc.* vol. 14, p. 389. This step is not necessary if the proof of § 6 is used.

number among these that vanish at B also. Let κ be the number of independent $(n-3)$-ics vanishing at B and k the number among these that vanish at A also.

If ϕ, ψ are of the degrees m, $n-3$, we have

$$\Sigma\, \alpha_r \phi(x_r, y_r)\, \psi(x_r, y_r) = 0 \quad \ldots\ldots\ldots\ldots(11),$$

and by taking ϕ to vanish at A we have $\lambda - l$ homogeneous linear relations among the values of ψ at B. It is here supposed that none of the coefficients $\alpha_1, \alpha_2 \ldots$ vanish. Hence

$$\kappa \geqslant \tfrac{1}{2}(n-1)(n-2) - \tfrac{1}{2}m'(2n-m-3) + \lambda - l \ldots(12),$$

and similarly by taking ψ in (11) to vanish at B we have

$$\lambda \geqslant \tfrac{1}{2}(m+1)(m+2) - \tfrac{1}{2}m(m+3) + \kappa - k \ldots\ldots(13).$$

By addition

$$k + l \geqslant \tfrac{1}{2}(m-n+1)(m-n+2) + 1 \ldots\ldots\ldots(14).$$

Similarly

$$i + j \geqslant \tfrac{1}{2}(n-m+1)(n-m+2) + 1 \ldots\ldots\ldots(15),$$

if i, j are the numbers of polynomials of degrees $m-3$, n respectively which vanish at all the points (x_r, y_r). The desired conclusion can often be deduced from (14) and (15). For instance, if $m = n$, so that $i = k$, $j = l$, we have $l \geqslant 2$ if $k = 0$; that is, the m^2 points are common to two m^{ics} unless they lie on an $(m-3)^{\text{ic}}$, and similarly in other cases. The relations (14), (15) moreover, as equalities, are those satisfied in general when the mn points are the intersections of an m^{ic} and an n^{ic}. Still, this is not the only possible consequence of the condition (9), as the following cases shew.

I. Take $m = 8, n = 7$, so that there are 56 points, and any curve of degree 12 through 55 of them contains all. Take 56 of the intersections of two curves of degrees 5, 12. Then any 12^{ic} through 54 of these will contain all, and therefore they will satisfy two conditions such as (9) and yet will not in general be the complete intersection of a septimic and an octavic.

II. Take $m = n = 9$, so that there are 81 points. Choose these among the 90 intersections of two curves of degrees 15, 6. Then a 15^{ic} through 80 of them will generally pass through the other one, and a condition of the form (9) is satisfied, but the points are not the complete intersection of two nonics.

Thus the *converse of Jacobi's theorem* appears to consist in the statement that (14) (15) follow from (9).

9. If u, v are of the same degree, $p+1$, and p of their intersections are collinear we may take w to be $\phi\delta$ where δ is the line containing p intersections and ϕ is an arbitrary $(2p-2)^{\text{ic}}$. Then a relation

$$\Sigma \alpha_r \phi(x_r, y_r) = 0 \quad \ldots\ldots\ldots\ldots\ldots(16)$$

holds among the values of ϕ at the p^2+p+1 other intersections, α_r having the value

$$\delta(x_r, y_r)/J(x_r, y_r).$$

This case is rather exceptional, in that u, v are not the only independent $(p+1)^{\text{ics}}$ through the points involved. We may write

$$u \equiv \delta V - \epsilon U, \quad v \equiv \delta W - \zeta U$$

where U, V, W are p^{ics} and δ, ϵ, ζ linear; then $\epsilon W - \zeta V$ is an independent $(p+1)^{\text{ic}}$ through the p^2+p+1 points, and the equations satisfied by these points are*

$$\begin{Vmatrix} U & V & W \\ \delta & \epsilon & \zeta \end{Vmatrix} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (17).$$

In applying the converse theorem of §8 to this case it is convenient to take (16) in the form

$$\Sigma \alpha_r (ax_r + by_r + c)^{2p-2} = 0$$

and integrate, say with respect to b. Thus

$$\Sigma \alpha_r (ax_r + by_r + c)^{2p-1}/y_r = \text{a function of } a, c \text{ only},$$

which must be homogeneous and of degree $2p-1$, and can therefore be represented by the sum of p terms of the form

$$\beta(a\xi + c)^{2p-1}.$$

We have, then, an identity of the form

$$\Sigma \beta_r (ax_r + by_r + c)^{2p-1} = 0,$$

containing p^2+2p+1 terms, in p of which $y_r = 0$. The theorem of §8 is therefore applicable, m, n being each $= p+1$.

* A relation of the same kind as (16) can be found among the points where the determinants (17) vanish, even when neither row is linear.

On the phosphorescence observed on the glass of vacuum tubes when the pressure is not very low. By Sir J. J. Thomson, Cavendish Professor of Experimental Physics.

[*Read* 14 March 1910.]

If an electric discharge is sent through a vessel from which the air is gradually exhausted, at a certain stage of the exhaustion the whole of the walls of the tube will be found to be phosphorescent. This phosphorescence is of quite a different colour from that produced by the cathode rays and occurs at a much higher pressure, the pressures at which it is brightest vary with the dimensions of the vessel and with the gas inside it, but they are of the order of 1 mm. of mercury. The colour of the phosphorescence with soft soda glass is an olive-green, quite distinct from the yellowish-green of the phosphorescence due to cathode rays, with lead glass this phosphorescence begins by being greenish but gets blue as the pressure diminishes. The following experiments show, I think, that the cause of the phosphorescence is ultra-violet light produced by the electric discharge. A large vessel was divided into two parts A and B, separated by an opaque screen which was perforated with a long narrow channel, thus if any light were produced in A it would enter B as a fine pencil. The discharge was sent through A and when the stage was reached when the walls of A phosphoresced the place where a pencil going through the channel would strike the walls of B became phosphorescent with a well-defined spot, a little powdered millemite placed on the glass greatly increased the brilliancy of the spot. Various substances were placed in the path of the pencil, glass was found to be fairly opaque to these rays, although the amount of phosphorescence produced after the pencil has passed through a cover slip is quite appreciable, and with care the pencil can be detected after it has passed through the walls of a vacuum tube. Quartz is much more transparent to the pencil than glass, and white fluorite than quartz. The refraction of the pencil by the fluorite was quite marked: a plate of fluorite with parallel sides supported by a glass rod working in a ground-glass joint was put in the way of the pencil, as the plate was rotated the spot of light due to the pencil on the screen moved backwards and forwards just as a spot due to visible light would have done.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On the Mobilities of the Ions produced in Air by Ultra-Violet Light. By A. Ll. Hughes, M.Sc., 1851 Exhibition Research Scholar, Scholar of Emmanuel College, Cambridge. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 9 May 1910.]

Since the discovery of the ionisation of gases by X-rays, several investigations have been made to determine whether there is any increase in the conductivity of gases when exposed to ultra-violet light. It is well known that a large current of negative electricity can be obtained from metallic and other surfaces when ultra-violet light falls upon them. Yet when the same ultra-violet light is passed through a gas it is difficult to get evidence of any increased conductivity. Absorption of ultra-violet light by a gas does not necessarily mean a corresponding ionisation, as Henry * and Whiddington † found for iodine vapour which absorbs ultra-violet light considerably but shows no increased conductivity due to the light.

Lenard ‡ carried out some experiments which appeared to show that air was made conducting under the action of a very absorbable kind of ultra-violet light. He determined the mobilities of the ions so produced, and found for the negative a mobility of 3·13 cm./sec. and for the positive a mobility of ·0015 cm./sec. The latter is of quite a different order to the mobilities of ions produced by X-rays and the radiations from radio active substances. This led to the suggestion that the positive carrier in Lenard's experiments is not a positive ion but a dust particle which has

* Henry, *Proc. Camb. Phil. Soc.* IX. p. 319, 1897.
† Whiddington, *Proc. Camb. Phil. Soc.* XV. p. 189, 1909.
‡ Lenard, *Ann. der Physik*, I. p. 486, III. p. 298, 1901.

lost negative electricity in accordance with the ordinary photo-electric effect at the surfaces of solids or liquids. This view was confirmed by the experiments of Bloch*, who repeated Lenard's experiments in such a way that the air could be rendered free from dust. With ordinary air he obtained Lenard's results, with dust-free air he was unable to detect any conductivity in the air due to the action of the ultra-violet light.

Professor Sir J. J. Thomson† has carried out some experiments in which he found that the ultra-violet light from the glow in the lime cathode discharge produced a small conductivity in a stream of air, increasing the current to about six times the natural leak.

In a letter to *Nature*, Palmer‡ stated that he found that air was rendered conducting by ultra-violet light of wave length shorter than about λ 1850 (Ångstrom units). No details are given.

Stark§ has investigated the effect of ultra-violet light on the conductivity of gases and has obtained positive results with certain complex organic vapours—anthracene, diphenylmethane, diphenylamine and α naphthylamine. Stark concludes from his experiments on air that Lenard's results do not indicate a true ionisation of the air by ultra-violet light. He points out in discussing researches on the ionisation of air by ultra-violet light, that, before one can say that a conductivity in air is really due to ionisation of air molecules, the absence of dust particles and droplets must be proved and that positive as well as negative ions must be shown to exist in the air.

I was unable to obtain any definite indication of ionisation of air due to ultra-violet light which had passed through thin quartz plates. The sources of light used were the mercury arc, the glow in a lime cathode discharge and the discharge in hydrogen.

There was reason to believe that, if the ionisation of air depended upon the wave length of the light employed, the shorter the wave length, the more ionisation one would expect.

Professor Lyman has carried out several important researches upon ultra-violet light especially in the region of very short wave lengths. He found that the ultra-violet spectrum of hydrogen was full of closely packed lines extending down to λ 1030‖. Another research¶ of his was carried out to find whether there was any substance more transparent to ultra-violet light than quartz. Many substances were tried, but only one—fluorite—was found to transmit ultra-violet light of shorter wave length

* Bloch, *Le Radium*, p. 240, 1908.
† Sir J. J. Thomson, *Proc. Camb. Phil. Soc.* xiv. p. 417, 1907.
‡ Palmer, *Nature*, 77, p. 582, 1908.
§ Stark, *Phys. Zeits.*, Sept. 15, 1909.
‖ Lyman, *Astrophysical Journal*, xxiii. p. 181, 1906.
¶ *Ibid.*, xxv. p. 45, 1907.

than that transmitted by quartz. Thin quartz was found to transmit down to a wave length λ 1450 while some specimens of clear colourless fluorite transmitted down to λ 1230.

It therefore seemed more promising to use fluorite instead of quartz. Several crystals of clear colourless fluorite were obtained and plates about 2 cms. thick were cut from them. Most of them in the process of cutting and grinding broke up into bits too small to be of any use. One plate, however, perfectly free from flaws was obtained with a clear space of 2 cms. in diameter. This was the piece used in the following experiments.

The apparatus used is shown in fig. 1. The light is obtained from the discharge tube *L*. The two electrodes were separated by a glass tube with a narrow opening facing the fluorite plate. In this way, great concentration of luminosity was obtained at the end of the inner tube. The electrodes were connected to the secondary of a small induction coil used as a transformer. The primary was connected to the electric light mains (alternating) using as a resistance a few lamps in parallel. With hydrogen in the tube at a pressure of 2 or 3 mm., a steady source of light was easily obtained.

The air is drawn in through a wide plug of cotton-wool and passes up a narrow tube until it reaches the fluorite window. The end of the tube is within 1 mm. of the fluorite window, which insures that all the air which passes through the apparatus comes under the action of the easily absorbed short wave length ultra-violet light. The air then passes into the ionisation cylinder through a short glass tube suitably bent to prevent reflection of the ultra-violet light on the electrodes. For the mobility experiments two electrodes *N* and *F* along the axis were used.

It was found that when the light was produced in *L*, the stream of air flowing through the ionisation cylinder was highly conducting and contained both positive and negative ions, the latter predominating. Control experiments were made (1) with the air in motion but without the light, and (2) with the light but no motion of the air. Under these conditions there was no leak in the ionisation cylinder.

To make sure that the positive carriers were not positively charged dust particles, the air after leaving the ionisation cylinder was drawn through a Wilson expansion apparatus where samples could be tested. With expansions of less than 1·25 no nuclei were obtained, while above that expansion there was no increase over the ordinary effect obtained with dust-free air. This showed that the cotton-wool plug was efficient in removing dust particles.

There seems to be no way of explaining the presence of these positive ions other than by an actual ionisation of the air by the ultra-violet light.

In view of Lenard's results, it appeared advisable to measure the mobilities of these ions. An absolute determination would have been more difficult than a comparative one, and for the purpose of the research hardly as conclusive as a direct comparison with X-ray ions. The method adopted was a modification of Zeleny's* or Rutherford's†.

If ionised air be drawn through the ionisation cylinder (fig. 1) and the quantities of electricity received in unit time by the "far" electrode F be plotted against the potentials on the cylinder, then a curve will be obtained cutting the abscissa at the smallest potential required to drive all the ions into the "near" electrode N. The form of this curve will depend upon the distribution of the ions in the stream of air. One would expect a fairly uniform distribution on consideration of the way in which the air streams up against the fluorite window.

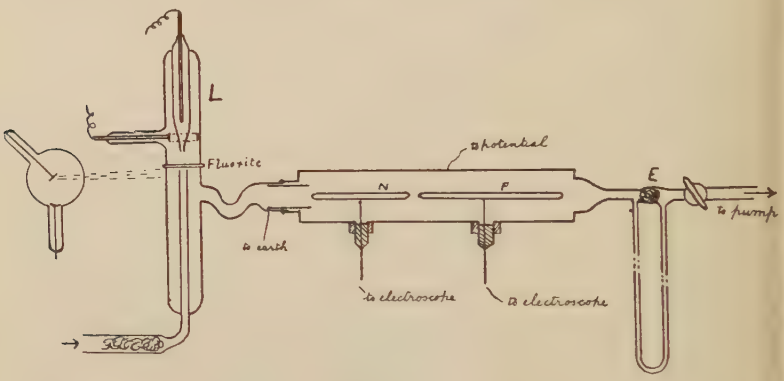


Fig. 1.

In the comparative experiments the air was ionised by X-rays in as nearly as possible the same place as when ionised by ultra-violet light. A beam of X-rays 2 cms. wide was directed at that portion of the apparatus just under the fluorite window, all other parts of the apparatus being shielded by thick lead screens. The X-ray bulb was placed at a distance of 25 cms. from the fluorite window. An idea of the effect of the light may be obtained from the fact that the *positive* leak in the apparatus was of the same order when the ionisation was produced by the light as when produced by the X-ray bulb working vigorously. (The X-rays had to pass through the glass tube, the walls of which were 1 mm. thick.) The negative leak when the ionisation was produced by the light was about twenty times as large as the positive.

* Zeleny, *Phil. Trans.* A. 195, p. 193, 1900.
† Rutherford, *Phil. Mag.* Vol. 47, p. 109, 1899.

In order to obviate time measurements and to be independent of any variations in the intensity of the light or of the X-rays, the quantity of electricity received by the "near" electrode N was measured as well as that received by the "far" electrode F. The total quantity received by N and F was taken as a standard and the actual value received by F was divided by this. This was the quantity plotted in the curves.

The velocity of the air was indicated by the pressure drop across a small plug of cotton-wool at E and was kept at approximately the same value throughout each series of experiments.

Both electrodes were connected to Wilson tilted electroscopes and to capacities of about 400 cms. The electroscopes were calibrated after each reading by means of a potentiometer.

TABLE I. *Ionisation produced by Ultra-Violet Light.*

n = quantity of electricity received by "near" electrode.
f = " " " "far" "
$F = \frac{f}{n+f}$ = quantity of electricity received by "far" electrode for a given number of ions passing into the ionisation cylinder.

Potential on cylinder	n	f	F	Potential on cylinder	n	f	F
− 8 volts	·75	·44	·370	+ 10 volts	·87	·61	·412
−10 "	1·05	·43	·291	+ 12 "	·68	·35	·340
−12 "	1·22	·33	·214	+ 14 "	·82	·26	·241
−14 "	1·21	·21	·148	+ 16 "	·96	·20	·174
−16 "	2·14	·24	·110	+ 18 "	·82	·12	·128
−18 "	1·07	·073	·067	+ 20 "	·98	·10	·093
−20 "	1·20	·070	·055	+ 26 "	·98	·03	·029

TABLE II. *Ionisation produced by X-rays.*

Potential on cylinder	n	f	F	Potential on cylinder	n	f	F
− 8 volts	·465	·27	·370	+ 10 volts	·463	·285	·379
−10 "	·660	·27	·290	+ 12 "	·500	·220	·306
−12 "	·800	·095	·107	+ 14 "	·538	·180	·250
−14 "	·530	·069	·115	+ 16 "	·538	·140	·180
−16 "	·545	·056	·093	+ 18 "	·565	·083	·128
−18 "	·575	·0	·0	+ 20 "	·535	·083	·134
				+ 26 "	·620	·058	·048

The quantities F are plotted against the potentials in figs. 2 and 3.

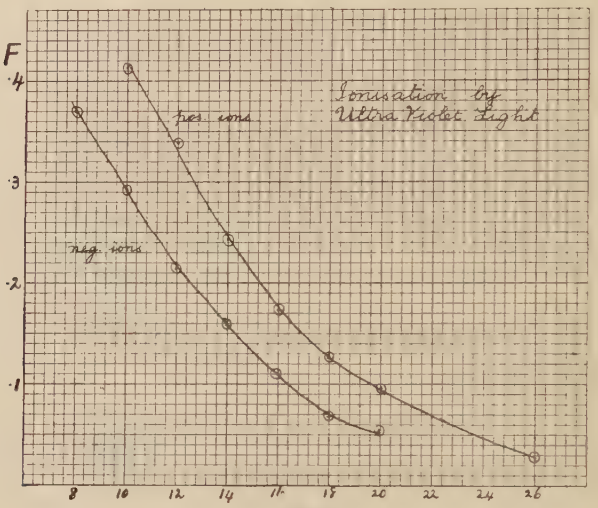


Fig. 2.

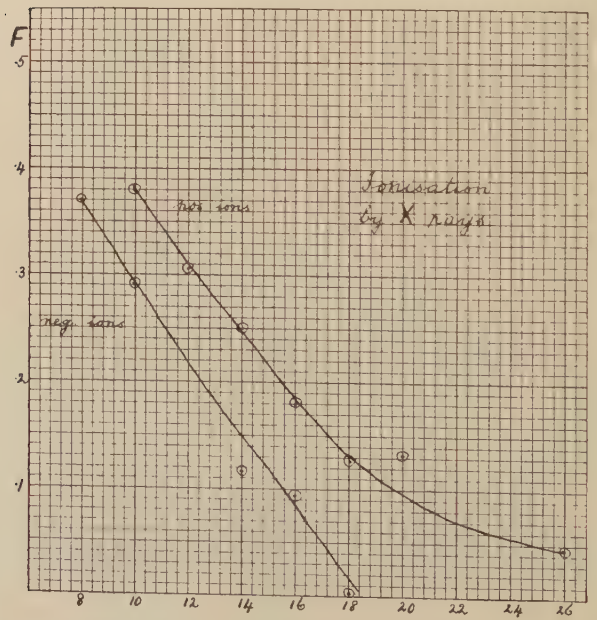


Fig. 3.

The curves are in fair agreement, showing at any rate that

there is not much difference between the mobilities of the ions produced by X-rays and those produced by ultra-violet light. The figures at the end of the tables are not so reliable as those at the beginning, for example the F corresponding to -18 volts in Table I is ·073 and this was calculated from an electroscope reading of ·8 of a division, which from its small size is liable to considerable error.

Absolute agreement between the two sets of curves could only be expected if the ions were distributed in the stream of air in precisely the same way. It was thought, however, that considerations of this sort could not wholly explain the differences between the results given above and that a better agreement was possible.

As nearly all the negative ions in the experiment may be put down to surface ionisation, it was not necessary to investigate their mobilities, for Rutherford* has shown that surface ions are identical with negative ions produced by X-rays, etc. The results for the positive and the negative ions produced by ultra-violet light and the positive ions only produced by X-rays are given in Tables III and IV.

During the short time occupied in taking a reading, the velocity of the air did not alter appreciably, but it was difficult to adjust the velocity each time to exactly the same value. A small correction was therefore made in the figures of Tables III and IV. The pressure drop across the cotton-wool plug E was about 150 cms., and it was assumed that the variations in the velocity of the air were proportional to the small deviations from this value and that the quantity of electricity F was increased in the same proportion. Therefore, to obtain the value of F for a standard velocity corresponding to a pressure drop of 150 cms. the value was *diminished* in proportion to the *excess* of the pressure over 150 cms. and conversely. This correction, which was never more than about 5 % or 6 % brought the results into better agreement.

TABLE III. *Ionisation by Ultra-Violet Light.*

Potential	F	Potential	F
− 6 volts	·522	+ 8 volts	·415
− 8 „	·312	+10 „	·367
−10 „	·225	+12 „	·256
−12 „	·156	+14 „	·217
−14 „	·117	+16 „	·151
−16 „	·059	+18 „	·112
−18 „	·038	+20 „	·073
		+26 „	·041

* Rutherford, *Proc. Camb. Phil. Soc.* IX. p. 401, 1898.

TABLE IV. *Ionisation by X-rays.*

Potential	F	F from Table III	F (ultra-violet), $-F$ (X-rays)
+10 volts	·348	·367	+·019
+12 „	·248	·256	+·008
+14 „	·210	·217	+·007
+16 „	·158	·151	−·007
+18 „	·132	·102	−·020
+20 „	·078	·073	−·005
+26 „	·082	·041	−·041

The results given in these tables are plotted in fig. 4. The

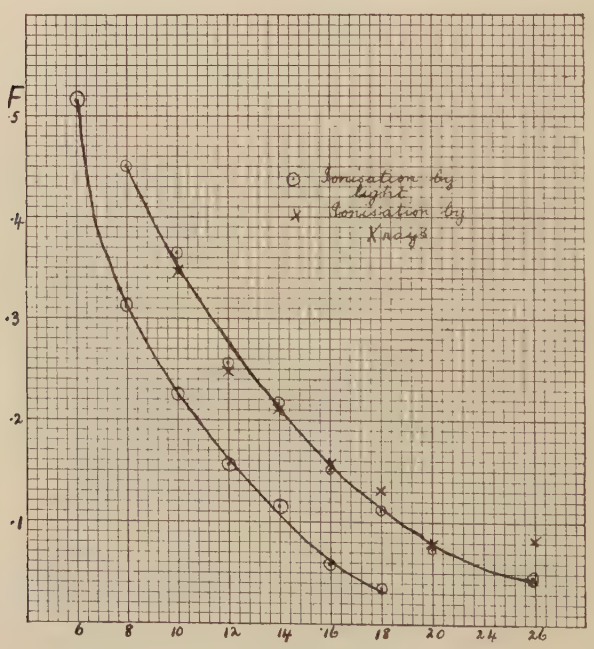


Fig. 4.

third column in Table IV is copied from Table III for comparison with the second column and the outstanding differences are given in the last column. From these results, it may be concluded that the mobilities of the positive ions produced by ultra-violet light and by X-rays are identical.

The piece of fluorite used in the experiment was the only piece which would transmit ultra-violet light capable of producing positive ions in any quantity. Many other pieces cut from different crystals seemed opaque to the radiation producing the positive ions, but in every case the negative (surface) ions were obtained though in less quantity than before. In this connection Lyman's work shows that the limit of the spectrum transmitted by fluorite varies considerably in different specimens. The shortest wave length transmitted was λ 1230. These results suggest that the ionisation of air by ultra-violet light sets in at some wave length between λ 1230 and λ 1450 and increases very rapidly with decreasing wave length.

I have great pleasure in thanking Professor Sir J. J. Thomson for suggesting the investigation to me and for his interest in the course of the work.

On a Dissymmetry in the Emission of the Cathode Particles which are produced by Homogeneous Röntgen Radiations. By R. T. Beatty, M.A., B.E., Emmanuel College. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 9 May 1910.]

When Röntgen radiations fall upon certain metals, homogeneous secondary radiations are produced. If one places a thin metallic sheet in the path of such a homogeneous radiation, cathode particles will emerge from both surfaces of the sheet. The object of the present research is to find out what ratio exists between the quantities of cathode energy leaving the front and back surfaces.

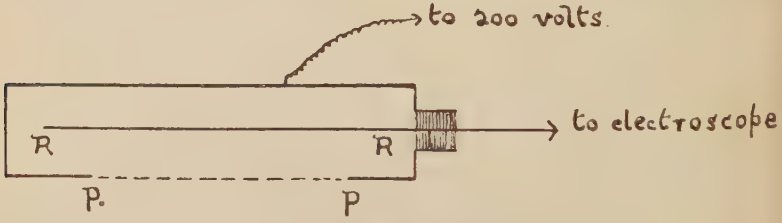


Fig. 1.

A shallow cylindrical brass ionisation chamber (fig. 1) received the radiations through a thin parchment window PP in its lower side. RR is a ring cathode. Two concentric circles were cut out of a sheet of cardboard and an annulus was thus formed which had an external diameter equal to that of the ring electrode, while its internal diameter was about 2 cms. less. A silver leaf (equivalent in weight to 2 mm. of air) was laid upon two sheets of thin paper (each sheet being equivalent to 1 cm. of air) and the whole was tightly gripped between two cardboard rings made as described, and gummed at the edges.

A second specimen was now made of the exact dimensions of the first, but in this case the silver leaf was placed between the two sheets of paper, the latter absorbing all the cathode particles from the silver.

These rings were placed in turn on RR, and the ionisation measured when a given radiation entered through the window PP. A second electroscope was used to standardise the radiation.

Now in one cardboard ring the arrangement was paper—silver leaf-paper. Let us call this ring O since no cathode particles can escape through the paper. The second ring was made up of silver leaf-paper-paper. We may call this C since cathode particles *do* escape from one side of the leaf.

Now if we take a reading with O in position, and another with O replaced by C, the silver leaf being uppermost, the difference will represent the ionisation due to cathode particles on the emergent side of C. The direct ionisation due to Röntgen radiation will be the same in both cases, since O and C absorb the radiation equally.

If now we reverse C, we can in the same way find the ionisation due to the cathode particles on the incident side of the silver leaf. We must make a correction in the emergent case since the radiation passing through the leaf has been absorbed to some extent by the paper and the leaf itself.

The cathode particles due to soft radiations (Fe, Cu) emerge only from a small fraction of the thickness of the leaf. Hence we may assume that in the emergent case, the radiation has suffered absorption by the whole thickness of the leaf.

This absorption was found to be 14·8 °/₀ for the Fe radiation. Hence the emergence values were multiplied by $\frac{100}{85{\cdot}2}$ to bring them up to the value which they would have had if the Fe radiation had suffered no absorption.

TABLE I. $Ratio = \frac{Energy\ of\ emergent\ cathode}{\text{„}\qquad incident\quad \text{„}}$.

Silver leaf.

Radiator	Uncorrected ratio	Mean	Corrected ratio
Fe	{·846, ·860, ·901}	·869	1·02
Cu	{·925, ·934, ·931}	·930	1·01
Se	{1·09, 1·08}	1·085	1·10
Ag	{1·30, 1·28}	1·29	1·29
Sn	{1·30, 1·306}	1·303	1·303
Al	{1·44, 1·43}	1·435	1·435

Similar corrections were made for the Cu and Se radiations. The corrections for the harder radiations were negligible. The change in the secondary radiation from the leaf when O and C were

interchanged was also negligible. The radiation from Al was excited by very penetrating rays from the bulb, and was passed through Al sheets to cut out the softer portions.

On repeating the experiments with a Cu leaf replacing that of Ag, similar results were obtained (Table II).

TABLE II. *Comparison of ratios for Ag and Cu.*

Radiator	Ratio, Ag leaf	Ratio, Cu leaf
Fe	1·02	—
Cu	1·01	—
Se	1·10	1·08
Ag	1·29	—
Sn	1·303	1·319
Al	1·435	1·42

In the case of the Cu leaf the cathode energy excited by the Fe and Cu radiations was too small to measure.

It will be seen that:

1°. A dissymmetry exists in the amount of energy due to cathode particles emerging from opposite sides of a metal sheet. This dissymmetry increases as the radiation becomes more penetrating.

2°. For a given radiation the dissymmetry is the same for Ag and Cu leaves.

Bragg and Glasson* have shown that a similar dissymmetry exists in the case of secondary Röntgen radiations. They state, however, that when the secondary radiator is Fe or Cu, both of which give out a quantity of homogeneous soft secondary radiation, the dissymmetry is greatly reduced.

In the present research Cu and Ag leaves were used because in the former case the homogeneous radiation is much more easily excited. The dissymmetry in the cathode distribution appears to be independent of the presence or absence of excited homogeneous radiation.

I beg to thank Professor Sir J. J. Thomson for his interest in this investigation.

* "On a Want of Symmetry shown by Secondary X-Rays," *Philosophical Magazine*, June, 1909, p. 855.

On Right- and Left-Handedness in Barley. By R. H. Compton, B.A., Frank Smart Student of Gonville and Caius College.

[*Received* 6 June 1910.]

A seed of barley produces on germination a tubular sheath through which the first green leaf emerges. This first leaf is so folded that one margin overlaps the other, at first throughout nearly its entire length; later on, after more leaves have been produced, only the lower sheathing portion retains the original fold. In some cases the right-hand margin overlaps, in others the left-hand. A convention is necessary in the use of these terms: the one used here is illustrated by figure 1. The observer is supposed to be

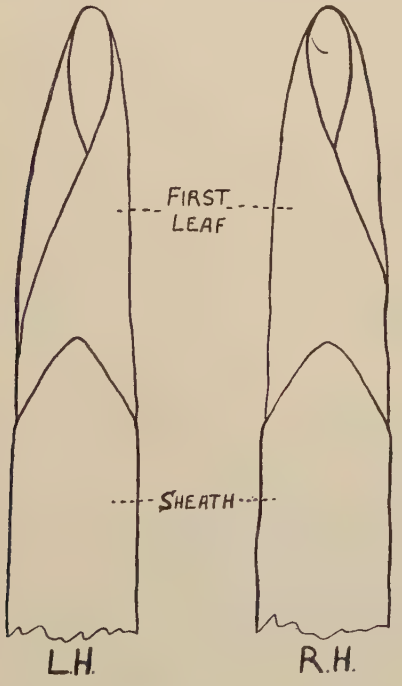


Fig. 1.

Diagram of the two "stereo-isomeric" forms of Barley seedling.

standing in the axis of the plant, facing outwards through the open side of the leaf, and folding one hand over the other to

imitate the leaf-margins. The condition with the right margin overlapping is called right-handed; that in which the left margin overlaps is called left-handed*.

Leaves subsequent to the first are also twisted: the normal arrangement appears to be that R.H. and L.H. leaves alternate all the way up the stem; i.e. if the first leaf be L.H. the second will be R.H., the third L.H., and so on. But this rule is subject to frequent exceptions in which the arrangement may be irregular and two or more successive leaves may have the same twist. The frequency of these exceptions, as well as the difficulty of accounting for them, suggest that the characters of right- and left-handedness are largely under the influence of slight "accidental" causes, of whose nature we are ignorant†.

The fertile axis bears a series of alternate notches in which are inserted the flowers, three to each notch. In six-rowed barley all three flowers set seed; in two-rowed barley only the middle flower of each group of three. I shall speak of the "odd" and "even" notches in the spike; the odd being numbers 1, 3, 5, ..., and the evens being numbers 2, 4, 6, ..., reckoning from the base upwards. The first few flowers are usually sterile.

On flowering the anthers may be extruded, and pollen scattered; but the flowers do not open, and the stigmas never come outside the glumes. Consequently, self-pollination is regularly effected. Prof. Biffen tells me that he has never detected any signs of crossing. It is possible, though not without much difficulty, to perform emasculation and cross-pollination of the flowers: and I have made a few such crosses. The results were meagre, however, and as, for reasons to be explained below, they throw no light on the problem, no further mention will be made of them.

I have worked almost entirely with two-rowed varieties of barley, because some of the manipulation is thereby simplified.

The following points will be considered:

* There is never any real doubt whether the seedling is R.H. or L.H. Occasionally plants are found in which both margins of the leaf appear to be inrolled: but inspection shows that at the base of the leaf the normal overlapping occurs. Among 4000 seedlings 15 such were found.

In one case a single seed produced on germination two shoots of equal size: of these one had the first leaf R.H. the other L.H.

The auricles at the junction of leaf-blade with sheath follow the twist of the leaf: that auricle being lower which is on the overlapping margin.

† See Stratton and Compton, "On Accident in Heredity," in the present number of these *Proceedings*, p. 507.

This overlapping of leaf-margins is entirely different from the rolling of the leaf blades of grasses which is used as a taxonomic character in agriculture. See for example Percival, *Agricultural Botany*, p. 511, where barley, wheat and rye are said to have the "leaf-blades rolled to the right," while in oats the leaf-blade is "generally rolled to the left." See also Hackel, in Engler and Prantl's *Nat. Pflfam.* "Gramineae," p. 4.

(1) The ratios of lefts to rights found in the first leaves of large groups of seedlings of different varieties.

(2) The ratios obtained from individual spikes of seed.

(3) The question whether the twist of the last leaf below the spike has any effect on the twist of the seedlings produced from the spike.

(4) The question as to the effect of the position of a seed in the spike on the twist of the seedling produced from it.

(5) The question whether the twist of the first leaf is inherited.

I. The seeds of certain varieties of barley were sown in large numbers in the ground or on wet canvas or blotting-paper, and the seedlings were counted with a view to determining the ratio of rights to lefts among the first leaves. The following are the tabulated results:

TABLE I.

Variety of Barley	L.H.	R.H.	Ratio L.H./R.H.	Percentage L.H.
Plumage Corn (pure line) .	2408	1604	1·501	60·02
Archer: Danish (pure line)	765	508	1·506	60·09
Archer	819	586	1·389	58·29
Chevalier: Kinver	730	546	1·337	57·21
Chevalier: Prize Prolific...	405	299	1·354	57·53
Goldthorpe (pure line).....	608	463	1·313	56·77
Goldthorpe: Guinness.....	740	587	1·261	55·77
New Binder (pure line) ...	762	571	1·335	57·16
Total.....	7237	5164	1·401	58·36

From the above table it is clear that there is a considerable excess of left-handed over right-handed first leaves in large groups of seedlings. Adding all the plants examined together, we find that out of a total of 12,401 seedlings 58·36 % had the first leaf twisted in the left-handed fashion. The number of seedlings examined, and the constant occurrence of the excess of lefts in all the varieties studied seem entirely to preclude the assumption that this preponderance of lefts is fortuitous.

The ratios obtained in different kinds of barley vary to some extent. It appears probable that the difference between Plumage Corn and Guinness' Goldthorpe, for example, is significant: it seems to be outside the limits of probable error, when we consider the number of plants counted—4012 in Plumage Corn,

1327 in Guinness' Goldthorpe. With regard to the intermediate ratios it is unwise to dogmatise: but it appears likely that differences should exist between different races of barley in this respect.

It is difficult to know how to interpret these results. It is interesting, however, to compare the ratios obtained for barley with those recorded in certain other cases of right- and left-handedness.

In a paper on the inheritance in man of the mode of clasping the hands, Lutz* states that of the total population investigated 61% of the males and 58% of the females habitually cross the right thumb over the left. Here the same percentage occurs as in barley.

In *Anableps anableps*, a Cyprinodont fish, the anal fin is turned either to the right or to the left in different individuals. Garman† found that about $\frac{3}{5}$ of the males are dextral, $\frac{2}{5}$ sinistral: in the females the ratios are reversed. Here again each sex shows a percentage of 60 as between rights and lefts.

Though the repeated occurrence of this percentage is remarkable, there are cases in which practical equality of rights and lefts is found. For instance, in the males of the Fiddler Crabs, *Gelasimus pugilator* and *G. pugnax*, where the chelae are unequally developed, Yerkes‡ states that out of nearly 3000 specimens he found almost equal numbers of rights and lefts.

In species of cotton, right- and left-handed plants may be distinguished according as the accessory bud is on the right- or left-hand side of the median axillary bud. Leake§ found that nearly equal numbers of both sorts of plants are produced, and considers that right- and left-handedness do not follow the ordinary laws of inheritance.

In Teleost fishes the optic nerves cross one another at the optic chiasma. Parker‖ examined a hundred specimens of each of ten species of teleosts, and found 514 with the nerve from the right eye dorsal, 486 with it ventral. Individual species all approximated fairly closely to the ratio of equality. Larrabee¶ investigated the matter further, and found that a random collec-

* *Amer. Naturalist*, XLII. 1908, p. 195. My own experience, however, seems to indicate a preponderance of persons who cross the left thumb over the right.

† *Amer. Naturalist*, XXIX. 1895, p. 1012.

‡ *Proc. Amer. Acad. of Arts and Sci.* XXXVI. 1901, no. 24. Yerkes considers that the occurrence in equal numbers of right- and left-handed Fiddler Crabs shows that these characters are not hereditary, but are due to chance (p. 440). This is clearly a mistake, for the ratio equality in a population may readily occur in cases of Mendelian inheritance: but in any case Yerkes' data are not competent to settle the question of the application of heredity to this instance of right- and left-handedness.

§ *Journ. and Proc. Asiatic Soc. Bengal*, v. 1909, p. 23.

‖ *Bull. Mus. Comp. Zool. Harvard*, XL. 1903, no. 5, p. 219.

¶ *Proc. Amer. Acad. of Arts and Sci.* XLII. 1906, no. 12.

tion of 650 brook trout included 365 with the right-eye nerve dorsal, 285 with it ventral: this being 56·1% of right-handed individuals. Among 4615 trout, including the above numbers and also the offspring of several selected matings, Larrabee found that 56% had the nerve of the right eye dorsal. Since he showed that the dimorphism is not hereditary, it seems probable that this percentage is significant.

The case of Gryllus, investigated by Lutz*, is interesting as an instance of a species typically of one only of the two "stereoisomeric" forms, in which accident causes a certain proportion of individuals to assume the reverse form.

The extraordinary results obtained by Mayer†, in some species of the land-snails, Partula from Tahiti, may be mentioned here, as showing that the proportion of rights and lefts may vary with the geographical race.

In many asymmetric Mollusca the occurrence of the twist the reverse of normal is sporadic, the abnormal specimens never reaching a high proportion‡. In the Achatinellidae Gulick§ states that some species are normally dextral, others normally sinistral, while others again are both dextral and sinistral: but the proportions of rights and lefts in the last cases are not, so far as I know, on record.

In the other cases which I have been able to collect from the very scattered literature the number of rights and lefts were too small to fix the ratio with exactness.

* * * * *

The constancy of the ratio in successive generations of barley is attested by the following table, giving two years' counts in the same family of Kinver Chevalier.

TABLE II.

Year	Twist of first leaf		Ratio L.H./R.H.	Percentage L.H.
	L.H.	R.H.		
1909.....	730	546	1·337	57·21
1910.....	379	259	1·463	59·40
Expectation for 1910 on ratio 1·337	364	274		

* *Canadian Entomologist*, XXXVIII. p. 207.

† *Mem. Mus. Comp. Zool. Harvard*, XXVI. 1902, no. 2.

‡ A useful summary of such cases is given by Sykes, in *Proc. Malacol. Soc.* VI. 1905, p. 253.

§ "Evolution, Racial and Habitudinal," *Carnegie Institute Publications*, XXV. 1905.

The discrepancy between the two generations is only such as would be removed by a turn-over of fifteen lefts to rights, assuming the 1909 ratio to be absolutely accurate.

In this connection it should be remarked that stability of population with regard to certain characters occurs, in the absence of selection, as a result of the Mendelian inheritance of those characters*. Hence the occurrence of a stable ratio in this case is not in itself incompatible with the inheritance of the characters concerned in a Mendelian fashion: though of course it does not necessarily tend towards such a view, other alternatives being possible.

II. The ratios for whole populations having been obtained, the question arises as to the ratios given by individual spikes of fruit on germination. In order to determine this a number of ears of "Plumage Corn" barley were laid whole on wet blotting-paper, and on germination the seedlings of each spike were counted for rights and lefts among the first leaves. Each seedling was classified according to:

(i) The twist of the last foliage leaf below the spike, whether *right* or *left*.

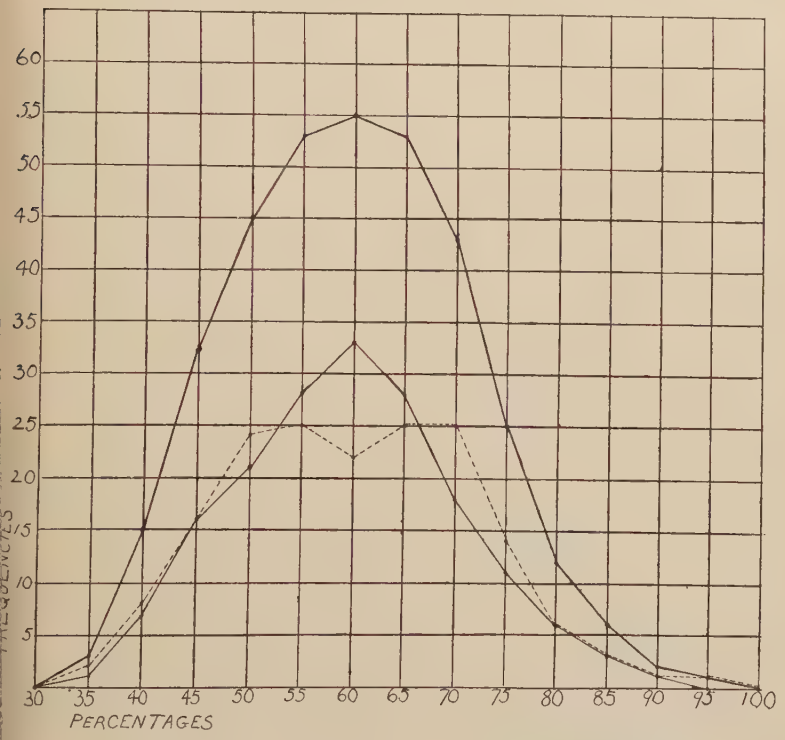
(ii) The row of seeds in the ear from which the seedling arose, whether *odd* or *even*.

The percentages of left-handed seedlings for each individual spike, in which a reasonable number of seeds (more than ten) germinated, were calculated; and curves were drawn expressing the relation of these percentages to the frequency of each percentage. The curves were obtained by plotting, at intervals corresponding to every $5°/_{\circ}$, the number of percentages included in a range of $5°/_{\circ}$ on either side. Thus the number plotted for $60°/_{\circ}$ is that of the percentages between $55°/_{\circ}$ and $65°/_{\circ}$: the number plotted at $65°/_{\circ}$ is that of the percentages between $60°/_{\circ}$ and $70°/_{\circ}$: and so on. The curve for spikes above L.H. last leaves is drawn as a continuous line (the lower in the diagram); that for those above R.H. last leaves as a dotted line: there were 86 spikes of each class. The taller continuous curve is given by all the 172 spikes taken together.

The curves for the spikes above L.H. and R.H. last leaves respectively differ somewhat in shape. The former is steeper and narrower, and indicates less range of variability than the latter. The curve for R.H. top leaves shows two slight maxima, with a minimum where the other curve reaches its maximum. It would be unwise to lay stress on these differences, however, as they are not very pronounced. The total curve is of a form very usual in studies of variability. It is practically symmetrical about its

* Hardy, *Science*, N.S., XXVIII. 1908, p. 49.

mode, which is at 60°/₀. This percentage is the same as the average given by the whole population of seedlings. There is no sign (except possibly in the curve for R.H. top leaves) of any segregation, or of a separation into "pure lines." It would be interesting to study other varieties of barley, not of pure lines,



or which give different ratios of rights and lefts in the total population, and to determine in what way the curves of percentages differ from those given by the pure line "Plumage Corn."

* * * * *

The results of this portion of the investigation are summarised below in tabular form, which displays certain facts not exhibited by the curves. (Table III.)

III. Table III shows conclusively that the twist of the last foliage leaf, immediately below a spike, has no effect whatever on the ratio of left- to right-handed seedlings borne by that spike. The correspondence between the two classes of spikes is indeed extraordinarily close, and tends to confirm the accuracy of the ratio 1·5 as expressing the proportion of lefts to rights in "Plumage Corn."

TABLE III.

Twist of last foliage leaf below the spike	Odd Seeds			Even Seeds			Total		
	L.H.	R.H.	Ratio L.H./R.H.	L.H.	R.H.	Ratio L.H./R.H.	L.H.	R.H.	Ratio L.H./R.H.
Class I. L.H. ..... (87 spikes)	653	417	1·566	581	406	1·431	1234	823	1·499
Class II. R.H. ..... (105 spikes)	621	406	1·529	553	375	1·475	1174	781	1·503
Total.....	1274	823	1·548 *60·75 °/₀*	1134	781	1·452 *59·22 °/₀*	2408	1604	1·501 *60·02 °/₀*
Expectation, if ratio 1·5 applies to both odds and evens	1258	839		1149	766				

IV. The question as to a possible regularity in the arrangement on the parent plant of seeds bearing definite characters is of great interest. Prof. Bateson considers that there is a strong *a priori* presumption that some regularity exists: but as he and Miss Killby were unable to discover any such regularity in Peas, he is "disposed to think that the process of oögenesis in which the dominant or recessive character of the egg-cell is determined must be liable to disturbance by accidents*."

In the present case it appeared probable that some such arrangement might be discovered; though, as will be shown later, there is here no evidence of heredity. But the character under consideration is one of symmetry, and is determined early in embryogeny; and it seemed not unreasonable to expect that the symmetry of the parent spike might have some clear influence over the symmetry of the embryos borne upon it.

The attempt to discover such an order of arrangement of the right- and left-handed offspring on the parent ear gave, however, inconclusive results. Table III shows the aggregate numbers for the odd and even rows of seeds on the spike. It will be seen that the ratio $\frac{\text{L.H.}}{\text{R.H.}}$ in seeds from odd rows is slightly higher than in those from even rows. The total difference is 1·53%. This discrepancy might be considered insignificant, were it not that the accuracy of the counts, as judged by the agreement between the two classes of spikes, appears to be very high. The fact that both classes of spikes gave a difference between odds and evens in the same direction is also slightly in favour of regarding this difference as significant.

Clearly, however, the evidence is inconclusive; and it would be rash to try to found upon it any theory of the influence of the position of the seed on the parent on the character of the seedling developing therefrom.

No more significant result was obtained by a study of individual spikes. Many single ears of "Plumage Corn," "Kinver Chevalier" and "Goldthorpe" were examined carefully with a view to discovering any regularity which might exist in the arrangement of the seedlings borne upon them. No such regularity was discernible, however.

In a paper by Mackloskie† the following statement is made: "The grains arising on adjoining rows in the ear of corn [maize] are of different castes, and produce 'antidromic' plants (that is, growing up in opposing curves), and that the same property belongs to all the Gramineae." So remarkable is this assertion in the light of the above results for barley, that I repeated Mackloskie's experiment on maize; but completely failed to confirm

* *Reports to the Evolution Committee of the Royal Society*, II. 1905, p. 69.

† "Antidromy in Plants," *Amer. Naturalist*, XXIX. 1895, p. 973.

his statement. Here as in barley there appeared to be no regularity in the arrangement of the two sorts of seeds on the ear.

V. *Inheritance.* In the case of the mode of clasping the hands, Lutz* found that the marriages of R.H. by R.H. gave a considerable preponderance of R.H. among the offspring; those of L.H. by L.H. gave an excess of L.H. children; while those of R.H. by L.H. gave intermediate ratios. Thus the mode of clasping the hands appears to be inherited in some way†.

In the case of the optic chiasma in Teleosts, Larrabee‡ showed that inheritance of the dimorphic condition cannot be detected either by Mendelian or Galtonian methods.

With respect to this subject of the inheritance of characters of symmetry it is almost impossible to form an *a priori* opinion; and it is an open question which is the more remarkable fact—the inheritance of the mode of clasping the hands or the non-inheritance of the dimorphism of the optic chiasma. Much more investigation is necessary to determine the essential distinction that produces the different behaviour of the two similar sets of characters.

The case of barley agrees with that of the optic chiasma in that no inheritance is discernible. This appears to be the conclusion that must be drawn from the following statistics.

The seeds produced in 1909 by 36 plants, the twist of whose first leaf was known, were sown in 1910, each ear of grain being treated separately. The following are the tabulated numbers of right- and left-handed first leaves among the seedlings; all the seedlings produced being included, whether they were borne on the main spike or on "tillers."

TABLE IV. (Kinver Chevalier.)

	L.H. first leaf	R.H. first leaf	Ratio L.H./R.H.
Total seedlings from plants with L.H. first leaf, 1909	134	97	1·381
(Expectation on ratio 1·463	137	94)	
Total seedlings from plants with R.H. first leaf, 1909	245	162	1·512
(Expectation on ratio 1·463	242	165)	
Grand total.....	379	259	1·463

* *Amer. Naturalist*, XLII. 1908, p. 195.

† See Stratton and Compton, "On Accident in Heredity," in the present number of these *Proceedings*, p. 507.

‡ *Proc. Amer. Acad. of Arts and Sci.* XLII. 1906, no. 12.

It is true that this table does not reach the same degree of precision as that attained in the previous discussion, the numbers recorded being smaller. But there appears to be no reason to doubt its substantial accuracy. The ratios given by the offspring of right- and left-handed seedlings respectively are so nearly the same that it seems impossible to detect any hereditary nexus between successive generations. The fact that the proportion of R.H. seedlings produced from (self-fertilised) left-handed plants is slightly greater than that produced from (self-fertilised) right-handed plants seems to emphasise this negative answer to the problem.

* * * * *

The absence of influence of the first leaf on the ratio given by the spike, taken with the similar absence of influence of the last leaf (p. 501), leads to the conclusion that these vegetative characters of the parent plant do not govern the ratio of rights to lefts among the offspring.

Conclusion.

We are confronted with the fact that a random collection of a reasonable number of barley seeds always produces, so far as known, an excess of seedlings with the first leaf twisted in what may be called the left-handed direction. The ratio of lefts to rights in the case of "Plumage Corn" (the variety most accurately studied) approximates very closely to 1·5; in other varieties similar or smaller ratios are found.

Statistics show that:

(i) The twist of the last leaf below a spike has no influence whatever on the ratio of right- to left-handed seedlings produced from that spike.

(ii) The same ratio subsists among the seedlings whether produced from the odd or even rows of seed on the parent ear: and no orderly arrangement of seedlings with respect to the twist of the first leaf could be detected on the ear.

(iii) The ratio among the seedlings is not governed by the twist of the first leaf of the parent plant: i.e. the characters of right- and left-handedness in barley appear not to be hereditary.

Thus the reason for the production of the excess of left-handed seedlings remains an unsolved problem. It seems clear that the cause of this constant excess must be one of some subtlety, such as is usually classed as "accidental." It is possible, of course, to assume that barley is genetically left-handed, and that a constant proportion (in "Plumage Corn" 40%) of "accidental" reversals occur in every population and in each generation. This explanation would cover the observed facts: but it would

hardly change the real problem, which is the nature of the factor determining the assumed accidental reversal.

The twist of the first leaf is determined early in embryogeny; so that the period during which the accidental influence must act is quite limited. Thus it seems possible to conduct experiments on the maturing ears by manipulating the orientation or the environment: and this I hope to do in the future. It has been thought worth while to publish the above results, as marking a definite step taken in analysing the problem under consideration.

Finally, I wish to express my gratitude to Prof. Bateson, who suggested the investigation; to Prof. Biffen, who has most kindly supplied me with seed and with information; to my parents and sister, and to several friends at the Botanical Laboratory, who have greatly assisted me in the labour of counting the seedlings; and to Mr F. J. M. Stratton who helped me with the curves.

On Accident in Heredity, with special reference to Right- and Left-Handedness. By F. J. M. Stratton, M.A., Gonville and Caius College, and R. H. Compton, B.A., Gonville and Caius College.

[*Received* 6 June 1910.]

The characters which an organism exhibits are the product of the action of the environment upon the potentialities inherited from its ancestry. The external conditions may have the effect of preventing the manifestation of an inherited character. For instance, a plant grown entirely in the dark is unable to develop chlorophyll in its plastids. The chlorotic condition induced in this way, though much the same in outward appearance, is of a different nature to that which occurs in a quarter of the seedlings produced by the self-fertilisation of the variegated "aurea" variety of Antirrhinum, where the "presence of chlorophyll" is a Mendelian factor segregating against "absence of chlorophyll*."

It is thus readily possible for two organisms to exhibit the same peculiarity, in the one case as a result of characters inherited from the parents, in the other case owing to the direct action of external conditions.

The case of right- and left-handedness seems to afford a striking example of the possibility of the inversion of inherited characters owing to unknown causes, which we may provisionally call "accidents." The difficulty in analysing the facts in human right- and left-handedness is extreme: especially is this the case in discussing the main functional asymmetry in man, since it is almost impossible to allow for the varying influences of education. But this is not the only source of difficulty. Right- and left-handedness are characters which, though sharply distinct, are yet often of an extremely unsubstantial nature as regards heredity. For example, the mode of clasping the hands with fingers interlocking, varies in different persons: and though the distinction between a person who crosses the right thumb over the left and one who clasps hands in the reverse way is perfectly sharp, yet the real difference between the two persons is obviously trifling. Such an asymmetry we may well expect to be highly susceptible to accidental influences; and it appears probable that a change of character may readily occur. So that a person naturally right-handed in respect of clasping the hands may through accident become apparently left-handed, and *vice versa*.

Two examples may be mentioned in order to emphasize this probability of inversion of character in cases of right- and left-handedness.

The leaves of many Gramineae (e.g. barley, maize) are folded

* Baur, *Ber. d. d. Bot. Gesellsch.* xxv. 1907, p. 442.

so that one margin overlaps the other*. Accordingly there are possible two kinds of folding, which may be distinguished as right- and left-handed. Considering the series of leaves on a stem, the rule is that the twist of successive leaves is alternately right- and left-handed. If the first leaf is R.H. the second will normally be L.H., the third R.H., and so on. But a disturbance of this regular sequence often occurs, with the result that two or more leaves of the same twist arise in succession. This inversion of the normal twist of a leaf or leaves appears to be the result of some accidental condition, so slight as to escape notice, but nevertheless sufficient to effect the change from R.H. to L.H., or *vice versa.*

The second case was investigated by Lutz†, in an American species of Gryllus, in which one tegmen overlaps the other. Among the ♀s Lutz found a total of 742 with the right, 370 with the left, tegmen uppermost. In the ♂s, however, he found 685 rights to only 13 lefts. The reason for this distinction between ♀ and ♂ appears to be the smaller amount by which the tegmina overlap in the ♀ than in the ♂: hence a change in the position of the tegmina is more readily effected in the ♀ than in the ♂. Reversal can be produced by manipulation in the ♀, but in the ♂ discomfort is caused to the insect and the tegmina are soon moved back to their original position‡. Here we have a clear example of the effect of accident in producing left-handed members in what appears to be a normally right-handed race of organisms. The difference between the ♂ and ♀ Gryllus in susceptibility to accident is very instructive.

To return now to the mode of clasping the hands in man. Lutz§ analysed statistics which had been collected by Prof. J. A. Thomson with a view to discovering whether this character is inherited. The following are his results, condensed from his table:

Parents	No. of Families	Offspring		Percentage of R.H.
		R.H.	L.H.	
♂ R. × ♀ R.	75	166	63	72·5
♂ R. × ♀ L.	49	61	49	55·5
♂ L. × ♀ R.	53	86	64	57·3
♂ L. × ♀ L.	36	46	63	42·2

* See Compton, "On Right- and Left-Handedness in Barley," in the present number of these *Proceedings*, p. 495.

† *Canadian Entomologist*, XXXVIII. p. 207.

‡ The ♂s, however, can be made to reverse tegmina while still soft after the moult; and this is perhaps the origin of the few left-handed ♂s.

§ *American Naturalist*, XLII. 1908, p. 195.

Clearly the mode of clasping the hands is inherited in some way: it remains to find a reason why neither the right- nor the left-handed character breeds true.

It seems probable to us that the explanation is to be found in the "accidental" conversion of genetically right-handed individuals into the left-handed condition, and *vice versa*. If this were the case, among a number of apparent rights there would be a certain proportion of genetic lefts; so that their failure to breed true to the right-handed character would in part be due to the admixture of genetically left-handed persons with the apparently right-handed. Further, the same possibility of accidental reversal would apply to the offspring, so that, even if genetically pure right-handed individuals interbred, a certain proportion of the children would appear to be left-handed. All this would of course apply equally to the mating of the left-handed, real and apparent.

* * * * *

Making certain assumptions as to the way in which accidental causes affect the appearance of a character it is easy to see how the inheritance of the character will be modified. The simplest assumptions seem to be that a constant proportion of individuals are affected, and that the change takes place indifferently in either direction. Let the population consist of true homozygous dominants (DD), heterozygotes (DR), and homozygous recessives (RR) in the proportion

$$DD : DR : RR = p : 2q : r.$$

Then for stability through successive generations the equation

$$pr = q^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

must be satisfied*.

Now let θ be the fraction of accidental changes taking place either way, so that the apparent distribution of dominants to recessives will be given by

$$\frac{(p+2q)(1-\theta)+r\theta}{r(1-\theta)+(p+2q)\theta} = \frac{\text{number of dominants } (D)}{\text{number of recessives } (R)} \quad \ldots(2).$$

The value of this ratio is of course known from a count of the population.

We assume that the mating is random, and examine first the offspring resulting from the mating of apparent dominants. Since the proportion of true dominants to factitious ones in the apparent dominants is

$$(p+2q)(1-\theta) : r\theta,$$

the true matings will be in the proportion

dominant × dominant : dominant × recessive : recessive × recessive

$$= \{(p+2q)(1-\theta)\}^2 : 2r(p+2q)(1-\theta)\theta : r^2\theta^2.$$

* G. H. Hardy, *Science*, N.S., XXVIII. 1908, p. 49.

Now the offspring of a cross *true* $D \times D$ will be

$$\text{dominant : recessive} = p^2 + 4pq + 3q^2 : q^2,$$

i.e. $\dfrac{p^2 + 4pq + 3q^2}{(p + 2q)^2}$ of the offspring will be D's.

The offspring of a cross *true* $D \times R$ will be

$$\text{dominant : recessive} = p + q : q,$$

i.e. $\dfrac{p+q}{p+2q}$ of the offspring will be D's.

The offspring of *true* $R \times R$ will be all R's.

The proportion of D's to R's in the offspring of the *apparent* $D \times D$ crosses will be

$$(p^2 + 4pq + 3q^2)(1-\theta)^2 + 2r(p+q)(1-\theta)\theta : q^2(1-\theta)^2 + 2rq(1-\theta)\theta + r^2\theta^2,$$

or say $$X : Y.$$

These offspring will have a constant proportion transformed, and we shall have an apparent distribution of the offspring in the ratio

$$\frac{X(1-\theta) + Y\theta}{Y(1-\theta) + X\theta} = \frac{D}{R} \quad \text{.....(3).}$$

In a similar way we can obtain the ratio of the true D's to true R's resulting from the *apparent* $D \times R$ crosses. They are

$$(p^2 + 4pq + 3q^2)(1-\theta)\theta + r(p+q)(1 - 2\theta + 2\theta^2) : q^2(1-\theta)\theta + rq(1 - 2\theta + 2\theta^2) + r^2(1-\theta)\theta,$$

or say $$X' : Y'.$$

The proportion of apparent D's to R's in these offspring will be

$$\frac{X'(1-\theta) + Y'\theta}{Y'(1-\theta) + X'\theta} = \frac{D'}{R'} \quad \text{.....(4).}$$

The remaining equation comes from the offspring of the *apparent* $R \times R$ crosses, which give D's to R's in the ratio

$$(p^2 + 4pq + 3q^2)\theta^2 + 2r(p+q)(1-\theta)\theta : q^2\theta^2 + 2rq(1-\theta)\theta + r^2(1-\theta)^2,$$

or say $$X'' : Y''.$$

The apparent offspring will be in the ratio

$$\frac{X''(1-\theta) + Y''\theta}{Y''(1-\theta) + X''\theta} = \frac{D''}{R''} \quad \text{.....(5).}$$

Statistical examination of families will give in any particular case the values of the right-hand side of equations (2), (3), (4) and (5). It remains to determine the ratios $p : q : r$, and the value of θ.

In the first place it must be noted that it is not in general possible to satisfy all the five equations (1)—(5) with only three unknowns, viz. θ and the two ratios $p:q:r$ at our disposal. Probably the simplest method of attempting to satisfy the equations is to assume in the first place a value of θ and substitute this value for θ in (2). This gives a linear relation between p, q and r. Multiply this equation by r and substitute $pr=q^2$ from (1), and we are left with a quadratic equation in $\frac{q}{r}$. Thus for any assumed value of θ there will be at most two values for $p:q:r$, and it is an open question whether these values will satisfy the equations (3), (4) and (5). In fact in the one case in which an attempt to apply this method has been made it was found quite impossible to find a value of θ which would fit into the observed facts, so long as the more frequent of the two classes observed was assumed to exhibit the dominant character. Only after a change in the assumption as to which of the characters was dominant was a good fit obtained, and then the approximation was very close.

Taking the case of the mode of clasping the hands investigated by Lutz as a test example, equations (1)—(5) assume the following form:

$$pr = q^2 \quad \text{.....(6),}$$

$$\frac{(p+2q)(1-\theta)+r\theta}{r(1-\theta)+(p+2q)\theta} = \frac{40}{60} \quad \text{.....(7)*,}$$

$$\frac{D}{R} = \frac{57{\cdot}8}{42{\cdot}2} \quad \text{.....(8),}$$

$$\frac{D'}{R'} = \frac{43{\cdot}6}{56{\cdot}4} \quad \text{.....(9),}$$

$$\frac{D''}{R''} = \frac{27{\cdot}5}{72{\cdot}5} \quad \text{.....(10).}$$

These equations apply when the condition with the left thumb uppermost is regarded as the dominant character. Under the contrary assumption the fractions in (7)—(10) would have to be reversed.

Now if we assume a turn-over of 20 °/₀, due to "accidents," from either position, and substitute in (7) $\theta = 0{\cdot}2$, we get

$$2p + 4q - r = 0,$$

or multiplying by r and substituting from (6)

$$2q^2 + 4qr - r^2 = 0,$$

i.e.

$$\frac{q}{r} = \frac{-2 \pm \sqrt{6}}{2}.$$

* Lutz gives 61 °/₀ ♂s and 58 °/₀ ♀s with right thumb uppermost. We take 60 °/₀ as a mean.

Rejecting the negative solution we get $\frac{q}{r} = 0{\cdot}22$, and from (6) $\frac{p}{r} = 0{\cdot}048$.

We may take the numbers $3 : 14 : 65$ as approximately representing the ratios $p : q : r$. Then we get

$$\frac{D}{R} = \frac{55}{45},$$

$$\frac{D'}{R'} = \frac{44}{56},$$

$$\frac{D''}{R''} = \frac{27}{73},$$

a surprisingly close fit to the observed figures (p. 511).

It is not desired to press this particular case too strongly, save as affording an instance in which the admission of an "accidental" factor brings an otherwise discordant inherited character into harmony with known laws.

* * * * *

The above calculations can easily be adapted to suit cases in which the reversal of a character only takes place in one direction. Such a case would be that of the functional asymmetry to which the terms right- and left-handedness are usually applied. Here the effects of education seem to tend steadily to the conversion of left-handed persons into right-handed, and probably never act in the opposite direction.

The object of this note is to show that it is possible, by the exercise of assumptions which seem to be justifiable, to account for certain cases to which the usual Mendelian formulae do not at first seem to apply. The case we have considered specially is an exceptionally favourable one from the present point of view: for educational influence seems to be absent; accidental change of character seems readily to be possible, and would, so far as we can see, occur with equal ease in either direction.

Discontinuities in Light Emission. II. By NORMAN CAMPBELL, M.A., Fellow of Trinity College.

[*Read* 6 June 1910.]

§ 1. The following pages are a continuation of a former paper presented to the Society under the same title*. In the former publication it was stated that the experiments had been unsuccessful in attaining their main object: they have continued unsuccessful.

The difficulty which had prevented the attainment of the desired results was that of finding a source of light which should have at the same time great intensity per unit area and great constancy. The first characteristic was necessary in order that it might be possible to split the light from the source into two beams of sufficient intensity by means of an optical system of lenses and mirrors. The second was necessary because it was found that the relative intensity of the two beams into which the light from the source was split changed with a change in the total intensity of the light. Accordingly, unless the total intensity was quite constant, fluctuations due to alterations in the total intensity would be imposed upon and would mask the fluctuations which it was desired to observe.

Throughout the earlier experiments the source of light used was a Nernst lamp†. The town supply had been used as a source of current, and it was thought that the fluctuations in the intensity of the lamp might be due to the changes in the P.D. of the supply. Arrangements were made to use accumulators as a source of current, but even with this source the intensity of the light, as measured by the resistance of the filament, was far too variable for the purpose of the experiments. The average intensity of the light over a long period was very constant, but there were considerable fluctuations of short period, which were probably due to the cooling of the filament by air currents, but could not be prevented by enclosing the lamp in an almost air-tight enclosure.

It had been found, on the other hand, that the constancy of the light from an Osram wire lamp carrying the current from

* *Proc. Camb. Phil. Soc.* XV. 1909, p. 310.

† An attempt had also been made to use the sun shining in a cloudless sky as a source of light, the two beams being divided by means of a half-silvered mirror. The requisite constancy and intensity appeared to be obtained, so far as could be judged with the use of the inadequate heliostat which was employed in the preliminary experiments. But it would be impossible to carry through the research with such a source of light in a climate where the necessary conditions are fulfilled so seldom.

accumulators was amply sufficient for the purpose: preliminary observations had been made on the fluctuations obtained when the two photoelectric cells were illuminated by two independent lamps of this nature, one of which was placed in front of each cell. But in order that the main experiment might be carried out, it was necessary to divide the light from a single lamp into two beams: and such a division was rendered very difficult by the comparatively large area from which the light was emitted. It was impossible, for instance, to obtain a parallel beam by means of a lens and to divide that beam by reflection, as had been attempted in the case of the Nernst lamp. Since, again, it was necessary that the two beams, falling on the two cells, should proceed originally from the same part of the incandescent surface, it was impossible to obtain such beams as were required by the simple expedient of placing the lamp between the two cells, so that light emitted from the lamp in different directions should fall on the cells: for, with a wire lamp, the light emitted in different directions does not come from the same part of the incandescent filament.

§ 2. An attempt was then made to attain the desired object by means of the following device. A 110-watt Osram lamp with a bulb 12 cm. in diameter was silvered thickly on the outside, so that the walls became very nearly perfect reflectors. Two holes of 1 cm. diameter were made in the silver, and the light emergent from these holes was allowed to fall on the two cells. It is easy to see that the light emerging from either of the holes is made up of rays emitted from every part of the wire, reflected in most cases many times from the silvered walls. Accordingly the two beams emerging from the two holes are "dependent" in the sense of the word employed in the previous paper. The device was so far successful that, on looking into the bulb through one of the holes, no sign of the wire could be seen: the centre of the bulb appeared to be occupied by a uniformly incandescent body, the shape of which, in accordance with theory, was approximately that of the surface enveloping the wire filament.

The device was, however, unsuccessful, for the bulb became so hot that it was necessary to keep it cool by immersion in a water bath. After a few hours in such a bath the silver began to detach itself from the walls, whatever method of silvering was used: covering the silver with several kinds of varnish did not improve matters materially.

§ 3. An attempt was then made to substitute mercury for silver. The lamp with the walls carefully cleaned was immersed in an iron vessel containing clean mercury. Through two holes in the iron vessel passed two glass tubes with ground ends held against the surface of the bulb by springs. These tubes provided

passages for the light through the mercury, and corresponded to the holes in the silver in the earlier arrangement. The mercury acted also as a cooling bath, the heat being removed by means of a stream of water flowing through a jacket covering the top and surrounding the upper part of the iron vessel. Since mercury is not nearly so perfect a reflector as silver, the filament could just be seen on looking in through the tubes, but it could be distinguished only with difficulty from the bright background furnished by reflection.

With this arrangement it was found that the relative intensity of the beams emerging from the tubes, as indicated by the position of balance of the measuring instrument, did not vary appreciably with the total intensity of the light. A change of 10 °/₀ in the current through the lamp was not accompanied by any change in the position of balance which could be observed. The explanation of the change in the relative intensity of the beams from a Nernst lamp with a change in the total intensity of the light was now obvious. The beams into which the light is divided do not come entirely from the same parts of the filament, owing to slight errors in the adjustment of the optical train*. When the resistance of the filament varies, all parts of it do not undergo the same change of temperature, and the light emitted by one part of the filament increases relatively to that emitted from another. But, with the arrangement now adopted, light from every part of the filament emerges through both tubes, and changes in the relative intensity of the light from different parts of the filament have but little influence on the relative intensity of the light emergent through the two tubes.

§ 4. But though the main difficulty which had beset the work hitherto was thus overcome, a solution of the problem had not been attained. For, when an attempt was made to measure the fluctuations obtained on balancing the currents through the two cells, due to the light emergent from the two tubes and incident upon the cells, it was found that these fluctuations were very much larger than those which had been obtained under similar conditions with light from two wholly independent lamps. $\overline{\theta_T'^2}$ was some 12—18 times larger for the two "dependent" than for the two "independent" sources: and, moreover, the magnitude of the fluctuations was much more variable, so that it was impossible to obtain measurements of them which showed any useful agreement. Since on neither of the theories of light which are being examined the fluctuations due to "dependent" sources should be greater than those due to "independent," it was clear that some important influence had been overlooked.

* See § 19 of the previous paper, where it is noted that the effect depends on the adjustment of the optical train and not on that of the measuring system.

A little consideration soon discovered an explanation for the results obtained so obvious that it ought to have been foreseen. Equation (9) § 7 of the previous paper gives

$$\overline{x^2} = N(\omega^2 + \overline{\eta^2}),$$

where $\overline{x^2}$ is the mean square fluctuation in the number of electrons emitted, N the number of light impulses per second, ω the number of electrons liberated by a single impulse and $\overline{\eta^2}$ the mean square fluctuation in ω. The reason for the very large value of $\overline{x^2}$ when the reflecting lamp is used is to be found in the large value of $\overline{\eta^2}$.

Some further considerations as to the value of $\overline{\eta^2}$ when a plain open lamp is placed before the cell will be given later. Here it need only be noted that, in this case, all light impulses which fall upon the active surface of the cell have very nearly the same power of setting free electrons. The only variations in this power arise from slight differences in the distance of different parts of the filament from the cell, or slight differences in the angle at which the light from different parts of the filament is incident about the cell. The greater part of the variation in ω for different light impulses probably arises from effects connected with the nature of ionisation: it is unlikely that the value of $\overline{\eta^2}$ would be much reduced if all light impulses had exactly the same power of forming electrons.

But in the case of the lamp nearly surrounded by a reflecting surface matters are very different. Part of the light which passes out through the tubes has travelled a much longer distance within the bulb than other parts, and has undergone many more reflections. Its intensity is reduced both by the increase in distance and by the loss suffered in reflection by an imperfect reflector. Even if each point of the filament emits in each elementary cone light of the same intensity, the intensity of the light emitted from different points and in different directions will not be the same when it arrives at the active surface. Accordingly, superimposed on the variations of ω due to variations in the process of ionisation, there will be variations, probably of far greater amount, due to variations in the intensity of different light disturbances when they arrive at the cell.

It would be possible by suitable calculation to obtain some measure of the amount of the latter fluctuations, but even for very simple cases, differing considerably from the enclosure of complex shape which surrounds an Osram lamp, the work is very laborious and complicated: it has not been thought worth while to make even an approximation in the absence of any knowledge as to how close the approximation would be.

We have also to take into account the fact that the convection

currents in the mercury probably cause slight changes in the reflecting power of the walls at various points. Additional fluctuations will be due to this cause, to which may also probably be attributed the great irregularity of the magnitude of the fluctuations for the reflecting lamp as compared with those of the simple lamp. These irregularities are so great that it has not been thought worth while to give the results of any of the very large number of measurements which were made on the fluctuations with a lamp of this nature. In some cases measurements made at different times under apparently similar conditions differed by more than 100 %.

§ 5. When the reflecting lamp proved a failure the ingenuity of the author was insufficient to devise any new form of source of light which appeared to offer any reasonable chance of success. The difficulty which has rendered the experiment impossible was not foreseen when the work was started, but has proved insuperable. Its nature may be described briefly once more.

The object of the experiment was to compare the fluctuations due to two beams from independent sources, i.e. beams from different incandescent surfaces, with those due to two beams from dependent sources, i.e. beams proceeding from the same incandescent source and divided, either by being emitted in different directions or by some such process as partial reflection and transmission. In order to be free from fluctuations due to changes in the total intensity of the source, either a source of absolutely constant intensity must be found, or some means devised of splitting the beam in such a way that different parts of it arrive at the cells without any changes in their relative intensity.

§ 6. Though the main object of the research had to be abandoned, some observations were made on the fluctuations due to independent sources. It was hoped that these might throw some light, both on the theory of the measurement of such fluctuations and also, possibly, indirectly on the main problem which had inspired the research.

The independent sources were two 8-volt 16-watt Osram lamps placed one opposite each cell. The total intensity of the light falling on the cell was varied by changing the distance between the lamp and the cell. Compensating resistances were introduced, so that a change in the E.M.F. of the battery from which the lamps were run in parallel did not cause any change in the relative intensity of the light from the two lamps. The remainder of the apparatus was exactly as described in the previous paper. The observations were made and the fluctuations calculated as has been described with one exception (§ 17). In place of series consisting of 200 observations, series of 20 observations were taken. The object of the change was to eliminate errors arising from the constant drift of the position of balance; it was difficult

to obtain series of 200 (13 minutes) without some sign of such change. Since the calculations gave incidentally a computed value for the drift, all series were rejected in which that value showed any material change in consecutive series.

Blank experiments were performed in which the fluctuations were observed while no light fell on the cells. For such blank experiments $\overline{\theta_T'^2}$ had a remarkably constant value, 0·8 scale divisions. The value seemed quite independent of the values of a, b, p, C and R. The fluctuations in such experiments arise probably from two sources. (1) From the fact that the scale was read only to the nearest division, so that, even if the spot was moving perfectly uniformly there would be apparent fluctuations. It is easy to show that for fluctuations arising from this cause $\overline{\theta_T'^2}$ should have the value $\frac{1}{6}$. (2) From chance mechanical disturbances in a building which is by no means completely free from vibration. The independence of the blank fluctuations of the electrical constants of the system seems to show that electrical disturbances were sufficiently excluded. A correction equal to the value of the fluctuations found in the blank experiments was subtracted from all other values of the fluctuations.

It was remarkable that consistent results could be obtained for the value of the mean fluctuation of less than a single scale division: but it must be remembered that the means are never taken for less than 300 observations—sometimes for more than 1000.

§ 7. The first point to be investigated is the variation of the fluctuations with the intensity of the light: this intensity is measured by $N\omega$. Table I gives some of the results.

The variation of the fluctuations with the intensity of the light is important because it might throw some light upon the two theories, the consideration of which is the chief object of the work. On the "spherical wave" theory, N, the number of light impulses falling upon the cell in unit time, is independent of the distance of the source from the cell: the decrease in illumination with increase of distance is due to a decrease in ω, the number of electrons liberated by each impulse. On the "bundle of energy" theory, it is N which changes with the distance, while ω remains constant. Accordingly, if we take only N and ω into account, equation (9) of the previous paper shows that on the first theory $\overline{\theta_T'^2}$ should be proportional to ω^2, i.e. proportional to the square of the intensity of the light, whereas on the second theory $\overline{\theta_T'^2}$ should be proportional to N, i.e. proportional to the intensity simply*.

* A similar argument for the case of γ rays has been given by E. v. Schweidler (*Phys. Zeit.* xi. 1910, p. 225). The considerations about to be given apply also to that argument.

But, if we take account of the quantity $\overline{\eta^2}$, the matter becomes rather more complicated. On the bundle of energy theory the value of $\overline{\eta^2}$ is of no importance for comparative measurements, for ω and therefore, presumably, $\overline{\eta^2}$ are the same for each bundle, whatever the intensity of the light. But on the spherical wave theory the value of $\overline{\eta^2}$ is of great importance, since changes in the intensity of the light are due to changes in the value of ω.

The evaluation of $\overline{\eta^2}$ cannot be made until some knowledge of the mechanism by which the electrons are liberated is obtained. But the general theory of probability indicates that $\frac{\overline{\eta^2}}{\omega^2}$ will increase as ω decreases. On two simple assumptions, which give results that probably differ from the truth in opposite directions, we can calculate the value of $\overline{\eta^2}$. Thus, if we assume that the light disturbance liberates an electron whenever, in passing through a layer of constant thickness, it meets a system in some special condition, it is easy to show that $\overline{\eta^2} = \omega$. But that assumption involves the conclusion that a single light disturbance might liberate an indefinite number of electrons, if only it met a sufficient number of suitable systems—a statement which is obviously untrue, since the liberation of electrons doubtless requires the expenditure of energy and the amount of energy in a single disturbance is limited.

Taking an assumption which represents the other extreme, we may suppose that each disturbance can liberate only one electron, and that it will only liberate one electron if it chances to meet a suitable system before its energy is rendered unavailable for the purpose by passing through the photoelectric layer, or by reflection or by some other cause. ω is now the chance that a disturbance will meet such a system before its energy is lost, and a well-known formula shows that

$$\overline{\eta^2} = \omega(1 - \omega).$$

Hence on the first assumption

$$\overline{x^2} = N(\omega^2 + \omega),$$

on the second assumption

$$\overline{x^2} = N\omega.$$

In the latter case, whatever the value of ω, being, of course, less than 1, $\overline{\theta_T{}^2}$ will be proportional to ω, that is, to the intensity of the light, and no distinction can be made between the two theories. In the former case a distinction can be made only if ω^2 is not small compared to ω.

Now, if Planck's theory of the "elementarwirkungsquantum"

be accepted (and its author holds it to be compatible with the spherical wave theory), the measurements on the photoelectric effect in zinc, quoted in the former paper, indicate that the number of electrons which can be liberated from zinc by a single light disturbance is about 3, and the number of such electrons is likely to be of the same order of magnitude in the case of the alloy of sodium and potassium. But, on the same theory, the fraction of that number which are liberated within any given photoelectric cell is the solid angle subtended by the surface of the cell at the source.

In our case this solid angle was never as much as $\frac{1}{20}$, and hence, on this theory, ω should be considerably less than unity and $\overline{\eta^2}$ large compared with ω. It should be expected therefore that, whichever of the two theories of light emission is true, $\overline{\theta_T'^2}$ should be proportional to the intensity simply. If Planck's theory be not accepted, there is no apparent method of calculating the number of electrons liberated by a single disturbance.

Table I.

160 volts on Electrometer Needle. i and R are given in electrostatic units, $\overline{\theta_T'^2}$ in scale divisions. The value of the last quantity is deduced from not less than 300 observations in each case.

(A) $R = 0{\cdot}155$

i	=	89	54	38	26
$\overline{\theta_T'^2}$	=	21·4	11·6	7·5	4·4
$\overline{\theta_T'^2}/i$	=	0·240	0·233	0·197	0·169
$\overline{\theta_T'^2}/i^2$	=	0·0027	0·0043	0·0052	0·0065

(B) $R = 0{\cdot}205$

i	=	88	40	26
$\overline{\theta_T'^2}$	=	29·5	15·1	9·1
$\overline{\theta_T'^2}/i$	=	0·335	0·375	0·350
$\overline{\theta_T'^2}/i^2$	=	0·0038	0·0094	0·0134

(C) $R = 0{\cdot}304$

i	=	86	49	29
$\overline{\theta_T'^2}$	=	50·1	24·8	15·2
$\overline{\theta_T'^2}/i$	=	0·584	0·506	0·522
$\overline{\theta_T'^2}/i^2$	=	0·0068	0·0103	0·0180

It will be seen from Table I that the value of $\overline{\theta_T'^2}$ varies apparently as some power of the intensity between the first and the second, but considerably nearer to the first. This result shows that, contrary to the hopes of the author, no evidence has been produced *against* the "spherical wave" theory. Indeed the fact that the power of the intensity is slightly greater than unity might be held to be favourable to that theory as opposed to the "bundle of energy" theory. But it must be remembered that all sources of error tend to increase the fluctuations at the higher intensities in comparison with those at the lower and so to give a divergence from a proportionality between $\overline{\theta_T'^2}$ and the intensity in the direction observed.

In the first place there is a possible error from small changes in the magnitude of the constant "drift" of the position of balance, which increases rapidly with the total intensity of the light falling on the cell. The cause of the drift was never discovered. It was always, on the average, in the same direction while the light was acting, and may probably be due to some slight "photoelectric fatigue," notwithstanding the high vacuum, which was different in the two cells.

In the second place the considerations mentioned in § 5 above must be remembered. The finite area of the source of light is of less importance when the source is further from the cell: hence $\overline{\eta^2}$ is likely to be somewhat less at greater distance.

It is concluded, accordingly, that these experiments can make no decision between the two theories: they can only say that the "spherical wave theory" is inconsistent with a large value for ω—a conclusion which is to be expected on other grounds.

§ 8. The second point requiring consideration is the variation of the fluctuations with the instrumental constants. These constants were altered in two ways:—(1) By changing the potential of the electrometer needle and so altering at the same time the electrometer constants a and b, the sensitiveness s and the capacity C. (See previous paper.) (2) By changing the resistance R.

(1) Table II shows the calculated and observed values of the ratios of $\overline{\theta_T'^2}$ for different values of the potential on the electrometer needle, the resistance being kept constant during any one set. a and b were never less than six times p: hence the expression (11′) of the previous paper reduces without sensible error to

$$\int_0^T f^2(t)\,dt = \frac{\epsilon^2 s^2}{C^2} \cdot \frac{1}{2p}.$$

It will be observed that the agreement between calculation and observation is quite as good as could be expected. Since the

period of the electrometer varied in the ratio of 2·5 to 1, it is clear that the prediction of theory that the fluctuations are independent of the period of the measuring instrument, so long as it is sufficiently small, is justified.

Table II.

V is potential on electrometer needle, s sensitiveness, C capacity; $\overline{\theta_T'^2}$ is in scale divisions, deduced from not less than 300 observations in each case.

V	Ratio of s^2	Ratio of C	$\overline{\theta_T'^2}$ (R=0·304	0·336	0·0316)	Ratio calc.	Mean ratio obser.
160	1·00	1·00	16·0	15·6	5·3	1·00	1·00
120	0·52	0·81	9·2	12·0	3·1	0·64	0·66
80	0·24	0·64	5·4	7·0	1·9	0·37	0·39

(2) Table III shows the variation of $\overline{\theta_T'^2}$ with p, when p is changed by alterations in the resistance R. Only one set of measurements is given, but other sets with other values of the remaining constants show exactly similar features. The last column gives the value of the quantity which should be constant, if the theory which has been given is correct: it will be observed that it is approximately constant for the higher values of R, but shows a marked increase for the lower values.

Table III.

160 volts on electrometer needle. $s = g \times 10^5$, $i = 84$.

R	C	$\overline{\theta_T'^2}$	$\overline{\theta_T'^2} \cdot R/2C$
0·336	395	52·0	$1·22 \times 10^5$
0·304	410	50·9	$1·34 \times 10^5$
0·205	400	29·5	$1·18 \times 10^5$
0·155	405	21·4	$1·12 \times 10^5$
0·110	460	22·0	$1·84 \times 10^5$
0·096	520	17·2	$1·86 \times 10^5$
0·032	710	7·3	$3·15 \times 10^5$
0·027	750	4·9	$2·73 \times 10^5$

§ 9. The explanation of this deviation from theory has not been discovered, but it appears to be connected with an increase in the measured capacity of the system with a decrease in resistance, a change to which reference was made in the previous paper. The resistance R was measured by passing a known current through

the resistance and measuring the potential difference between its ends. The quantity p, the logarithmic decrement of the charge on the electrode system, was found by observations on the decay of the deflection of the electrometer needle: the capacity was then found from the relation $C = \frac{1}{Rp}$. Measurements made in this way showed that the capacity of the electrode system increased notably with a decrease of R, while it was expected that C would be almost independent of R. The only change made in the apparatus when R was altered consisted in a change of the distance apart of the electrodes immersed in the high resistance fluid, and the calculated capacity of these electrodes at their nearest approach was only 15 cms., less than $\frac{1}{20}$ of the whole capacity. Moreover, it was found that a variation in the resistance made no difference in the ratio of the capacities of the electrode system with different potentials on the electrometer needle: it would seem then that the change of capacity must lie in the electrometer and not in the rest of the system.

When the resistance was sufficiently high, or when additional capacities were inserted, measurements could be made of the capacity by other methods. In all cases these agreed with the capacities measured by the decrement method: but in those cases when the measured capacity appeared abnormal, when measured by the decrement method, no other method of measurement of sufficient accuracy could be devised, whereby the results might be checked.

§ 10. It will be observed that, if the theory of the measurements which has been given is correct, a knowledge of the number of electrons liberated by a single light disturbance can be found from the relation

$$\left(\omega + \frac{\overline{\eta^2}}{\omega}\right) = \overline{\theta_{T'^2}} \cdot \frac{2pC^2}{\epsilon i s^2},$$

where i is the total photoelectric current.

In making use of this relation, only those observations made with resistances greater than 0·15 have been used: it has been assumed, for the reasons given at the end of the last paragraph, that the values for smaller resistances are abnormal. The mean value of $\left(\omega + \frac{\overline{\eta^2}}{\omega}\right)$ is then found to be 4·1, if the value of ϵ, the charge on an electron, is $4{\cdot}65 \times 10^{-10}$.

The resulting value of ω depends on the assumptions made in calculating $\overline{\eta^2}$. It will be seen that the value is inconsistent with the second assumption made in § 7, when discussing the value of this quantity, for on that assumption ω could not be greater

than 1. On the first assumption, which leads to $\overline{\eta^2} = \omega$, the value of ω is 3·1, a quantity which agrees well in order of magnitude with that found by totally different experiments. (See § 7.) It also agrees, at least so far as order of magnitude is concerned, with that to be expected from Planck's theory of radiation, if, and only if, the bundle of energy theory is true. If the spherical wave theory is true, and the energy in a single light disturbance is spread over a sphere surrounding the source, the above value for ω must be multiplied by more than 100 in order to take account of the angle subtended at the source by the photoelectric cell.

It appears to be the chief importance of this work that it has shown that it is possible, by the fluctuation method of v. Schweidler, to obtain directly measurements of quantities agreeing well with those deduced on other and much more indirect methods.

Summary.

1. The paper is a continuation of that presented lately to the Society under the same title. The difficulties which had hitherto prevented the attainment of results are described.

2, 3. A description of new sources of light designed to overcome these difficulties, and an explanation of their nature.

4. Reasons are given for the failure of the new devices to attain the desired object.

5. With this failure all hope of carrying out the main purpose of the research was abandoned.

6. It was thought, however, that some results of value might be obtained by the observation of the fluctuations of two independent sources of light. The method of measurement is described.

7. The variation of the fluctuations due to such sources with the total intensity of the light is first examined. It is pointed out how far such measurements could be used to distinguish between the rival theories of light. The conclusion is reached that the experiments recorded here afford no grounds for distinguishing between these theories. They can only fix a limit to certain values which is in accordance with deductions from other work.

8, 9. The variation of the fluctuations with the instrumental constants is examined. It is concluded that the agreement between experiment and the theory worked out in a previous paper is satisfactory, except in cases where the value of the resistance R is small. Reasons are given for thinking that these

values are abnormal, and that the theory of the measurements may be held to be established for the larger resistances.

10. If the theory of the measurements examined in §§ 8, 9 is accepted, the observations lead to an estimate of the average number of electrons liberated by a single light impulse. The number obtained is 3·1. This result agrees well with that which has been deduced on other grounds, and also with that to be expected from Planck's theory of radiation, if the bundle of energy theory is correct.

The Absorption of Bromine by Lime. By W. A. R. Wilks, B.A., Gonville and Caius College. (Communicated by Dr Fenton.)

[*Read* 23 May 1910.]

The action of bromine on slaked lime was first investigated by Berzelius. The product obtained—often called "bromine-bleaching powder"—appears to have been very little studied since. Owing to the fact that many adsorption compounds, as in the well-known instance of iodide of starch, have peculiar colours it was suspected that this might be a similar case. The product has a reddish brown colour, and as this appears to be anomalous, at Dr Fenton's suggestion the author undertook an investigation on the subject.

A series of experiments was carried out in the following way. A gram of pure freshly prepared slaked lime (dried in a vacuum desiccator) was weighed into a stoppered glass bottle and 100 c.c. of a carbon tetrachloride solution of bromine (of known strength) added. Solutions of different strengths were added to the same quantity of lime in other bottles, the volume of solution being always 100 c.c. After equilibrium was attained the strengths of the solutions were determined by means of potassium iodide. The concentration of bromine in the lime was then calculated. The following are some of the results obtained.

10 c.c. of bromine solution required in c.c. of "thio"	Amount of bromine in 1 gm. of lime expressed in terms of "thio" solution
4·45	44·6
13·2	46·2
22·05	46·8
31·05	46·0
39·85	47·0

The thiosulphate contained ·02108 grm. of $Na_2S_2O_3$ per c.c. Temperature of experiment 15° C. In this experiment the concentration of bromine in the lime was constant while the concentration in the solution varied. The most probable interpretation of this result is that a chemical compound is formed, probably analogous to bleaching powder. The ratio $Ca(OH)_2 : Br = 4{\cdot}42 : 1$.

Much weaker solutions of bromine were used in the next experiments which were carried out in the same way as before.

10 c.c. of bromine solution required in c.c. of "thio" $= C_2$	Amount of bromine in 1 gm. of lime expressed in terms of "thio" solution $= C_1$	$\frac{C_1}{C_2^{\frac{1}{3}}}$
5·3	42·6	2·443
13·1	50·2	2·130
21·8	68·2	2·441
30·6	76·4	2·443

10 c.c. Iodine solution (1 c.c. = ·01282 I) = 51·85 c.c. "thio."

Here the concentration of the bromine in the lime increases with the increase in concentration in the solution. The third column gives the numbers calculated from the expression $\frac{C_1}{C_2^n}$ when n is made one-third. These numbers are sensibly constant. This result might be explained either by assuming that the bromine is in solid solution in the lime or that it forms an adsorption compound. On the first hypothesis the bromine in solid solution would be dissolved as a fraction of an atom (assuming that bromine in carbon tetrachloride dissolves as Br_2—a most probable assumption since iodine has been shown to dissolve in the same liquid as I_2). It seems fairly certain therefore that we are dealing with an adsorption phenomenon.

In both the above experiments the slaked lime was freshly prepared from pure quicklime and dried in a vacuum desiccator for two days. In the following experiment it was dried for four weeks.

10 c.c. of bromine solution required in c.c. of "thio"	Amount of bromine in 1 gm. of lime expressed in terms of "thio" solution
2·48	22·66
6·85	26·42
11·70	35·38
25·35	31·16
34·55	34·18
44·0	34·60
53·6	33·52

37·2 c.c. of "thio" = 10 c.c. Iodine solution (1 c.c. = ·01282 I).

In this case when the concentration of the bromine in the solution is small the bromine is adsorbed by the lime, but the concentration of bromine in the lime soon reaches a constant so that it is not possible to calculate the values $\frac{C_1}{C_2^n}$. The constant concentration of bromine in the lime again indicates the formation of a chemical compound. The ratio $Ca(OH)_2 : Br = 14{\cdot}9 : 1$. The drier the lime therefore the less the ratio $Br : Ca(OH)_2$ becomes—a result quite similar to that obtained by other observers in the case of chlorine and lime.

In the last experiment the colour of the lime deepened from yellowish brown to reddish brown as the concentration of bromine in it increased. After a constant concentration was reached the colour was unaffected by increasing the concentration of bromine in the solution.

Note on the Reduction of Nitrosyl Chloride. By H. O. Jones, M.A., Clare College, and J. K. Matthews, B.A., Downing College.

[*Read* 23 May 1910.]

While examining the interaction of nitrosyl chloride and ethyl mercaptan in ethereal solution at $-80°$ C. the formation of small quantities of hydroxylamine hydrochloride was observed. This was produced by the reducing action of the mercaptan on nitrosyl chloride with the simultaneous formation of diethyl disulphide.

The action of other reducing agents on nitrosyl chloride was therefore studied.

Hydrogen sulphide and nitrosyl chloride, in the presence of an inert solvent such as ether or petroleum ether, interact to produce ammonium chloride and sulphur even at low temperatures. Some sulphur chloride is also produced, but this was found to be due to the action of nitrosyl chloride on the sulphur formed by the first action.

The main reaction may be represented by the equation

$$NOCl + 3H_2S = NH_4Cl + H_2O + 3S,$$

though attempts to establish this by quantitative experiments were not successful owing to the secondary reactions which take place.

In the gaseous state nitrosyl chloride and hydrogen sulphide interact to produce sulphur, together with some sulphur chloride, hydrochloric acid, nitric oxide, nitrogen and ammonium chloride.

Under no conditions was the formation of hydroxylamine hydrochloride observed.

The action of hydrogen on nitrosyl chloride in the presence of palladium, of finely divided platinum and of nickel was next examined.

At the ordinary temperature palladium charged with hydrogen was attacked by nitrosyl chloride with the formation of palladium chloride, nitric oxide and nitrogen.

When hydrogen and nitrosyl chloride were passed over palladium at temperatures between 100° C. and 230° C. the palladium was not appreciably attacked and considerable quantities of ammonium chloride were produced, but no hydroxylamine hydrochloride could be detected in the product.

A mixture of nitrosyl chloride and hydrogen was then passed over finely divided platinum, prepared by the reduction of platinum

chloride by means of an alkaline solution of glycerol. At the ordinary temperature a very vigorous reaction set in at once, the temperature of the platinum rose rapidly to a red heat and ammonium chloride was formed.

The platinum was therefore surrounded by a freezing mixture and the gases allowed to pass over it very slowly, and in this way some hydroxylamine hydrochloride was formed together with the ammonium chloride, but the quantity was small, hardly exceeding five per cent. of the ammonium chloride formed. Nitrogen was also produced.

Nickel was also found to bring about the interaction of hydrogen and nitrosyl chloride with the formation of ammonium chloride.

The development of Trypanosoma lewisi in the Rat Flea (Ceratophyllus fasciatus). By C. Strickland, B.A., and Dr N. H. Swellengrebel. (Communicated by Prof. Nuttall.)

[*Read* 6 June 1910.]

Little is known about the development of trypanosomes outside the vertebrate host, and what is known is liable to Patton's criticism, that asserts that most of the *Crithidia* forms found in different invertebrates and considered to be developmental forms of various trypanosomes are indeed independent forms and have no connection with trypanosomes.

This question being of the highest importance not only theoretically but also from a practical point of view in connection with sleeping sickness, we endeavoured to trace out the life-cycle of *T. lewisi* (provided there were any) in the rat flea (*Ceratophyllus fasciatus*). We chose the rat flea and not the louse (*Haematopinus spinulosus*) because it is clear from the papers of Nuttall, Manteufel, Minchin and one of us (C. S.) that the louse transmits the trypanosomes only mechanically, whereas the flea becomes infective after 7—14 days and remains so for a long time, which fact suggests that a true development takes place.

We infected 83 fleas out of an uninfected lot (bred in the laboratory), allowing them to feed during a period of 12 hours upon an infected rat (10th day of infection). The fleas were then removed from the rat and were fed at intervals of three days by biting non-infected rats through a gauze bag. A control series of experiments was made by feeding 60 fleas (out of the same box as the infected series) on a non-infected rat and by treating them afterwards in the same way. Each day, for 16 days after the first feeding, four to six of the infected and control fleas were dissected and observations were made *in vivo* and in stained preparations.

The control fleas were not always free from flagellated forms. Nothing was ever seen in the living preparations; but by making stained preparations of the different parts of the gut, we were able to detect flagellates in 3·3 °/₀ of the fleas. This fact was probably due to an accidental infection, an infected rat having broken into the flea box. But even if we consider the flagellates of the control fleas to be natural forms, we may safely conclude that most of the flagellates found in the fleas fed on infected rats are developmental stages of *T. lewisi*, because the rate of infection was 44 °/₀.

Twelve hours after feeding on the infected rat, *T. lewisi* may be found unaltered in the midgut, after 24 hours the blepharoplast leaves its terminal position and approaches the nucleus. The hind-end of the flagellate becomes larger so that the trypanosome appears club-shaped. The nucleus now wanders to the hind-end of the flagellate, and consequently the distance between the blepharoplast and nucleus diminishes gradually, and at a later stage the two pass each other so that the blepharoplast is situated in front of the nucleus, the trypanosome having now a *Crithidia* facies. This change is accomplished in the midgut in $1\frac{1}{2}$ days. The club-shape is now very well marked and the front-end of the flagellate becomes thinner and thinner, the flagellum often staining with difficulty. At last the flagellar end of the parasite atrophies, and the trypanosome is now metamorphosed into an oval-shaped cell, with a short flagellum, the blepharoplast being situated at the side or in front of the nucleus. These "large oval forms," as we may call them, pass into the anterior part of the hindgut, and are to be found there after $3\frac{1}{2}$ days in rosettes. During the following days the large oval forms become rounded, giving rise to the so-called "round forms," which begin to divide actively after 5—6 days. This development is first to be observed in the flea's rectum but afterwards also in the hindgut. We never observed any indication of multiple division, but the parasites arising from the repeated binary division of the round forms hold together with the front-ends, so that rosettes are formed. These rosettes are composed of "little oval forms" with short flagella, the blepharoplast being situated just in front of the nucleus. Gradually these become more elongated, the flagellum grows out further, and the blepharoplast passing the nucleus comes to lie in the posterior end of the cell. Since the flagellum follows the blepharoplast, the result is the formation of an undulating membrane. The parasite has now recovered the aspect of a small trypanosome with a particularly large blepharoplast, situated at the extreme posterior end. These forms begin to appear after 7 days but are abundant after 8—10 days and remain so for an indefinite time, probably till the host's death.

All these different types of parasites—little ovals, intermediate *Crithidiae* and small trypanosomes—may divide, so that preparations made during the last stage of infection exhibit a great variety of forms. The small trypanosomes and the *Crithidiae* from which they originate occur in two well-marked types: a slender one and a stout, but united by intermediate forms. We do not know if this is to be considered as a sexual differentiation, but we have never seen any sign of conjugation.

Not being able to find any further stage of development beyond that of "small trypanosomes," either in our series or in

the fleas taken haphazard from infected rats, we think it possible that these forms represent the final stages of development. If so, we are unable to say in which way these small trypanosomes return to the rats' blood. We have not been able to find any flagellated form either in the coelom or salivary glands of infected fleas.

At present the broad fact remains established by a proper system of controls that *T. lewisi* undergoes a morphological cycle of development in the gut of its invertebrate host.

The Development of Gnomonia erythrostoma, the Cause of Cherry Leaf Scorch Disease. By F. T. Brooks, M.A., Emmanuel College.

[*Received* 6 June 1910.]

[Abstract.]

The life-history of this Pyrenomycete may be summarised as follows: Infection of the foliage occurs in the early summer by means of the ascospores. Spermogonia and "coils" are formed in the diseased leaves. It was considered by Frank, who described the outlines of the life-history several years ago, that the spermatia produced in the spermogonia fertilise the "coils" by passing down the trichogynes which arise in association with the latter. The "coils" ultimately develop into perithecia. During the summer the vegetative mycelium passes down the leaf stalk and prevents the formation of the abscis layer. On account of this, the diseased leaves remain hanging on the trees throughout the winter and become the source of reinfection the following season.

The present investigation concerns an examination of the life-history of this fungus from the cytological standpoint, very little work on the Pyrenomycetes having hitherto been done from this point of view.

The vegetative mycelium of *Gnomonia* consists of multinucleate cells. It is intercellular and haustoria are not developed.

The spermogonia show similarities in structure to those of the Uredineae. The spermatia are long and thread-like and exhibit the cytological characters of male cells. It is considered, however, that they are now functionless, there being no evidence that fertilisation is effected by their agency as Frank supposed was the case.

The trichogynes occur in tufts of 2—5. They do not invariably arise in association with "coils." It is suggested that the trichogynes though originally receptive organs now perform a different function—serving possibly as a means of aeration.

The "coils" represent the first stages of perithecial development. In the centre of each coil one or more slightly differentiated hyphae are found. These are considered to be of the nature of ascogonia. No clear connection can be traced between the trichogynes and these cells. The "coil" undergoes a period of rest before developing further.

In the subsequent stages of development of the perithecium from the "coil" it is no longer possible to trace the ascogonia. There is evidence to show that the ascogenous cells arise by differentiation from ordinary cells. No fusion of nuclei has been observed prior to that which occurs in the young ascus.

The nuclear phenomena in the ascus are described in some detail. Only a single process of reduction has been observed which occurs at the time of the first nuclear division. It is not possible yet to decide whether this division is heterotypic or brachymeiotic though the analogies are with the former rather than the latter.

Thus only a single nuclear fusion and a single process of reduction have been seen to occur in the life cycle of this Ascomycete.

The Absence of Living Tubercle Bacilli from some Old Tuberculous Lesions in Man. By Louis Cobbett, M.D. Pathological Laboratory.

[*Read* 6 June 1910.]

Two theories of the origin of phthisis involve the hypothesis of latency—Baumgarten's which regards infection as occurring before birth, and remaining undeveloped, perhaps for years; and Behring's which regards infection as occurring usually during infancy, and remaining latent until some disturbance of health set up by the onset of puberty, by parturition or lactation, or by measles or some other of the specific fevers or some other cause weakens the natural powers of resistance and so favours the growth of the bacilli. "Then," to translate Behring's words, "we have the beginning of phthisis, but the real origin of the tuberculous infection is far back." Both theories then assume that the tubercle bacillus is capable of remaining alive quiescent in animal tissues for long periods of time, measured not by weeks and months but by years.

Behring in arriving at his views was largely influenced by the frequency with which lesions apparently tuberculous are found, often quite unexpectedly, in the bodies of those who die from all sorts of causes.

These lesions undoubtedly occur very frequently, though opinions may differ as to just how common they are. The difference of opinion is due to the difficulty of deciding in some cases whether the lesion is tuberculous or not. But in many cases there is no difficulty at all, there is definite caseation or calcification, and in some the characteristically acid-fast tubercle bacilli may even be seen under the microscope. In such cases very little attention seems to have been paid to the question whether the lesion was really quiescent, or healed, that is to say whether it did or did not contain living bacilli capable under favourable conditions of causing a tuberculous infection.

During the time when I was working for the Royal Commission on Tuberculosis I was more than once astonished to find that what I regarded as undoubtedly tuberculous material had no effect when injected into the guinea-pig; and, as this is one of the most sensitive of animals to tuberculosis, whether caused by bacilli of the human or bovine variety, I could not but regard the lesions from which this material had come as cured, that is to say incapable of giving rise to any further tuberculous trouble.

Since I left Stansted I have continued to investigate isolated and unsuspected tuberculous lesions, which have been from time to

time found in the bodies of those who died from other causes, and not a few of these have been found to be non-infective. These observations seem to me to be worth recording as a contribution to the data from which one may estimate the duration of the vitality of tubercule bacilli in the bodies of living animals, and because our views about the practically important question of the origin of phthisis must be largely determined according to the conclusion we come to as to the vitality of the bacillus. If any considerable proportion of these lesions are really not capable of infecting then the great frequency with which they are present is not so strongly in favour of latency as it is usually held to be.

The first case investigated by me was that of a child of eight, who died of diphtheria. Quite unexpectedly, at the post-mortem examination, was found a mass of enlarged mesenteric glands, four of which were caseous and softened at their centres. There was ample material for the purpose of inoculation and I was able to make as much as 25 c.c. of a creamy emulsion, in which as many as 170,000 tubercle bacilli per cubic centimetre could be seen. The whole of this emulsion was injected into two calves, two rabbits and two guinea-pigs, but not even the latter developed any trace of tuberculosis.

Naturally I considered the result suspicious, and every suggestion probable and improbable was considered which might account for it, without having to suppose that the bacilli were dead; but without success. It was even thought that diphtheria toxin might be fatal to tubercle bacilli, but this was tried and found not to be the case.

The next case was one in which tuberculous glands were removed by operation from a child of a little under four years of age. Fifteen months previously some glands had been removed from this child and sent to me, and proved to contain numerous virulent tubercle bacilli of the bovine type. The glands removed at the second operation contained two as large as ripe gooseberries, which were caseous and softened. An emulsion was made in the usual way and injected into guinea-pigs, but the animals remained unaffected, and cultures attempted from the emulsion failed to grow any tubercle bacilli.

The next case was one of tuberculosis mainly affecting the lungs and pleura of a child of eight. The bronchial glands were enlarged, and three contained small caseous foci, another contained a larger mass of the same material; a guinea-pig injected with all the caseous material which could be obtained from these glands remained unaffected. No tubercle bacilli were seen in the emulsion prepared for injection.

The fourth case was one of localised tuberculosis in a boy of fourteen. After a slight injury to the hand, which was slow in

healing, an almost painless slowly growing lymphatic gland appeared in the axilla, and about two months later it ulcerated through the skin and discharged a thin fluid. Some of this discharge was used for the injection of guinea-pigs but without effect. The quantity of it was small, but its tuberculous nature was demonstrated by finding a few tubercle bacilli in it.

The fifth case occurred in Sheffield. It was clinically a typical case of Addison's disease, in a young woman. No trace of the supra-renals could be found though carefully sought for, and no lesion of any kind was found, except a single mesenteric gland of stony hardness and entirely calcareous. No tubercle bacilli could be found in it by microscopic examination. The gland was emulsified and injected into guinea-pigs but no tuberculosis resulted. They were killed two months later and traces of the material injected were found bound to the omentum, but no tubercle bacilli could be found in this material and there was no cellular action around them.

In the next two cases living tubercle bacilli were found in the old lesions.

The sixth case occurred in Cambridge, it concerned a child of four who had tuberculous disease of the upper dorsal vertebrae. An abscess formed in the thorax which interfered with respiration, and the child died. At the post-mortem examination an old caseous mesenteric gland was found. It was about $\frac{3}{4}$ of an inch in diameter and consisted of thick gritty caseo-pus contained within pink-coloured fibrous walls.

Two guinea-pigs were injected with the caseous material, one developed tuberculosis, the other did not.

The seventh case, a youth of eighteen, died of heart disease. A caseous bronchial gland, as large as a bean, and filled with creamy caseous contents containing calcareous grains. No tubercle bacilli were found in it by microscopic examination, but it caused tuberculosis when injected into guinea-pigs.

Among these seven cases were five in which material which I think may be regarded as undoubtedly tuberculous was injected into guinea-pigs without producing tuberculosis.

Similar observations have been made by those working for the Committee of the Kaiserliches Gesundheitsamt, appointed to investigate the relation of bovine to human tuberculosis. It cannot be doubted then, I think, that tuberculous material taken from man is sometimes incapable of causing tuberculosis in susceptible animals. And it seems a fair inference that it contains no living bacilli.

It may perhaps be argued that these lesions were not really tuberculous, but this can hardly apply to case No. 2. And if these caseous and calcareous glands were not tuberculous what

were they? Some of them undoubtedly contained acid-fast bacilli.

It may again be suggested that even the guinea-pig is not entirely devoid of the power of resisting tuberculous infection, and to some extent can get over small doses of bacilli, as the calf and other animals, including man, undoubtedly can, and that in these cases the dose was too small to infect. To this it may be replied that doses apparently equally small have frequently been given with positive results, and that in some of the cases recorded, notably the first, the dose cannot be considered small since many thousand tubercle bacilli were present in each cubic centimetre of the emulsion injected.

I do not propose to discuss now the vitality of tubercle bacilli in actively living tissues—hitherto we have been considering their vitality in necrotic tissue caseous and calcareous. Many observers have found living tubercle bacilli capable of infecting guinea-pigs in enlarged tonsils, adenoid vegetations, or in cervical, mesenteric, or bronchial lymph glands. Work of this kind carried out by Weber and Baginsky for the Committee of the Gesundheitsamt, however, shows that such infection is not very common. Moreover, as these authors remark, one cannot tell in these cases how long the bacilli have been there. They might conceivably have only just got into the tonsil or lymph gland in which they were found.

If it is worth while speaking at all of probabilities, it seems hardly likely that bacilli could survive in living tissues which we know possess some degree of bactericidal power longer than in inert, dead, caseous, or calcareous matter.

Again it seems improbable that tubercle bacilli should remain alive in a quiescent state for great lengths of time. It is generally conceded that they do not form spores, and in our artificial cultures their vitality is measured by months rather than by years.

Very many more observations are needed before we can form any opinion as to the frequency with which living tubercle bacilli are present in old and quiescent tuberculous lesions. I have brought forward these few because much time must necessarily pass before I can greatly enlarge the number, and because few as they are they make one question, whether the frequency with which old tuberculous lesions are met with in post-mortem examinations really lends so much support to the hypothesis of latency as it is usually held to do.

Note on the Radium-content of the Waters of the Cam, Cambridge Tap Water and some varieties of Charcoal. By JOHN SATTERLY, B.A., B.Sc. (Lond.), St John's College. (Communicated by Professor Sir J. J. Thomson.)

[*Received* 24 June 1910.]

In view of the fact that the author has found more radium emanation in the air at Cambridge* than Eve found in the air at Montreal† it was thought that it would be of interest to examine quantitatively the water of Cambridge for radium.

Some experiments have already been made on the radioactivity of Cambridge tap water. J. J. Thomson‡ showed that air bubbled through this water became radioactive due to the removal from the water of some dissolved gas. Adams§ tested this gas and identified it with radium emanation. He also established that the water contained dissolved radium as well, for after boiling the water and clearing out the dissolved emanation a little emanation accumulated when the water was allowed to stand for some days; he states that the recovery was about one-tenth of its previous value. Adams evaporated quantities of water to dryness but failed to find any activity in the residue. Strutt‖, who employed Cambridge tap water in the aspirators which he was using in his work on the estimation of radium in rocks, states that no measurable quantity of emanation was generated in the water which had been previously boiled, although a close examination of his figures shows that two litres of water generated in a fortnight enough emanation to send up his air-leak about 4 %. None of the above experimenters however had at the time the means of expressing the amount of emanation in terms of the mass of radium which would generate it.

Recently Eve¶ has examined the water of the St Lawrence which is delivered as the town supply of Montreal without any filtration or other treatment, and found its radium-content to be $\cdot 25 \times 10^{-12}$ gm. of radium per litre of water, and Joly has found for the Nile $4 \cdot 2 \times 10^{-12}$. For sea-water from the North Atlantic, Eve¶ has found on the average $\cdot 9 \times 10^{-12}$ gm. of radium per litre,

* *Phil. Mag.* Oct. 1908 and July 1910.
† *ibid.* Oct. 1908.
‡ *ibid.* Sept. 1902.
§ *ibid.* Nov. 1903.
‖ *Roy. Soc. Proc.* A, Vol. LXXVII. 1906.
¶ *Phil. Mag.* July 1909.

and Joly 16×10^{-12}*, and more recently 12×10^{-12}† as an average of sea-water from all over the world.

The method of testing in my experiments was to take the water straight from the Cam or the tap and boil it in a flask connected to a condenser and aspirator. When the water was boiling air was rapidly drawn through it by the aspirator; the emanation was thus withdrawn and collected in the aspirator. The steam condensed in the condenser and ran back into the flask. The emanation-charged air in the aspirator was then passed into a previously exhausted testing vessel and the leak taken. This leak less the normal air leak is a measure of the emanation.

The waters of Cambridge contain much chalk in solution, so that unless acidified a deposit occurs on boiling. As this might affect the quantity of emanation given off two samples of each water were tried, one untouched and the other acidified with a little hydrochloric acid (the acid itself was tested and though not free from radium yet would, as only a small quantity was used, contribute practically no emanation, see Table II).

In the first set of experiments a litre of water was taken each time, and when acidified two c.c. of acid was used. After each boiling the flask was corked up and allowed to stand for any emanation to accumulate when the water was again reboiled and the emanation drawn off and tested. It was found that practically no emanation was accumulated on resting, so that practically all the emanation initially present in the water is just dissolved emanation.

Table I gives the first set of results; the amount of emanation being expressed on an arbitrary scale (the numbers are the leaks per minute in the testing vessel as read on the scale provided to the electrometer. The air leak which was about 1·0 has already been subtracted).

Table I.

Cambridge tap water (one litre).

Treatment	Amount of emanation
(1) No acid added	9·0
(2) 2 c.c. HCl „	9·0
(3) „ „ „	11·0

Cam water (one litre).

Treatment	Amount of emanation
(4) No acid added	·4
(5) 2 c.c. HCl „	·6
(6) No acid „	·2

* *Phil. Mag.* May 1908.

† *ibid.* Sept. 1909.

(1), (2), (3) were collected on separate days, (4) and (5) were collected at different times on the same day from the Cam at Sheep's Green just above the flood-gates, and (6) on another day from below the flood-gates. The water is broken up into foam on passing through the gates, so that it is quite natural that (6) should contain less emanation than (4) or (5).

There is not enough radium in a litre of water to grow sufficient emanation to give a readable leak. Therefore in order to test for the actual radium-content of the water larger quantities of water were taken. Two large clean tin cans were filled with nine litres of Cam water and tap water respectively. The water was then boiled, the dissolved emanation cleared out, the cans corked up and the water allowed to rest. After an interval of some days the water was boiled again and the emanation given off collected and tested. From the amount accumulated in the given period the amount accumulated in the first 24 hours was calculated*.

The results are given in Table II together with some results obtained with hydrochloric and sulphuric acids.

TABLE II.

	Period of rest	Accumulation of emanation in this period	Accumulation of emanation per day per litre
9 litres tap water	6 days	·7	·02
" " "	12 "	1·2	·02
9 litres Cam water	5 days	1·3	·04
" " "	13 "	2·3	·04
200 c.c. HCl + 300 c.c. distilled water	since manufacture	1·2	
	13 days	·15	·14
200 c.c. H_2SO_4 + 400 c.c. distilled water	since manufacture	0	

To express the radium-content of the water in terms of gms. of radium it was now necessary to use radium solutions of known strength. A radium solution containing $1{\cdot}57 \times 10^{-9}$ gm. radium was kindly presented to the author by Professor Rutherford. This was divided into two parts, one containing $\frac{2}{5}$ths and the other $\frac{3}{5}$ths and placed in two bottles. Each of these parts was treated in a manner similar to the above. Instead however of boiling the solutions, which was impossible as the solutions were in bottles, the bottles were heated in a bath of boiling brine (b.p. = 105° C.). The results are given in Table III.

* Suitable tables for this work are given by Kolowrat. *Le Radium*, 1909.

TABLE III.

Radium solution	Period of rest, hours	Growth of emanation		Mean growth of emanation per 24 hours
		in this period	per 24 hours	
$\frac{2}{5}$ of $1{\cdot}57 \times 10^{-9}$ gm. of radium	$20\frac{1}{2}$	7·1	8·2	8·0
	$74\frac{3}{4}$	19·8	7·6	
	$23\frac{1}{2}$	8·3	8·4	
	$24\frac{1}{3}$	8·4	8·3	
	$47\frac{1}{2}$	13·7	7·6	
$\frac{3}{5}$ of $1{\cdot}57 \times 10^{-9}$ gm. of radium	$23\frac{3}{4}$	11·4	11·5	11·8
	24	11·9	11·9	
	$23\frac{3}{4}$	11·8	11·9	

From Table III it follows that $1{\cdot}57 \times 10^{-9}$ gm. radium accumulates emanation at the rate of approximately 20·0 per day. From this and Table II it follows that

1 litre tap water	contains	$1{\cdot}6 \times 10^{-12}$	gm. radium
1 „ Cam „	„	$3{\cdot}2 \times 10^{-12}$	„ „
1 „ hydrochloric acid	„	11×11^{-12}	„ „

It may also be calculated that the emanation given off by the initial boiling of one litre of tap water is that which would be in equilibrium with 130×10^{-12} gm. radium, and of one litre of Cam water that which would be in equilibrium with $5{\cdot}0 \times 10^{-12}$ gm. radium.

For the water of the St Lawrence, as already stated, Eve* finds a radium-content of $\cdot 25 \times 10^{-12}$ gm. radium per litre, but he does not state whether this result is obtained from the initial or subsequent boiling. It follows, in either case, that the waters in the neighbourhood of Cambridge contain a much greater quantity of radium per unit volume than the waters in the neighbourhood of Montreal. Naturally therefore the air at Cambridge would contain a greater quantity of radium emanation than the air at Montreal. The proportion actually found† was 10 : 6.

It may be mentioned in conclusion that the Cam takes water which has drained through the Chalk deposits in the south of Cambridgeshire, and then runs over the clayey deposits at the base of the Chalk, while the water supply of Cambridge is taken from springs issuing at Cherryhinton and Fulbourn from the greensand which lies above the Gault and below the Chalk.

* *Phil. Mag.* July 1909. † *ibid.* Oct. 1908, July 1910.

Radium-content of some varieties of charcoal.

In my experiments* on the absorption of radium emanation from the air I have had occasion to measure the rate at which radium emanation is generated by the charcoal after the charcoal has been heated to a bright red heat and all its previous store cleared out. From these results, with the knowledge of the amount of emanation produced by a known mass of radium in a known time, it is possible to calculate the radium-content of the charcoal.

The results are given in Table IV.

TABLE IV.

Charcoal	Mass of charcoal tested	Emanation accumulated per day	Mass of radium in the charcoal	Mass of radium per gram of charcoal
Coconut	40	·2†	$1{\cdot}6 \times 10^{-11}$ gm.	$\cdot4 \times 10^{-12}$ gm.
„	140	·5‡	4×10^{-11} „	$\cdot3 \times 10^{-12}$ „
Brazil nut	30	·5	4×10^{-11} „	$1{\cdot}3 \times 10^{-12}$ „
Wood	15	·05	4×10^{-12} „	$\cdot3 \times 10^{-12}$ „

For purposes of comparison with the figures in the last column it may be mentioned that the average radium-content of rock and soil is $1{\cdot}4 \times 10^{-12}$ gm. per gm., and that wood charcoal contains 1—5 °/₀ of ash.

I wish to express my best thanks to Prof. Sir J. J. Thomson for his kindly interest in these experiments.

* *Phil. Mag.* Oct. 1908, July 1910.

† *ibid.* Oct. 1908, p. 597.

‡ *ibid.* July 1910, p. 7.

The Resolution of Externally Compensated Bases into their Optically active components. By Professor Pope and Mr J. Read.

[*Read* 23 May 1910.]

The authors show that when the resolution of an externally compensated acid (or base) is attempted by crystallisation with an optically active base (or acid), one of three different kinds of behaviour is to be observed. (1) The two salts, *d-B*, *d-A*, and *d-B*, *l-A*, which are capable of formation, are readily separable by crystallisation and show no tendency to form solid solutions one in the other; in this case an easy separation is effected by the method of Pope and Peachey. (2) The two salts, *d-B*, *d-A* and *d-B*, *l-A*, combine to form a partially racemic compound; in this case no resolution occurs. (3) The two salts, *d-B*, *d-A*, and *d-B*, *l-A*, form solid solutions one in the other, and separation by fractional crystallisation is slow and very incomplete.

Behaviour of the type (3) has not been previously distinguished and a number of cases of its occurrence were described.

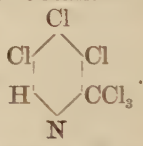
The Resolution of Dihydropapaverine. By Professor Pope and Mr C. S. Gibson.

[*Read* 23 May 1910.]

The resolution of externally compensated dihydropapaverine into its optically active components was effected by Pope and Peachey by crystallisation with *d*-α-bromocamphor-π-sulphonic acid; the less soluble salt, *l-B*, *d-A*, is crystalline and may be separated from the salt, *d-B*, *d-A*, which is resinous, by fractional crystallisation. It is now shown that the resolution is more easily effected by dissolving the externally compensated base in an equivalent proportion of *d*β-camphorsulphonic acid and adding to the aqueous solution one-half an equivalent of ammonium *d*α-bromocamphor-π-sulphonate; after separating the crystalline salt of the laevo-component of the alkaloid which is deposited the purification of the residual dextro-base is effected by crystallisation of its *l*α-bromocamphor-π-sulphonate.

Further study of the Products of Chlorination of α-Picoline. By Dr Sell.

[*Read* 23 May 1910.]

One of the most important compounds obtained by the chlorination of α-picoline has the formula  . This substance has been shown to produce by the action of sulphuric acid besides 3. 4. 5. trichloropicolinic acid and 3. 4. 5. trichloropyridine formerly described, a small quantity of an acid regarded as 3 oxy 4. 5. dichloropicolinic acid. This compound is resolved by heat into the corresponding oxydichloropyridine.

The above hexachloropicoline also reacts with alcoholic sodium hydroxide and yields 4 oxy 3. 5. dichloropicolinic acid, resolved by heat into 4 oxy 3. 5. dichloropyridine isomeric with the last-mentioned oxydichloropyridine.

It is also found that 3. 4. 5. trichloropicolinic acid gives with ammonia 4 amino 3. 5. dichloropicolinic acid which by heat is resolved into 4 amino 3. 5. dichloropyridine, and the same compound is also formed from 3. 4. 5. trichloropyridine by heating with ammonia. Finally when the amino group in 4 amino 3. 5. dichloropyridine is replaced by the hydroxyl group the same oxydichloropyridine is obtained as was shown to be produced by alcoholic sodium hydroxide on the original hexachloropicoline and heating the oxydichloropicolinic acid thus produced.

Formation of uric acid derivatives. By Dr Fenton and W. A. R. Wilks, B.A., Gonville and Caius College.

[Read 23 *May*, 1910.]

The condensation of urea with dihydroxymaleic acid, or its derivatives, would appear likely to be of interest from a physiological standpoint owing to the peculiar manner in which this acid is produced by oxidation in presence of traces of iron. The authors are making a study of these condensation-products and it has already been shown that the acid itself condenses with urea to give *isoiminazalone* and, probably, the aldehyde of hydantoic acid. When the acid is gently oxidised, mesoxalic semi-aldehyde is obtained; this might be expected to yield uric acid by action of urea but, under the conditions employed, the result is *glycoluril*, a substance closely related to *allantoin*. By oxidising the acid with bromine, dihydroxytartaric acid is obtained; this when acted upon by urea in presence of acetic anhydride is found to yield a product which appears to be identical with *acetyl dialuric acid*, recently obtained by Behrend and Friedrich from dialuric acid itself. [*Annalen* 1906 (344) 1.] It gives a violet precipitate with baryta water and, when evaporated with nitric acid, an intense murexide reaction. According to Ascoli and Izar [*Zeit. Physiol. Chem.* 1909 (62) 347], dialuric acid and urea under the influence of liver extract can give rise to *uric acid*, and it would therefore appear that the synthesis of the uric acid might be effected from dihydroxytartaric and urea by the changes indicated. In alcoholic solution, dihydroxytartaric acid and urea yield the crystalline salt $C_4H_6O_8 . 2CO(NH_2)_2$; when this is heated to about 100° it breaks up, probably quantitatively, into carbon dioxide, urea and *hydantoin*.

The fate of uric acid in the dog. By Harold Ackroyd, M.B. (Communicated by Mr W. E. Dixon.)

[Read 6 *June*, 1910.]

The liver of dogs when perfused with normal saline solution produces a small quantity of allantoin. When sodium urate is added to the perfusion fluid, a part of the sodium urate is destroyed: part of this can be recovered as allantoin but not the whole. No urea could be isolated from such a perfusion fluid. The liver of a dog has not the power to destroy allantoin.

The Adsorption of Acids by Carbohydrates. By F. Robinson, B.A. (Communicated by Dr Fenton.)

[Read 23 *May*, 1910.]

The action of acids, especially hydrogen chloride and hydrogen bromide, on carbohydrates has been studied by many chemists and it now appeared desirable to investigate the nature of the reaction of the various carbohydrates with the above-mentioned acids, in order to determine whether the initial phenomenon was one of adsorption, or chemical action, or both, and if possible to devise a method of characterising the various carbohydrates with reference to the capacity of each for absorbing the acid in question.

In 1809, Bouillon-Lagrange and Vogel (*Ann. de Chim.* 70, p. 91) showed that cane sugar, when boiled with aqueous hydrogen chloride, became coloured and that only a small percentage of the acid remained combined.

Similar experiments were conducted by Vogel (*Ann. de Chim.* 82, p. 158) with starch, milk-sugar and hydrogen chloride, but the product did not contain acid in combination.

Afterwards, Emil Fischer investigated the action of hydrogen chloride at low temperatures on various sugars, and isolated the substances known as 'chlorhydroses.'

In 1898, Fenton and Gostling (*Journ. Chem. Society*, 1898, p. 554) observed that levulose with ether and dry hydrogen bromide almost immediately gave an intense purple colour: further investigation showed that the ether had nothing to do with the formation of the colour, since the dry solid absorbed the gas to give the same purple colour. This reaction, with ether and hydrogen bromide, was repeated with various carbohydrates, which were found to fall into four distinct groups.

(*a*) With ketohexoses, such as fructose, sorbose, a purple colour appeared after a few minutes and became very intense in about one hour: and substances which, by hydrolysis, are capable of giving rise to these compounds [cane sugar, inulin] gave a similar effect but rather more slowly, the maximum intensity of colour being attained in about two hours.

(*b*) Hexaldoses (dextrose, galactose) and substances capable of giving rise to them (maltose, lactose, dextrin) showed in the first instance various shades of yellow, brown, or red and it was only after long standing that the purple colour was apparent.

The colour never approached that of the first group either in brilliance or intensity.

(*c*) Carbohydrates, whose molecules contain less than six atoms of carbon (arabinose, glycollic aldehyde) gave brown or red colours without any appearance of purple, and the polyhydric alcohols showed only a yellow colour.

(*d*) Carbohydrates like cellulose and starch gave a very faint red, or no colouration at all.

In a later communication (*J. C. S. Trans.* 1899, p. 423), the same authors showed that the product of the action of hydrogen bromide on the various carbohydrates was a colourless, crystalline substance, bromomethylfurfuraldehyde.

The action of hydrogen chloride was found by them to be similar to that of hydrogen bromide, but the reaction was much less violent, and the yield of chloromethylfurfuraldehyde not nearly so good.

Subsequent experiments shewed that all ketoses, or substances which give rise to these on hydrolysis, yield bromo or chloromethylfurfuraldehyde under this treatment; the aldoses give mere traces whereas the ketoses yield from 20—30 per cent., so that by estimation of the yield a ready method is afforded of distinguishing the two classes.

Principle of the Method.

A weighed quantity of the various carbohydrates, finely powdered, was treated with an excess of a standard solution of dry hydrogen chloride gas in dry chloroform or carbon tetrachloride, and the mixture was allowed to remain for some hours in tightly stoppered bottles, at a constant temperature.

The clear liquid was decanted from the solid, a known volume shaken with much water and titrated against standard baryta, phenolphthalein being used as indicator.

From this experiment the weight of hydrogen chloride adsorbed by the solid in each case was easily deduced.

It was found advisable to cover the solid with a known amount of pure solvent (generally 5 c.c.), in order to allow the former to absorb the maximum amount of liquid before the addition of the standard acid solution.

Experimental.

Experiment I.

Weight of each carbohydrate used, 0·25 gm.
Time of experiment, 18 hours.
Mean temperature, 14° C.

Volume of solution of hydrogen chloride in carbon tetrachloride added, 25 c.c.
Strength of solution. 1 cc. contains 0·00376 gm. of hydrogen chloride.
Weight of acid added, 0·0940 gm.

Carbohydrate	State of solid after experiment	Weight of acid left	Weight of acid adsorbed	
			by 0·25 gm.	by 100 gms.
Starch	Colourless	0·0363 gm.	0·0577 gm.	23·08 gms.
Raffinose ...	Black	0·0384 „	0·0556 „	22·24 „
Maltose ..	Colourless	0·0393 „	0·0547 „	21·88 „
Levulose ...	Black	0·0418 „	0·0522 „	20·88 „
Cane Sugar	Black	0·0525 „	0·0415 „	16·60 „
Cellulose ...	Colourless	0·0526 „	0·0414 „	16·56 „
Lactose ...	Colourless	0·0549 „	0·0391 „	15·64 „
Dextrose ...	Colourless	0·0565 „	0·0375 „	15·00 „

Experiment II.

Weight of each carbohydrate used, 0·5 gm.
Time of experiment, 21·5 hours.
Mean temperature, 14° C.
Volume of chloroform solution of hydrogen chloride added, 30 c.c.
1 c.c. of solution contained 0·00473 gm. of HCl.
Weight of acid added, 0·1418 gm.

Carbohydrate	State of solid after experiment	Weight of acid left	Weight of acid adsorbed	
			by 0·5 gm.	by 100 gms.
Starch	Colourless	0·0288 gm.	0·1130 gm.	22·6 gms.
Raffinose ...	Black	0·0298 „	0·1120 „	22·4 „
Levulose ...	Black	0·0308 „	0·1110 „	22·2 „
Maltose ...	Colourless	0·0368 „	0·1050 „	21·0 „
Cane Sugar	Black	0·0458 „	0·0960 „	19·2 „
Cellulose ...	Colourless	0·0488 „	0·0930 „	18·6 „
Dextrose ...	Colourless	0·0588 „	0·0830 „	16·6 „
Lactose ...	Colourless	0·0658 „	0·0760 „	15·2 „

Experiment III.

Weight of each carbohydrate, 0·25 gm.
Time of experiment, 18 hours.
Mean temperature, 14° C.
Volume of standard solution added to each, 18·2 c.c.
Volume of solvent (CCl_4), 11·8 c.c.
1 c.c. of standard solution contained 0·00501 gm.
Weight of hydrogen chloride added, 0·0912 gm.

Carbohydrate	State of solid after experiment	Weight of acid left	Weight of acid adsorbed	
			by 0·25 gm.	by 100 gms.
Starch	Colourless	0·0317 gm.	0·0595 gm.	23·80 gms.
Fructose ...	Black	0·0324 „	0·0588 „	23·52 „
Raffinose ...	Black	0·0339 „	0·0573 „	22·92 „
Arabinose ..	Yellowish	0·0346 „	0·0566 „	22·64 „
Maltose ...	Colourless	0·0426 „	0·0486 „	19·44 „
Cane Sugar	Black	0·0456 „	0·0456 „	18·24 „
Dextrose ...	Colourless	0·0478 „	0·0434 „	17·36 „
Cellulose ...	Colourless	0·0495 „	0·0417 „	16·68 „
Lactose ...	Colourless	0·0536 „	0·0376 „	15·04 „

Experiments were now made in order to discover the relationship existing between the amount of acid adsorbed by the solid and the amount of acid remaining in solution.

For this purpose, one gram of starch was placed in each of seven tightly stoppered vessels, 10 c.c. of dry carbon tetrachloride were added to each, and the mixture was allowed to stand for about one hour, so that the starch might become fully saturated with solvent: various quantities of a solution of hydrogen chloride in the same solvent were added to each respectively, and the volume was made up to 100 c.c. with dry solvent.

After remaining at the ordinary temperature for eighteen hours, 25 c.c. of the clear liquid were withdrawn, shaken about with much water and the whole was titrated against baryta of known strength.

From these results, the amount of acid adsorbed by the starch, and hence the amount left in solution, was determined.

The results are given in detail below.

For purposes of calculation the well-known exponential formula has been used, namely

$$\frac{c_1}{a} = kc_2^n,$$

where n and k are constants.

c_1 is the amount of acid adsorbed and a is the weight in grams of the adsorbing substance, therefore $\frac{c_1}{a}$ = amount adsorbed per gm.

c_2 is the final concentration of the solution.

Using any two corresponding values for c_1 and c_2, dividing one equation by the other and taking logarithms, the following expression is obtained,

$$\log c_1 - \log c_1' = n\,[\log c_2 - \log c_2'].$$

Hence

$$n = \frac{\log c_1 - \log c_1'}{\log c_2 - \log c_2'}.$$

Starch.

No. of experiment	Volume of HCl solution added	Volume of dry solvent added	Weight of acid added	Weight of acid left	Weight of acid used
	c.c.	c.c.	gm.	gm.	gm.
H	90	10	0·4690	0·1850	0·2840
B	80	20	0·4147	0·1377	0·2770
C	70	30	0·3610	0·1475	0·2135
D	60	40	0·3080	0·1075	0·2005
E	50	50	0·2555	0·0770	0·1785
F	40	60	0·2044	0·0531	0·1513
G	25	75	0·1265	0·0176	0·1089

In experiments *H*, *B*, *C* the starch presented a pink colour after 18 hours, whereas in the others it remained colourless.

If, in this equation for n, c_1' and c_2', the respective values of the weight of acid used and the weight of acid left in experiment *G*, be regarded as fixed and the corresponding values of c_1 and c_2 in the other experiments respectively be substituted, the following values for n are obtained:

0·41, 0·45, 0·32, 0·35, 0·34, 0·30.

Mean value 0·36.

On substituting this mean value of n in the original equation, the following values are obtained for k:

2·74, 2·97, 2·23, 2·35, 2·36, 2·27, 2·45.

Mean = 2·48.

If the corresponding values of $\log \frac{c_1}{a}$ and $n \log c_2 + \log k$ be plotted against each other, the curve obtained is approximately a straight line.

These concentration experiments were repeated with cane sugar, a carbohydrate which blackens in contact with the acid and which gives fairly large quantities of chloromethylfurfuraldehyde when heated with hydrogen chloride under pressure.

One gram of finely-powdered cane sugar was treated with a known quantity of pure solvent, then with various quantities of a standard solution of the acid in dry carbon tetrachloride and the volume was made up to 80 c.c. with dry solvent.

The mixture was left at the temperature of the room for 18 hours and the amount of acid remaining in solution was determined as before.

1 c.c. of solution contained 0·0026 gm. of hydrogen chloride.

Cane Sugar.

No. of experiment	State of solid after experiment	Volume of solution added	Volume of solvent added	Weight of HCl added	Weight of acid used	Weight of acid left
		c.c.	c.c.	gm.	gm.	gm.
J	Blackened	70	10	0·182	0·0613	0·1207
E	,,	60	20	0·156	0·0606	0·0954
D	less ,,	50	30	0·130	0·0533	0·0767
C	less ,,	40	40	0·104	0·0431	0·0609
G	Colourless	10	70	0·026	0·0164	0·0095

Here again the exponential formula was used and n determined by substituting the values of c_1 and c_2 in the expression

$$n = \frac{\log c_1 - \log c_1'}{\log c_2 - \log c_2'},$$

where c_1, c_1' &c. are the various values of c_1; c_2, c_2' &c. are the corresponding values of c_2.

If n be calculated as before, the following results are obtained:

0·52, 0·57, 0·56, 0·52.

Mean = 0·54.

The corresponding values for k are

2·05, 2·30, 2·27, 2·08, 2·25.

Mean = 2·19.

If the values of $\log \frac{c_1}{a}$ and $\log k + n \log c_2$ be respectively plotted, the resulting curve is approximately a straight line.

Experiments with Hydrogen Bromide.

Experiment I.

Weight of carbohydrate in each case, 0·5 gm.
Mean temperature, 15·5° C.
Time of experiment, 19·5 hours.
Volume of solution of acid in chloroform added, 25 c.c.
Strength of solution, 0·00758 gm. of acid per c.c.
Weight of acid added, 0·1895 gm.

Carbohydrate	State of solid after experiment	Weight of acid left	Weight of acid adsorbed	Weight of acid adsorbed per gram
Raffinose...	Blackened	0·0077 gm.	0·1818 gm.	0·3636 gm.
Levulose ...	Blackened	0·0100 „	0·1795 „	0·3590 „
Starch	Almost colourless	0·0116 „	0·1779 „	0·3558 „
Cane Sugar	Blackened	0·0200 „	0·1695 „	0·3390 „
Maltose ...	Pink	0·0247 „	0·1648 „	0·3296 „
Cellulose...	Yellow	0·0403 „	0·1492 „	0·2984 „
Lactose ...	Yellow	0·0668 „	0·1227 „	0·2454 „
Dextrose...	Pink	0·0681 „	0·1214 „	0·2428 „

Hydrogen Bromide. Experiment II.

Weight of carbohydrate used, 0·5 gm.
Temperature, 17° C.
Time of experiment, 18·5 hours.
Volume of solution of acid in chloroform added, 30 c.c.
Strength of solution, 0·00632 gm. per c.c.
Weight of acid added, 0·1986 gm.

Carbohydrate	Weight of acid left	Weight of acid adsorbed	Weight of acid adsorbed per gram
Starch	0·0096 gm.	0·1890 gm.	0·3780 gm.
Levulose	0·0212 „	0·1774 „	0·3548 „
Raffinose	0·0231 „	0·1755 „	0·3510 „
Cane Sugar	0·0328 „	0·1658 „	0·3316 „
Maltose	0·0463 „	0·1523 „	0·3046 „
Cellulose	0·0728 „	0·1258 „	0·2516 „
Dextrose	0·0791 „	0·1195 „	0·2390 „
Lactose	0·1044 „	0·0942 „	0·1884 „

Hydrogen Bromide. Experiment III.

Weight of carbohydrate, 0·5 gm.
Temperature, 17° C.
Time of experiment, 18·5 hours.
Volume of solution of acid in chloroform added, 30 c.c.
Strength of solution, 0·00613 gm. per c.c.
Weight of acid added, 0·1839 gm.

Carbohydrate	Weight of acid left	Weight of acid adsorbed	Weight of acid adsorbed per gram
Starch	0·0154 gm.	0·1685 gm.	0·3370 gm.
Levulose	0·0193 „	0·1646 „	0·3292 „
Raffinose	0·0232 „	0·1607 „	0·3214 „
Cane Sugar	0·0366 „	0·1473 „	0·2946 „
Maltose	0·0405 „	0·1434 „	0·2868 „
Cellulose	0·0707 „	0·1132 „	0·2264 „
Dextrose	0·0838 „	0·1001 „	0·2002 „
Lactose	0·0870 „	0·0969 „	0·1938 „

Concentration experiments with Hydrogen Bromide.

One gram of cellulose, dried for several hours at 100° C., was weighed into each of five stoppered vessels, 10 c.c. of dry carbon tetrachloride were added to each and the mixture was allowed to stand for some time so that the cellulose might become fully saturated with solvent: various quantities of a solution of hydrogen bromide in the same solvent were added to each respectively and the volume was made up to 70 c.c. with dry solvent.

After standing for eighteen hours, at the ordinary temperature,

25 c.c. of the clear liquid were withdrawn, shaken about with much water and the whole was titrated against baryta of known strength, phenolphthalein being used as indicator.

In each experiment, the cellulose became darker in colour, the stronger solutions of the acid producing the deepest colour.

Strength of solution, 1 c.c. contains ·01161 gm. of HBr.

Cellulose and Hydrogen Bromide.

No. of experiment	Volume of solvent added	Volume of solution added	Weight of acid added	Weight of acid used	Weight of acid left
	c.c.	c.c.	gm.	gm.	gm.
F	50	20	0·2322	0·1820	0·0502
G	40	30	0·3482	0·2407	0·1075
H	30	40	0·4644	0·2888	0·1756
I	20	50	0·5804	0·3028	0·2777
J	10	60	0·6966	0·3328	0·3638

The exponential formula was again employed and the following values were obtained for n:

0·37, 0·37, 0·30, 0·30.

Mean = 0·33.

The corresponding values for k are

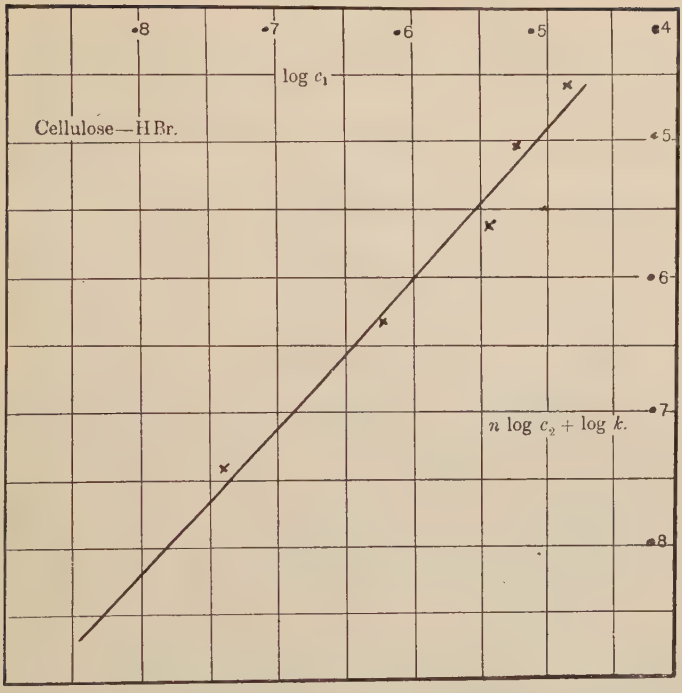
1·98, 2·04, 2·08, 1·88, 1·89.

Mean = 1·97.

If the corresponding values of $\log \frac{c_1}{a}$ and $\log k + n \log c_2$ be plotted against each other, the following results are obtained.

$\frac{c_1}{a}$ is the weight of acid adsorbed per gram of cellulose, c_2 is the final concentration of the solution.

$\log c_1$	$\log k + n \log c_2$
− 0·740	− 0·743
− 0·618	− 0·634
− 0·539	− 0·565
− 0·519	− 0·498
− 0·478	− 0·459



Concentration experiments were made with hydrogen bromide and cane sugar, a carbohydrate which readily blackens with this acid, and the following results obtained:

Weight of sugar, 1·0 gm.
Concentration of acid added, 0·01434 gm. HBr. per c.c.
Temperature 17° C.
Time, 18·5 hours.

No. of experiment	Volume of solvent added	Volume of solution added	Weight of acid added	Weight of acid left	Weight of acid used
	c.c.	c.c.	gm.	gm.	gm.
B	60	10	0·1434	0·0322	0·1112
G	50	20	0·2868	0·0785	0·2083
E	20	50	0·7170	0·2856	0·4314
D	10	60	0·8604	0·3605	0·5000
I	0	70	1·0038	0·4320	0·5718

In every case the solid was completely blackened, and with the stronger solutions the colour of the solution became yellow.

n was calculated in the same way as before with the following results:

0·70, 0·62, 0·62, 0·63.

Mean = 0·64.

The corresponding values for k are

15·3, 16·1, 14·6, 14·6, 14·8.

Mean = 15·0.

Summary of results.

(1) Carbohydrates adsorb hydrogen chloride and hydrogen bromide with great readiness at the ordinary temperature, but the quantity of acid adsorbed varies greatly with the carbohydrate considered.

(2) The relative order of adsorption seems to show no relationship to the chemical constitution and properties of the various carbohydrates and hence no method has been obtained for characterising them.

There appears to be no definite connexion between the adsorbing power for these acids and the production of bromo or chloromethylfurfuraldehyde, since starch gives an extremely minute yield of bromomethylfurfuraldehyde, yet it adsorbs most acid.

The hexaldoses always appear at the bottom of the table. Maltose and lactose differ widely in their powers of adsorption, although they bear a great resemblance in their chemical properties.

(3) The initial phenomenon agrees with that generally accepted for adsorption, and constant values for n and k have been determined by experiment.

(4) The process probably consists of a rapid condensation of the acid on the surface of the solid and afterwards it works its way into the interior: this is evidenced by fructose and sucrose and hydrogen bromide, in which case the sugar first becomes pink and eventually black: this black colour may be either due to chemical action or to the formation of an 'adsorption compound' as in the case of the so-called 'iodide of starch.'

In conclusion, I wish to tender my best thanks to Dr Fenton for suggesting this research, and for much kind help received during its progress.

The results of Sterilisation Experiments on the Cambridge Water. By G. Sims Woodhead, M.A., Trinity Hall, Professor of Pathology.

[*Received* 20 July 1910.]

Although the sterilisation of water has a special local interest at the present time, it is bound to come into greater general prominence as time goes on and as sources from which pure drinking water may be obtained become restricted in number, and as the collecting grounds for the present supplies are built over and sewers are carried through them. Without entering into any discussion as to the bacteriological purity of the Cambridge water, a question on which, however, I hold a very strong opinion, I may point out that in the interests of both consumers and water company, in view of the report of the Local Government Board* on the "potential danger" of contamination, it was necessary to adopt some method of eliminating any such potential danger by adapting or devising a method of sterilising the water or, failing this, of having recourse to a new supply.

Of the methods that have, from time to time, been suggested, some five or six in number, one was soon proved to be unsatisfactory—Clark's method of softening and sedimentation.

Another, sand filtration, was also set down as unsatisfactory and for the following reasons. The Cambridge water, beautifully clear and bright, contains a very small amount of organic matter, so little, indeed, that the formation of an organic filtering film can go on very slowly and, as is now fully recognised, until such film is formed on its surface no sand filter is reliable, that is, it will not keep back the necessary proportion of microorganisms. Experience has proved that filtration of chalk water,—unless such water contains a fair amount of organic matter in suspension and in solution, or argillaceous material in suspension,—is practically useless.

A third method, sterilisation by heat, though no doubt efficacious, appeared to involve too great expense and too large an amount of apparatus to be entirely satisfactory. No doubt satisfactory results could be obtained but it is impossible to test this method without a very large preliminary outlay, an outlay which, if the method were unsatisfactory or too costly to run, would of course be entirely lost.

* Thomson and Crosthwaite, Report to the Local Government Board on Cambridge Water Supply, London, 1908.

Another suggestion was that sterilisation by ultra-violet rays might be adopted, especially as the Cambridge water, beautifully clear, would allow of the ready passage of these rays through its substance. On making enquiries, however, we could then hear of only one small apparatus, an apparatus capable of sterilising some 132 gallons only per hour of an ordinary water supply, more or less according to the translucency of the water. The apparatus a Cooper-Hewitt lamp made from transparent quartz requires for its proper working 3 amperes of current at 110 volts. With this the process of sterilisation had been carried on on a purely experimental scale and the apparatus required would be somewhat costly as it had been determined by the Water Company that any sterilisation experiments carried out on their behalf must be on a sufficiently large scale to supply sterilised water in sufficient quantities for, say, a couple of thousand people.

Ozonisation of the water, the next method that was suggested, has passed somewhat beyond the experimental stage, but although ozonising installations for the sterilisation of water could be seen in various towns in France and Germany there were none to which we could gain access in this country. On making enquiries it was found that the water supplied to Paris is very like the Cambridge water in its chemical composition. We were told that it had been selected specially because of its high degree of hardness, the Commission appointed to find a water supply for Paris maintaining that the use of a hard water is attended with fewer dangers to the human subject than when a soft water is used. The Commission evidently had in view, in this connection, rickets, imperfect bone formation and lead poisoning. At the request of the Water Company Mr Heycock and I journeyed to Paris where we found that the pure chalk water becomes somewhat muddied and otherwise contaminated on its passage from its source by the Marne river to the intake of the Paris Water Company at St Maur where it contains a very large number of the *Bacillus coli communis*. In order to get rid of a large proportion of these organisms and to remove the organic matter, the presence of both of which would interfere greatly with the process of sterilisation by ozone, the water at St Maur is passed through sand-filters, by means of which process a certain degree of clarity—almost equal to that of the Cambridge water—the removal of a considerable quantity of organic matter and the reduction of the number of the *Bacillus coli* to such a point that, usually, not more than one *Bacillus coli* to 40 c.c. of water is obtained. As a matter of actual experience, however, at the time we visited St Maur, these filters were not working quite up to this standard, for although the whole of the organic matter, except some of that in solution and the whole of the inorganic matter in suspension was removed the water then

becoming clear and bright, the reduction in the number of the *Bacillus coli* was not up to the standard given above, a single organism being found sometimes in 10 c.c., sometimes in 20 c.c. and sometimes in 30 c.c. For all practical purposes, however, this was sufficient, for, on examining bacterioscopically the water that had been treated in a de Frise tower by ozone produced in a Siemens-Halske generating apparatus*, 500 c.c. contained not a single *Bacillus coli.* It is evident then that in ozonisation we have a sterilising method which, applied to the Cambridge water, will probably give excellent results. In order to obtain complete sterilisation a low amount of organic matter and a small number of organisms seemed to be essential as, on adding organic matter and increasing the amount of *Bacillus coli* the quantity of ozone required to bring about sterilisation seemed to increase out of all proportion to the amount of these substances added. In connection with this it should be remembered that when we speak of sterilisation of water we refer to the destruction of disease-producing germs or of organisms that are usually found alongside such disease-producing germs. Sporebearing organisms need not, here, be taken into account, as the presence of the sporebearing organisms usually found in water affords no indication of the presence of dangerous contamination. *Streptococci* and the *Bacillus coli communis*, especially the latter, are the indicators we search for as affording evidence of a possibly dangerous contamination, whilst any method of sterilisation that will effectually destroy these may be relied upon to destroy equally, or more, easily any of the disease-producing organisms such as the typhoid bacillus or the cholera bacillus that may have gained access to the water supply.

The Cambridge water is quite clear, contains a very small quantity of organic matter, and the number of organisms is exceedingly low, much lower indeed than in the majority of the water supplies in the United Kingdom; it is therefore an excellent water on which to try the various processes of sterilisation. From the sentimental point of view there seemed to be no objection to the treatment of any water by ozone; even those who object to the addition of chemicals to water would probably not object to the addition of the "health-giving" ozone. I mention this at this point because, curiously enough, when we came to consider the next method we found that certain waters containing organic matter acquired a peculiar taste when treated with chlorine and at first we supposed that this was something specially associated with the taste of chlorine. When, however,

* See de Frise, *Stérilisation de l'eau par l'ozone*, 1907; Daske, *Deutsche Vierteljahrzcshr. f. öffentl. Gesundheitspflege*, Braunschweig, Bd. 41, 1909, for description.

we came to treat the same water with ozone we obtained a similar taste. With neither of these substances, chlorine or ozone, was this taste obtained in the filtered Paris water or in the unfiltered Cambridge water, so that we put the taste down to the action of the chlorine or the ozone upon an excessive amount of organic matter present. These substances, which appear to be related to the amines, chloramines, etc. must be further studied*.

The next method of sterilisation suggested was the treatment of the water with chlorine. In 1894 Dr Cartwright Wood and I† were much struck by the remarkable sterilising power of Hermite's solution—electrolised sea water containing chlorine and hypochlorous acid. We found that very minute quantities of this solution rendered sterile clear sewage from which much of the organic matter had been removed and that in larger quantities it was a most powerful disinfectant even in the presence of considerable quantities of organic matter.

In 1897 I had to go into the question of the sterilisation of the reservoirs and water mains at Maidstone during the great typhoid epidemic‡. Having in view Professor Delepine's experiments on disinfection by means of "bleach" solution, I filled the whole of the Maidstone mains from the gravitation reservoir with a 1—300 solution of chloride of lime or bleaching powder. From the complaints received by the Water Company from all sources I was satisfied that we had attained our object.

In 1898 Kanthack, Robinson and Rideal§ carried out a series of experiments on a similar substance known as "Electrozone" by which "sewage tank effluent" or the filtrate from bacteria beds was rendered almost germ-free. They found, however, that the amount of organic matter was not diminished to any very great extent by this treatment.

In 1903 Lieutenant V. B. Nesfield, I.M.S.‖, was so impressed with the activity of chlorine in its sterilising power in relation to sewage that he applied it to the sterilisation of water, using one part of chlorine to 8,000 parts of water. This, of course, was invariably sufficient to sterilise water, even that to which a considerable quantity of organic matter containing the *Bacillus coli communis* had been added. He used a tablet containing one and a half grains of bleaching powder mixed thoroughly with half a grain of sodium bicarbonate, this latter being added to keep the

* It was found later that some of the taste was due to presence of oil blown over the ozonizing area along with the air from the air pump.

† *Lancet*, London, 1894, Vol. 1, p. 1321.

‡ Woodhead and Ware, *Trans. Soc. Engineers*, Session 1900—1901, London, 1901, p. 49.

§ See Rideal, *Sewage and Bacterial Purification of Sewage*, London, 1900, p. 151.

‖ *Public Health*, July, 1903.

tablet dry and to provide, when added to water, the acid radicle CO necessary to liberate the feebly combined chlorine of the bleaching powder. Water which may be teeming with typhoid or colon bacilli becomes sterile within five minutes of the solution in it of this tablet. He points out further that the addition of a quarter of a grain of sodium sulphite to the pint of water so treated is free from chlorine and is practically tasteless.

$$[Cl_2 + H_2O + Na_2SO_3 = Na_2SO_4 + 2HCl.]$$

This method of sterilisation of water has been put into practice on a considerable scale in India and has been very effective in preventing the spread of water-borne disease amongst troops on active service.

The relatively better results obtained in the sterilisation of water than in the treatment of sewage is accounted for by the fact that sewage contains such a relatively large amount of organic matter and so many spore-bearing organisms. Pathogenic water-borne organisms, however, are killed by chlorine with very great readiness even in sewage, the "filter effluents" of the Boston sewage, as pointed out by Professor E. B. Phelps*, being almost sterilised by 3·5 parts per million, 99·2 °/₀ of the coli organisms succumbing within two hours, whilst 2·2 parts of available chlorine per million is sufficient to act in the same way on the Baltimore filter effluent—a somewhat weaker effluent. Although complete sterilisation required a considerably larger amount of chlorine there are here indications of the ease with which sterilisation may be carried on in the presence of even large quantities of organic matter; water containing a small quantity of organic matter is of course more readily sterilised.

In 1905 Houston and McGowan† sterilised with chlorine the whole of the Lincoln water supplying a population of about 50,000. In the first instance they used one part of Chloros in 10,000 parts of water. As Chloros contains 10 °/₀ of available chlorine they were using 10 parts chlorine per 1,000,000 of water. Later only five parts per 1,000,000 and ultimately one part per 1,000,000 was used. In this case, however, the sterilisation was not complete though the number of *Bacillus coli communis* found in the treated water was undoubtedly small. They found that after treatment 77·3 °/₀ of the 163 samples contained no *Bacillus coli* in 100 c.c., and that 17·7 °/₀ contained *Bacillus coli* in 100 c.c. but not in 1 c.c. Drs Houston and McGowan were of course dealing with a river water and surface water from land drains of an agricultural country and from a series of old sandpits containing a consider-

* *The Disinfection of Sewage and Sewage Filter Effluents*, Washington, 1909.

† Supplement to 35th Annual Report of Local Government Board for 1905–6, London, 1908.

able quantity of organic matter and a large number of micro-organisms; the results obtained must therefore be looked upon as exceedingly satisfactory.

Then came Dr Phelps' experiments followed by Dr Thresh's paper in the *Lancet**. The latter, beginning where Dr Houston had left off, showed that one part of chlorine in a million of water was fatal to the *Bacillus coli* in 24 minutes. He followed this up with a series of experiments in which he found that 0·75 parts in 1,000,000 of water was still effective as a sterilising agent. Thresh suggested, too, that any chlorine not taken up by the micro-organisms or other organic matter present in the water should be neutralised by the addition of bisulphite of soda.

Repeating these experiments but using Cambridge water I found that all that was claimed for this method was under- rather than over-stated, and that the limit at which chlorine would act as a sterilising agent had not yet been reached, and after carrying out a number of laboratory experiments I came to the conclusion that here we had the key to the solution of the question of the sterilisation of the Cambridge water. Continuing I found that "bleach" solutions containing one part of chlorine in two millions of water or even one part in four millions of water if left to act for half an hour upon *Bacillus coli* added to the water killed these bacteria. Typhoid bacilli added were similarly destroyed. Again I found that on adding organic matter, such as broth, to water containing these bacteria it was necessary, in order to obtain complete sterilisation, to introduce a larger amount of chlorine and that until much of this organic matter was saturated with chlorine the chlorine did not attack the bacilli. As soon, however, as a certain degree of saturation was effected the bacilli were attacked and destroyed by any slight excess of chlorine. These laboratory experiments were repeated time after time and in each case with similar results. Continuing the experiments suggested by Dr Thresh on the use of bisulphite of soda as a "Chlorine killer" or neutraliser I found that where an excess of chlorine was used in the presence of non-putrifying organic matter the process of neutralisation both as regards taste and smell was effective, but that in the presence of considerable quantities of *decomposing organic matter*, although the chemical neutralisation of the chlorine was effected, a peculiar iodoform taste and smell remained and it was very difficult to get rid of either taste or smell by continued storage or by the action of light, heat, etc. Then it was found that Cambridge water to which chlorine was added in the proportion of one part to six, seven or eight, millions of water, though completely sterilised, retained not a trace

* *Lancet*, London, 1908, Vol. II. p. 1597.

of either taste or smell of chlorine; in the later experiments therefore no neutralisation by means of bisulphite was considered necessary, especially as it was a difficult matter to keep the bisulphite solution free from organisms which, of course, could remain alive and active in the neutralised chlorine solution.

It is generally accepted as indicated by Odling that dry "chloride of lime"—"bleaching" powder—on being mixed with water is split up into calcium chloride and calcium hypochlorite to the latter of which, in the presence of a weak acid in the water, the sterilising power of chloride of lime is apparently due. Probably the best acid for the purpose is carbonic acid, carbon dioxide, which, releasing the hypochlorous acid from its combination with the base, does not cause it to break up, at once, at any rate. Thus, bleaching powder on being dissolved in water is said to break up into calcium hypochlorite.

$$[2Ca\begin{matrix}Cl\\ ClO\end{matrix} = CaCl_2 + Ca(ClO)_2.]$$

This latter, in the presence of the dissolved carbon dioxide, becomes converted into calcium carbonate and active hypochlorous acid

$$[Ca(ClO)_2 + CO_2 + H_2O = CaCO_3 + 2HClO],$$

the hypochlorous acid combining with organic matter directly or breaking up into hydrochloric acid and oxygen which, in a nascent condition, acting upon micro-organisms as does ozone speedily deprives them of vitality.

$$[2HClO = 2HCl + O_2.]$$

Organic matter, ammonium salts, and even amido-compounds, according to Rideal are attacked by chlorine at once so that any excess of organic matter in water militates against effective sterilisation when small quantities of chlorine are used. Where, however, the amount of organic matter is small and the amount of carbon dioxide dissolved in the water is considerable the conditions for sterilisation by chlorine are ideal. Only when the reduction of the chlorine compounds is complete and they are converted into hydrochloric acid are the best results obtained, the largest amount of oxygen possible then being set free in a nascent condition, and, the hydrochloric acid being tasteless or combining readily with the earthy basic substances, there is little chance of either disagreeable odour or taste being left in the water. In the presence of ammonia, urea and similar substances, although oxygen may be set free and the chlorine may combine with the nitrogenous compound, a certain amount of nitrogen is liberated and

a series of disagreeable compounds described as "chloramines" may be formed, these imparting to the treated water a persistent and disagreeable taste and smell. Hence in the presence of large quantities of organic matter especially when decomposing or capable of rapid decomposition it may be essential that the water should be submitted to some preliminary treatment. In the case of the Cambridge water, however, this is certainly unnecessary.

Bearing on this point it may be noted (1) that the amount of chlorine sufficient to sterilise waters containing considerable amounts of organic matter can not be neutralised as regards taste and smell by the addition of bisulphite of soda; (2) that such taste and smell may be present even when the process of sterilisation is not complete and when no free chlorine can be demonstrated by the iodide of potassium and starch test, whilst on the other hand water that contains only small quantities of organic matter may be completely sterilised and a slight violet iodine-starch reaction indicating the presence of a faint trace of available chlorine still unused, may still be present without the slightest trace of either taste or smell of chlorine remaining in the water. If such water be exposed to light or to a slightly higher temperature this trace of available chlorine disappears and, in all probability, is rapidly converted into hydrochloric acid.

The experiments carried out in the laboratory were not of course on a sufficiently large scale on which to base final conclusions as to sterilisation. The Directors of the Cambridge Water Company and Mr Gray, the Manager, who all along were most anxious to place full facilities for carrying out these experiments at my disposal, requested Mr Hawksley to erect a plant with which the sterilising process might be carried out on a "practical" scale. This consists of (1) a chlorinating cylinder capable of holding about 7000 gallons, so that water passing through this cylinder at the rate of 7000 gallons per hour should remain in contact with any chlorine solution entering the cylinder along with the water for about one hour or thereabouts, the time of contact of course being halved when double the amount of water is passed through the cylinder*; (2) a series of mixing and settling tanks, each containing 700 gallons of water. In one of these tanks is prepared the bleach solution and in another the neutralising or bisulphite of soda solution, the other tanks are used as settling tanks, one for each solution; (3) a couple of pumps so regulated that measured quantities of the bleach solution may be pumped into the water as it enters the chlorinating cylinder, and of the bisulphite solution as the water leaves the cylinder. Measured

* As a matter of fact some of the water passed through the cylinder in about 18 minutes, but most of it remained in the cylinder for nearly an hour.

quantities of chlorine, say from one part in a million of water to one part in eight millions, may thus be pumped into the water entering the chlorinating vessel. Here the two are left in contact as they pass through the cylinder, the chlorine can then, if necessary, be wholly or partly neutralised by the bisulphite of soda solution which is thrown into the water as it leaves the cylinder. The water as it leaves the cylinder flows into (4) a galvanised iron tank over a slotted weir so graduated that the amount of water passing over it at any given time may be easily measured. All samples of the treated water were collected as it passed over this weir. With this apparatus, capable of treating 84,000 gallons per day of 12 hours, sufficient for a community of from 2000 to 2500 people or, if running for 24 hours for 5000 people, a series of experiments were made. At first there was some difficulty in obtaining a steady working of the pumps, then certain difficulties arose in connection with the sterility of the bisulphite solution and again with the balancing or neutralising of the chlorine by the bisulphite owing to the marked instability of the latter salt which seemed to become oxidised very rapidly indeed. Ultimately these difficulties were overcome and out of six experiments made between the 8th December 1909 and the 6th January 1910 samples varying from 110 to 150 c.c. of treated water collected at the weir were found to contain not a single *Bacillus coli* or any of its congeners. In all these experiments one part of chlorine per million of water was used and, to get rid of the taste of chlorine, it was always necessary to neutralise by means of bisulphite of soda. When complete neutralisation was obtained neither taste nor smell of chlorine remained. The amount of chlorine was then halved and the amount of water tested for the presence of coli increased. 300 c.c. of the treated water taken at the weir contained not a single *Bacillus coli* or any organism resembling it. Between the 10th January and the 2nd February 1910 fourteen further experiments were carried out, the amount of chlorine per million of water being again halved, only one part of available chlorine being added to four million parts of water. Here, again, larger quantities of water, in no case less than half a litre, were tested, and in every instance it was found that the organisms of the *Bacillus coli* group had been destroyed. The method of testing, though simple, was very efficient. A strong solution of McConkey's bile salts glucose litmus medium was prepared and a small quantity placed in a litre flask. This, plugged with cotton-wool, was thoroughly sterilised and taken out to the Fulbourn pumping station where the experiments were carried out. At the weir the cotton-wool plug was removed and half a litre of the treated water was allowed to flow directly into the flask; the lip of this flask had, of course, been carefully pro-

tected from dust by a cotton-wool plug covered with tough paper that would stand sterilisation by heat. Sometimes three or four of these large flasks were used during a single day, whilst smaller quantities of the treated water—50 and 100 c.c.—were tested in a similar fashion especially during the earlier experiments; latterly, however, when it was found that the sterilising process was so complete, only the larger quantities were used. These flasks were then incubated for forty-eight hours; if the *Bacillus coli* was present acid was formed, the litmus turning red and gas bubbles making their appearance at the surface. Where there is any doubt as to the formation of gas this can readily be determined by plunging a warm platinum wire into the medium; if there is no gas no bubbles make their appearance, whilst if gas is being formed a large number of small bubbles rise to the surface at the margin of the fluid. This test, known as the "presumptive coli test" is admirable where it is desired to prove a negative, but by itself it is not sufficient to indicate the presence of the true *Bacillus coli*; for our purposes, however, it was all that was necessary, and as we had an enormous number of samples to test we accepted the absence of acid and gas as a proof of the absence not only of the true *Bacillus coli* but of its congeners. Even where the *Bacillus coli* had been completely eliminated a few colonies of spore-bearing organisms sometimes made their appearance on agar- and gelatin-plates; these however were present in exceedingly small numbers and may be disregarded; they are not anthrax bacilli nor are they *Bacillus enteritidis sporogenes*, an organism we have never found in the Cambridge water.

The above experiments seldom extended over more than four or five hours each and were devoted mainly to getting the apparatus "tuned" and although such comparatively good results were obtained the experiments during this period were not looked upon as being either crucial or final. Only on one occasion—when the pumps were being regulated—was any sample found to contain a coliform organism in 50 c.c. of treated water.

In order to obtain prolonged tests under actual working conditions a second series of experiments was carried out. This series extended over 15 days, the pumping engine running from 5.30 a.m. to 5.30 p.m. each day. During this period half-hourly observations were taken and on certain days samples were sent to Professor Frankland, Dr Thresh, Dr Houston and Dr Otto Hehner. Here again on the first and second days of this series we were occupied in regulating the pumps and testing the process of neutralisation of the chlorine by the bisulphite of soda solution and the results obtained were somewhat irregular. At certain periods complete sterilisation of the water was attained whilst at others, especially when the pumps were being regulated and we had an excess of

bisulphite of soda with its contaminating organisms, acid and gas reactions were obtained in 50 c.c. of water, the results obtained by the other bacteriologists coinciding almost exactly with those obtained in the Cambridge laboratories; one of them finding organisms of the coli group in 50 c.c. of water, the other three finding none in 500 c.c. of water. On the next three days of the run every sample of the treated water taken and tested was found to be free from any of the coliform (or any other non-sporing) organisms. Alterations had then to be made in the pumps and in the neutralising solution, and the results were less satisfactory, but the following day "sterilisation" was again complete. Up to this point we had been working with one part of chlorine in 1,000,000 parts of water. The following day the strength was altered to one part of chlorine in 2,000,000 parts of water. The first sample taken after this change was made was found to contain acid-forming organisms though no gas was produced in 100 c.c. of water. From this point on to the end of the fourth day of this experiment sterilisation was complete in every instance, quantities from one to two litres being tested. The following day the chlorine was reduced to one part in 4,000,000 parts of water. Here again although sterilisation was complete on almost every occasion that samples were taken, twice—in one sample taken at twelve o'clock giving a development of acid and gas in 100 c.c. of water and a second sample taken at 5.30 in which acid only was developed—the process of sterilisation was incomplete. On the two following days complete sterilisation was obtained throughout the whole of the run. In all these experiments bisulphite of soda solution was used to neutralise any excess of chlorine and to remove any slight taste and smell of this substance from the treated water. It will be noticed that only on the first day on which any single strength of chlorine was used were any failures to effect complete sterilisation noted, whilst even on these days a failure occurred only at certain periods, periods during which, owing to irregularity of the action of the pump or imperfectly regulated valves, too large a quantity of bisulphite of soda solution, and with it probably certain contaminating organisms, was thrown into the chlorinating vessel. As we gained experience concerning the contamination of the bisulphite solution, of the working of the pumps, and of the falling off in strength of the bisulphite solution, irregularities in the results obtained were gradually eliminated.

Following these experiments the installation was run for a couple of days so as to introduce one part of chlorine into between 7,000,000 and 8,000,000 parts of water. Here the excess of chlorine unabsorbed was so slight that neutralisation was unnecessary as there was neither taste nor smell of chlorine in any

of the samples of water taken from the weir. Sterilisation, as regards the coliform organisms, was absolute.

At this time however I was satisfied that we were not obtaining the greatest possible amount of work from our chlorinating cylinder, and that a small proportion of the water was passing through the cylinder unnecessarily quickly and was therefore in contact with the chlorine for too short a time. Using permanganate of potash in place of chlorine it was found that a trace of colour made its appearance at the weir in about 18 minutes. In order to prolong the period of contact and to prevent this somewhat rapid and irregular passage of a portion of the water a number of simple baffle plates were inserted. By these the passage of the water was so far delayed and regularised that the first trace of an introduced colouring matter made its appearance in about 20 minutes, so that none of the water could pass through the cylinder without being in contact with chlorine for at least that period before it was neutralised; the bulk remained in the cylinder for about an hour.

After these alterations had been made another series of experiments extending over 12 days (12 hours each day) was carried out. Here again we had a run, first for three days with one part of chlorine per million parts of water, the chlorine afterwards being neutralised by a calculated amount of bisulphite of soda. The results obtained were exceedingly unsatisfactory on the first two days and, even on the third, the results were not good. On searching for the cause of this it was found that the bisulphite solution was not sterile and that the organisms contained in it were passing into the water taken at the weir. The chlorine had not time before it was neutralised to kill the organism present in the bisulphite of soda solution. This of course afforded an explanation of the irregular results obtained in the preceding series. Even on the third day, when excellent results were obtained during the greater part of the day, acid-forming organisms, not true *Bacillus coli*, were present at 12 noon and at 3.30 p.m. The results obtained by the other bacteriologists coincided exactly with our own—sometimes the treated water was free from *Bacillus coli* in 500 c.c., at others the presence of this organism could be demonstrated. On the next two days the amount of chlorine was halved. During the early part of one day acid-forming organisms were present in 500 c.c. but, from this time onwards, sterilisation was complete. During the next three days one part of chlorine in four million parts of water was used and only once during the whole of this time was the *Bacillus coli* found in 500 c.c. of water. On the last four days of the experiment one part of chlorine in seven to eight million parts of water was used but no bisulphite of soda was added. Of 13 samples of 500 c.c.

each, taken during this period, every one was sterile, no coliform organisms being found in $6\frac{1}{2}$ litres of the treated water. Moreover on no single occasion was there either taste or smell of chlorine; the water was clear and bright, and fresh and palatable.

From these experiments I am satisfied (*a*) that sterilisation of the Cambridge water by bleaching powder is not only efficient but is easily carried out; for given the faintest trace of chlorine in the treated water as it comes from the chlorinating cylinder at the end of a somewhat indefinite period of treatment (more than 20 minutes) sterilisation was complete; (*b*) that in the case of the Cambridge water it is unnecessary to add bisulphite of soda, the process thus being enormously simplified; (*c*) that the trace of chlorine remaining at the end of treatment disappears very rapidly as the water passes through the pipes or as it is exposed in the reservoir.

The amount of chlorine remaining at the end of treatment may be readily measured by any intelligent labourer supplied with a bottle of iodide of potassium crystals and a flask of filtered starch; a crystal of iodide of potassium, a few drops of acetic acid and a tablespoonful of starch solution added to a litre of the water in a glass jug held over a white tile or a sheet of white paper enabling the observer to determine at once whether it has a blue or a violet tint or is clear. If there is a deep blue tint there is too much chlorine present; a violet tint is the proper "end reaction" showing the presence of a faint trace of chlorine, whilst, if there is no colour at all the amount of chlorine present is not sufficient to ensure sterilisation. This work was carried out mainly by my assistant, Mr Mitchell, to whom I am greatly indebted for the careful manner in which these observations were made. He remained at Fulbourn during the whole time over which the experiments extended, but after a time the engineer who looked after the pumps was detailed to make these colour estimations. These I compared with my own and Mr Mitchell's estimations and found that they were in all respects most satisfactory. This, of course, is an exceedingly important practical detail for it is evident that the chlorine may be made to serve as an indicator of the presence of organic matter in water. Instead of waiting for a bacteriological examination, the simplest and shortest of which takes several days to carry out, an estimate of the amount of chlorine absorbed may, if necessary, be made from hour to hour, the amount of chlorine required to give the violet reaction at the end of contact being the amount necessary to ensure complete sterilisation.

From these experiments then I am satisfied that from the point of view of ease of application, certainty of action, and absence of interference with the physical characters of the water,

this method of sterilisation is admirably suited for the treatment of the Cambridge water and indeed of all waters where the amount of organic matter is small but which are open to suspicion on other grounds. Even in those chalk waters in which the amount of organic matter is variable this method may be utilised, though in cases where the amount of decomposing organic matter is greater the difficulty of getting rid of taste and smell may be increased and some modification of the process may have to be devised.

It may be asked what is the effect of this treatment on living animal organisms. Sometime ago F. S. Locke* following up experiments carried out by Naegeli and by Ringer and Phear† found that very minute quantities of copper dissolved in distilled water had a toxic effect on tadpoles, killing them in from two to twenty-two hours, one part of sulphate of copper in two millions of water proving fatal. It was noted that several other heavy metals acted in the same way. Naegeli had already pointed out that one part of copper in seventy-seven million parts of distilled water was fatal to *Spirogyra*. Chlorine in the minute quantities used in the experiments above recorded seems to act equally powerfully on vegetable protoplasm, but on the health of tadpoles one part of chlorine in two million parts of tap-water has absolutely no effect, the tadpoles remaining exceedingly lively at the end of nine or ten days. Further it was noted that chlorine added to distilled water actually neutralises the action of copper, iron, and tin. It does not prevent the formation of a carbonate of lead, and tadpoles kept in chlorinated distilled water in which has been placed a bright strip of one of these metals remained active and vigorous whilst tadpoles that are placed in distilled, but non-chlorinated, water in which are placed similar strips of metal soon die. It is interesting to note that water drawn from the laboratory tap which has remained in the pipes for some little time proved fatal to tadpoles in 24 hours, showing that these creatures are very susceptible to slight changes in the water in which they are placed. Water drawn directly from the main and which has not remained in the pipes for any length of time has no effect upon the tadpole, which remains perfectly lively just as it remains in any dilution of chlorine up to one part in two millions of water whether bright metal has been added or not. This chlorinated water therefore is less deleterious to tadpoles than is water that has passed through the laboratory supply pipe, water that we constantly drink and use for all ordinary purposes.

It is evident then that in the Cambridge water very minute quantities of the oxy-chlorine compound are sufficient to bring

* *Journ. of Physiol.* London, Vol. xviii, 1895, p. 319.
† *Journ. of Physiol.* London, Vol. xvii, 1894–5, p. 423.

about most efficient sterilisation, especially if the following conditions are observed. The chlorine should remain in contact with the water as long as possible; the ideal method of treatment, therefore, would be by means of an automatic "injector" to throw "bleach" solution into the mains leading from the pumping wells to the reservoir. During dry weather, the necessary amount of chlorine once determined, the work once started and the chlorine flow regulated, the process may be allowed to go on almost without attention. During rainy weather rough tests should be made as to the amount of chlorine absorbed from time to time. When no colour reaction is obtained with iodide of potassium and starch it would be necessary to increase the strength of the chlorine solution until a reaction is obtained. This of course may be done either by increasing the amount of "chloride of lime" added to the solution already in the chlorine supply tanks, or by increasing the speed of the pumping engines injecting the fluid. The test is made in three or four minutes and any increase of organic matter is determined at once and, in our experience, is so small that it may always be neutralised even after the heaviest rainfall observed during the time our experiments were being made by an increase of about 20 °/₀ of chlorine, this addition invariably ensuring complete sterilisation. The chlorine and water should be thoroughly mixed in the rising main and should be conducted to a reservoir of sufficient capacity to contain a couple of days supply. Equalisation of the chlorine would thus be obtained and the conversion of any slight excess of chlorine into hydrochloric acid would be ensured. Water so treated is absolutely free from organisms of the *Bacillus coli* type and therefore from bacilli of the typhoid and paratyphoid groups.

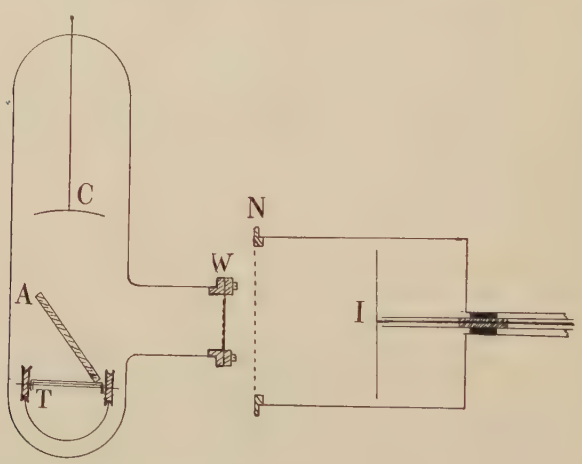
Preliminary note on the properties of easily absorbed Röntgen Radiation. By R. WHIDDINGTON, B.A., Allen Scholar, St John's College. (Communicated by Prof. Sir J. J. Thomson.)

[*Received* 2 August 1910.]

The present note is intended to give a brief outline of results obtained by the writer during the course of investigating the properties of very soft Röntgen radiation and to indicate the direction which the research is taking.

A study of the literature bearing on primary Röntgen radiation revealed the fact that what little work has been done on the subject did not include quantitative investigations into the properties of such soft radiations as are to be referred to below. Apparently the softest radiation ever quantitatively studied is the characteristic chromium radiation of mass absorption coefficient in aluminium $\left(\frac{\lambda}{\rho}\right)$ 136*.

The form of apparatus used in the experiments on the softest radiation is indicated in the figure†.



When the tube is evacuated to a convenient point a potential applied between C and A—C being the cathode—produces a stream of cathode rays which strike the anticathode A. A is one

* Barkla, *Phil. Mag.* 1909, vol. XVII. p. 749.

† For potentials higher than 4000 volts a different form of tube is found to be convenient.

of a series of metallic plates mounted on the magnetically controlled carriage T, so that any one can be brought under the action of the cathode stream. The velocity of the cathode rays can be varied by altering the potential applied to the tube.

The Röntgen rays produced are allowed to stream out through the aluminium window W and are measured by the ionization they produce in the ionization chamber I.

Absorbing screens can be introduced between N and W.

The following are some of the more important results.

1. There is a cathode ray velocity below which no Röntgen radiation can be detected.

2. Using cathode rays produced by a potential of 2500 volts and over, the anticathodes arrange themselves in the following descending order as radiators: aluminium, platinum, silver, lead, tin, nickel, cadmium, iron, antimony, copper, zinc. Antimony has a tendency, at greater potentials, to take a higher position in this scale.

Aluminium at these low potentials is about five times as efficient an anticathode as copper.

There is strong evidence in favour of the view that aluminium and platinum give out characteristic secondary radiations. The mass absorption coefficient $\left(\frac{\lambda}{\rho}\right)$ in aluminium is about 600.

3. There is reason to believe that this secondary radiation can be excited by a primary radiation somewhat less penetrating than itself.

4. The emission of the secondary radiation from aluminium and platinum is accompanied by corpuscular emission.

The investigation is being continued to higher potentials.

Further notes on the procession of Cnethocampa pinivora. By H. H. Brindley, M.A., St John's College.

[*Read* 6 June 1910.]

[Plates XIII, XIV.]

In October 1906 I described to the Philosophical Society some observations made at Arcachon in the previous April on the processional habit of the larvae of the Eupterotid moth *Cnethocampa pinivora* (*Proc. Camb. Phil. Soc.*, 1907, xiv. Pt. i. p. 97). In this paper I referred to the work already published on the subject from the pioneer observations of Réaumur in 1736 to the account of the life history of the moth given by Fabre in 1898 (*Souvenirs Entomologiques*, Série vi.), which is our chief source of information. It is unfortunate that this work, so delightful for its literary beauty, often fails the reader in exactness and statistical detail. The moth lives entirely among the pines of the Landes (where the tree is chiefly *Pinus pinaster*) and other districts of southern France. The eggs are deposited in cylindrical clusters on the young leaves of the pine, hatching occurs in September and the offspring of one parent construct a silk nest in the branches. Feeding and enlargement of the nest take place during the winter months, and on warm days in late March and early April excursions are made in procession from the nest tree over the sand. Burrowing for pupation concludes the last procession. The imagos appear in August and live only a short time, how long exactly is still uncertain, but Fabre's account indicates that the eggs are laid a few days after the adult stage is reached. The procession is in single file and each larva secretes a thread which Fabre holds to be the guide for return to the nest tree.

The accounts of Réaumur, Ratzeburg (1840), and Fabre and my own observations in 1906 left certain points about the procession in doubt, and last year Mr T. G. Edwards of Emmanuel College, at my suggestion, went to Arcachon and examined many processions between March 18th and 31st. His observations are recorded in the *Proceedings*, 1910, xv. Pt. v. p. 433. I visited Arcachon again from March 29th to April 5th of the present year, but was unfortunate in the weather, for it was unusually cold for the region in April, two days presenting the exceptional event of snow squalls, which kept the larvae in their nests. On no day could I discover any processions in the woods, though, except on the coldest days, larvae were to be found on the march in the Allée de Turenne, which is a sheltered road of villas in gardens

with many pines, descending to the entrance of the actual forest. Mr Edwards informs me that most of the processions and the longest individual ones he found were in the Allée de Turenne in the much more favourable weather he experienced. The comparative frequency of processions in this inhabited road is curious, as the pines in the villa gardens are nearly all high and nests seem relatively scarce, while they are very numerous in the large plantation of saplings, averaging 12 ft. high, surrounded by old pines destitute of nests a short distance within the forest. The accompanying photographs show nests in this plantation. I examined a large number of the nests on the saplings and did not find one which contained more than 16 living larvae, and this number was exceptionally high. As there was no evidence that the nests were temporarily deserted by larvae in procession it seemed that their occupants must have buried for pupation before my visit, which on the whole agrees with Edwards a fortnight earlier finding many more processions than I did. But if the forest larvae had buried themselves it appears probable that some condition holds for the Allée de Turenne which causes certain or all the events of the life cycle to fall later than in the forest proper. The longest processions I found in the Allée were 33 and 68 larvae respectively. These two were close together and there were reasons for thinking that they were a procession of 101 broken. Edwards records one of about 260 larvae, and Fabre observed one of 300.

As stated above I found no processions in the pine forest itself, and as prolonged observations in the Allée de Turenne were difficult from the traffic I made collections of larvae there and placed them on the sand in the forest. This treatment did not interfere at all with the formation of processions, and when the larvae were placed on a table indoors processions were made as quickly as on the sand in the forest: thus in one case a group of 7 larvae placed on a tablecloth were in regular march in 8 minutes.

It will be noted that the circumstances caused my observations this year to be made under less natural conditions than those by Edwards. Again, some of them were made on members of single processions and others on larvae of different processions mixed together. Thus some of the features noticed may have been due to the larvae being from different nests and of different ages, but I did not find any distinction between the behaviour of what may be called "pure" and "mixed" processions. Larvae certainly from different nests formed processions as readily as those belonging to one procession. But the great tendency of processions to break up into daughter processions recorded by all observers and the readiness with which stray larvae will join a procession render it impossible to regard any procession seen first in march as belonging to one nest. This raises the important

question as to how far the families remain separate after they have commenced the processional habit. Continuous observation of the members of one nest from the first day of procession is necessary to be certain on this point. Fabre does not deal with this question definitely. He observed the larvae emerging from the nest for a march, but does not say whether the integrity of the family was maintained from day to day. He states however (p. 339) that daughter processions return to the nest tree by wandering till they find the thread. The observations by Edwards and myself suggest that this must very often fail to happen. (*Before* the processional excursions begin it is probable that the larvae remain in separate families, since it is rare to find more than one nest in a tree.) In fact we remain in great ignorance regarding the peculiar habit of forming processions which return to the nest tree. That the larvae leave the tree in a procession which may cover much ground and which ends in burrowing for pupation is quite certain, but it remains uncertain whether processions which return to the tree are the rule or only exceptional. Fabre gives no statistics on this matter, but states merely that in winter and early spring such processions do occur on fine mild days, hazarding the suggestion that they represent "une promenade hygiénique, un pèlerinage de reconnaissance aux environs, peut-être un examen des lieux où se fera plus tard l'ensevelissement dans le sable pour le métamorphose." *Primâ facie* such processions appear disadvantageous, for instance, apart from other accidents such an exposed proceeding might be expected to engender, oviposition by Tachinid flies is likely to occur, and this event I observed in 1906, as described in my previous paper. In spite of some degree of protection by the glandular hairs of the larvae, which were obviously dreaded by the flies, the latter succeeded every now and then in inserting their ovipositors. In 1909 and this year Edwards and myself saw no Tachinids, but the sunless bleak days of my visit this year were probably unfavourable to their activity. In the limited time at my disposal and in the absence of opportunity of following a procession from its departure from the nest till its return to it, I endeavoured this year to obtain circumstantial evidence as to the occurrence of processions which return to the nest tree. One step was the examination of saplings bearing nests. In all cases the branches near the nest were coated profusely, in fact matted, with the aggregation of threads formed by the larvae, which is in accordance with Fabre's statement that in their nocturnal excursions from the nest to feed on the young leaves the larvae separate, each spinning a thread which he holds is a guide to returning to the nest towards sunrise. Below the nest the matting over with threads becomes less profuse, and on the trunk of the sapling threads are rare, though in places some are seen extending

towards the ground. In spite of much search I never found one reaching to the ground and very rarely within a yard from it. There is little difficulty in seeing the thread, which glistens as it lies adhering to the flakes of bark. Moreover it is very durable and does not seem to suffer much from wind or rain, as shown by the matting over of the upper branches and the durability of the nest itself, which is essentially a winter dwelling. Again, the thread left by a procession is the aggregate secretion of all the larvae; Fabre calls attention to its thickness when formed by a long procession, and Edwards records that 150 larvae may make a thread nearly 3 mm. wide. Such a thread is very strong and I have lifted the thread of a procession clear of the ground for two feet without breaking it, though it was loaded with adhering sand particles. Now the threads which I found running down the trunks of saplings were thin ones resembling those secreted by single larvae, and certainly did not suggest processions frequently leaving and returning to the tree. There seemed no ground for thinking that threads near the base of the trunk are especially likely to be swept off accidentally, by the movements of undergrowth in wind for instance. It must be admitted that the examination was on trees bearing in most cases deserted nests and perhaps some unknown cause had removed the threads near the ground since the final procession had left the tree, but if a thread near the base lasts as long as those on the branches, that of the final procession should have been found. But on the whole the impression left by the threads on the bark is that the larvae do not habitually leave the tree and return to it.

If the thread is really a guide to return to the nest tree the leader of a homeward bound procession must make use of it by coming across its own thread and recognising it as the one to follow. Edwards sought for information on this point by placing a frayed-out piece of artificial silk or the natural thread of another procession in the leader's path—this was disregarded entirely. I repeated Fabre's experiment of picking up the thread of a procession with forceps and looping it round so as to be encountered by the leader. On each occasion, and the experiment was made with different processions, the leader crossed the thread without regarding it more than any other small obstacle he encountered. This accords with Fabre's experience (*loc. cit.* p. 339), when endeavouring to make the larvae march in a closed circle. If a larva can distinguish its own thread among the numerous ones which are seen on fine days crossing the sand in all directions, it appears probable that it would show some recognition of it so soon after its formation as this, even if its psychological condition were not in favour of a return to the nest tree at the moment. It can only be said that much more

evidence is required before we can feel certain that Fabre's suggestion as to the function of the thread formed during procession can stand. It may well be that his far more extensive study of the larvae compared with other observers justifies his conclusion, but he does not present the evidence in the *Souvenirs*.

As regards the thread being a less important aid to maintaining the integrity of a procession than head-and-tail contact of the larvae, all that I observed is in agreement with Edwards, who made many experiments to test the point and came to the general conclusion that in the case of breaks in a procession joining occurs more readily if the thread is intact, though if the interval is short the presence of the thread seems negligible. Only in one instance of many did it appear to me that joining up really resulted from the larvae finding and following a thread secreted ahead of them. A procession of 6 larvae came across a thread formed by a single larva, when the leader of the 6 closely followed it, a proceeding which was more obvious because the single larva went round in a circle. At the same time the distance was only a few inches, and sight may really have been the guide. On all the other numerous occasions on which two processions became one or single larvae joined processions there was no suggestion that the thread was regarded, while on several occasions the impression that sight was the chief aid in effecting joining up was strong. Moreover, a single larva joining a procession does not always make contact with the last in the procession, but just as often comes parallel to the marshalled larvae and quickly intercalates itself between any two, being accepted without hesitation.

In considering the value of the thread in maintaining the integrity of a procession it is necessary to refer to the "circulating mass," so frequently formed by larvae on the march. This condition has been already described by Fabre (*loc. cit.* chap. XX, p. 332), Edwards (*loc. cit.* p. 434), and myself (*loc. cit.* p. 100), and it is enough to say here that the chain breaks up into a heap of larvae walking over each other without the heap as a whole making way over the sand. The mass formation was held by Fabre to be encouraged by cold and darkness, and to be very likely a mode of resting. Edwards has discussed the subject (*loc. cit.* p. 434). It is certainly the usual condition of a procession about to burrow, but it is frequently adopted temporarily, the procession reforming sooner or later. Artificial interference with a procession will often cause the formation of a mass, but it is quite unknown why it is so frequent a condition of a group of larvae on the march. While in a circulating mass the individuals continue to secrete threads, but the continual crossing over each other by the larvae necessarily prevents the fusion of the threads which

occurs when they are in procession, so the animals are very soon in a tangle of silk. Edwards is inclined to think that the threads secreted in the mass may be a help towards a procession when reformed keeping the same leader. In five different processions, the largest of which consisted of 57 larvae, he found that the new procession had the same leader as the old one.

I endeavoured to obtain light on the matter by observing short processions, distinguishing the individual larvae by powdered chalk of different colours, which, like the flour used by Edwards, adheres well to their hirsute dorsal surface. If only a small number of larvae are observed it is quite possible to preserve their identity by this method; with large numbers a colour scheme naturally breaks down. It is believed that the observations set forth below were accurate; others which appeared faulty from one cause and another are omitted. The times at which changes occurred are given to convey an idea of the rapidity with which the larvae alter their procedure. The numbers indicate individual larvae, 1, 2, 3, etc., being allotted according to their order in the first procession of one continuous observation. ... indicates that two larvae were not quite in contact. Each observation commenced by placing the larvae in a heap on sand with little or no vegetation.

I. Batch A. 5 larvae. April 2nd.

5.0 p.m.	larvae put in heap.
5.5 p.m.	procession 12345.
5.15 p.m.	larvae again put in heap.
5.22 p.m.	procession 1352...4.
5.25 p.m.	larvae again put in heap.
5.30 p.m.	procession 253...4...1.

II. Batch A. 5 larvae. April 3rd.

10.40 a.m.	larvae put in heap.
11.0 a.m.	procession 12345.
11.10 a.m.	larvae again put in heap.
11.15 a.m.	procession 35142.
11.28 a.m.	larvae again put in heap.
11.30 a.m.	procession 32145.
11.34 a.m.	procession 321405, 0 being a larva belonging to another batch which had buried itself shortly before and had just been dug up a few inches away from the procession, which it joined voluntarily in the position shown.
11.35 a.m.	larvae again put in heap.
11.45 a.m.	procession 462513.
11.50 a.m.	larvae again put in heap.
11.59 a.m.	procession 4235 (0 and 1 had strayed away).

III. Batch A. 6 larvae. April 3rd.
- 4.45 p.m. larvae put in heap.
- 4.55 p.m. all burying.
- 5.5 p.m. burying relinquished, procession forming.
- 5.15 p.m. procession 123456.
- 5.20 p.m. larvae again put in heap.
- 5.25 p.m. procession 543621.
- 5.40 p.m. a stray larva (0) from another batch was followed and the procession became 0...543621.
- 5.45 p.m. larvae massed voluntarily.
- 5.54 p.m. procession 2643105.

IV. Batch A. 7 larvae. April 4th (the only indoor experiment quoted).
- 1.25 p.m. larvae put in heap on white table-cloth.
- 1.40 p.m. procession 1234567.
- 1.45 p.m. larvae massed voluntarily.
- 1.51 p.m. procession certainly with another leader, but which exactly not ascertained.

V. Batch B. 6 larvae. April 2nd.
- 3.25 p.m. larvae put in heap.
- 3.38 p.m. procession 123456.
- 3.45 p.m. larvae massed voluntarily.
- 3.50 p.m. procession 1342 ? 5 ? 6.
- 3.55 p.m. larvae again massed voluntarily, during which four larvae (LLLL) from another batch came into the group.
- 4.30 p.m. procession forming as 1423LL.
- 4.40 p.m. procession 12356LLLL (No. 4 had commenced burying).
- 4.45 p.m. procession 1235? 6L4LLL (No. 4 having relinquished burying and intercalated).

VI. Batch C. 8 larvae. April 3rd.
- 4.50 p.m. larvae put in heap.
- 4.55 p.m. some or all burying.
- 5.5 p.m. burying relinquished, procession forming.
- 5.15 p.m. procession 12345LLL (last three not marked with colours).
- 5.18 p.m. larvae put in heap.
- 5.41 p.m. procession 1L2L35L.

A summary of the above is

	Number of cases		
	After voluntary massing	After artificial massing	Total
Same leader as in previous procession	2	4	6
New leader	2	4	6

In this there is little or no suggestion that the threads secreted in the circulating mass help to preserve the order of a procession, for even if the threads of individual larvae are secreted continuously in a voluntary mass they are certainly broken when the mass is made artificially. Edwards (*loc. cit.* p. 435) suggests that the formation of a mass may be a means of altering the order of the larvae while retaining the same leader, but there seems to be little evidence for this view.

Moreover the mass formation may continue for an hour or two before a new procession is formed, and as threads continue to be secreted the larvae are in a tangle of silk after a prolonged massing, in which it seems impossible any individual thread can be distinguished by them. It is true that the number of cases in which the old leader was retained is, both in Edwards' and my own cases, higher than mere chance would give, and a possible explanation is that in any procession one larva has a greater tendency to stray than the others, and this one is thus more likely to act as leader. I found that it is not at all uncommon for one larva to stray from a mass and to repeat this again and again when put back artificially. Such specially erratic larvae secrete a thread but it is disregarded by the others. These independent larvae sometimes return to the mass voluntarily after an excursion several inches away. In this case and also if put back artificially they are often seen to relinquish the wandering and to take any place in a new procession. It may be suggested that the procession is reformed from a mass by a larva with the wandering habit relatively marked tending to leave it, while the other larvae, as can always be seen when a procession forming is watched, in the endeavour to retain contact become parallel with it, at the same time falling behind because it walks faster. One of its fellows thus soon finds its head in contact with the leader's tail, which is the processional position. The same thing happens for all the larvae in succession and these fall from two or three abreast into single file till the new procession is fully arranged. Processions are invariably formed by the larvae coming into head-and-tail contact *in succession*. I have never seen them make head-and-tail contact simultaneously with their bodies bent and then the procession gradually straighten. What happens is that the procession emerges as a straight line from a continuously diminishing circulating mass.

The above observations suggest that the thread is of very little or no value in the formation of a procession, and that its help in maintaining the integrity of a procession is slight. It seems to be secreted all the time that larvae are in active movement, whether on the march, in mass or burying. It forms the nest in the tree and the cocoon of the pupa state. Why its secretion

should continue when the larvae are feeding and when they are away from the nest tree remains obscure, and at present Fabre's ingenious suggestions as to its value can be regarded only as surmises. The great expenditure of material and energy which its formation involves we have no right to call useless. It is possible of course that it is to some extent an excretory process, and we know that sericin and fibroin, which occur in silk, like the other skeletins of invertebrate animals, contain nitrogen (Schäfer, *Text Book of Physiology*, 1898, I. p. 76).

As far as I could ascertain a larva marching alone secretes a thread almost invariably. That of a single larva is very fine, but careful examination of the sand in its track with a lens shows it: even this single thread is strong enough to be lifted for an inch or two without breaking in spite of the particles of sand adhering to it. A fairly extensive enquiry on this point revealed only three or four cases of larvae marching without making the thread.

The details of processions given above are quite in agreement with the original statement by Fabre and the observations of Edwards that there is no special leader: when once a procession is formed the leading larva may be removed without interference with the progress of march. Leadership in a particular sense exists only in the formation of a procession from a circulating mass. All that therefore need be said on the subject here is to quote an experiment I made to find out with some exactness if the removal of the leader checked the progress of a procession at all. In the case of a procession of 6 larvae I reversed each larva, placing it with forceps as quickly and carefully as possible in head-and-tail contact with its neighbours, thus effecting an artificial countermarch. The procession proceeded at once in the direction opposite to the original one. Immediately it was well on the march I removed the leader and placed it in contact with the last larva, repeating this each time immediately the march was resumed. The times given below are those when the order was changed as stated.

5.0 p.m.	←123456	5.9 p.m.	456123→
5.2 p.m.	123456→	5.14 p.m.	345612→
5.5 p.m.	612345→	5.17 p.m.	234561→
5.7 p.m.	561234→	5.19 p.m.	123456→

The interval of five minutes between 5.9 and 5.14 was not due to special hesitation of the leader, but because removing it was delayed in order to watch the behaviour of the larvae to sunlight, which at this time broke through the clouds. Each time the leader was removed the new leader cast about with its

head in the manner characteristic of all isolated larvae and of any larva not in contact with the tail of another, and thus it checked the progress of the procession. In only one case did the hesitation last for more than two minutes. In all cases the old leader placed in head-and-tail contact with the last larva joined up immediately. This and similar observations made on other groups also suggest how little necessary is the thread for keeping the integrity of a procession. The extent to which the leader for the time being takes the initiative has been discussed by Fabre (*loc. cit.* p. 337), and Edwards (*loc. cit.* p. 432).

The influences which determine the direction of a procession on the march remain very obscure. Fabre (pp. 367, 368) states his belief that sunlight is the guide, but adds "les points d'où vient le plus de chaleur sont les préférés." In the case of processions I have seen in the woods the general direction appeared to be towards the greatest sunlight through the foliage, and the sidewalks and roadway of the Allée de Turenne at Arcachon in which processions are so often seen are more sunlit than the gardens from which they come: again in the case of processions indoors the course taken was usually towards the window. Edwards (*loc. cit.* p. 432) describes how the leader of a procession followed rays reflected artificially on to him alone. This year, April 4th afforded the only favourable opportunity I had of observing the possible effect of sunlight, as that was the one day of sunshine, bright intervals occurring between showers. Two short processions of three larvae each and a single procession resulting from artificial mixing of these certainly did not march towards the sun. The general direction taken was either at right angles to the path of sunlight or away from the light. In one case the sudden breaking out of light from between clouds was at once followed by the procession of six turning away from it. In this and on other occasions attempts to make the processions march towards the sun failed altogether. These processions were on sand clear of trees, but from 10 to 20 feet away from a zone of saplings. The latter were away from the sun and the general direction of the larvae was towards them. Whether smell or other sense attracted the larvae towards the trees cannot be asserted, but they certainly did not follow the path of greatest light. We remain in ignorance of the stimuli which influence the behaviour of a procession. The remarkable dorsal papillae of the larvae may be "appareils de météorologie" as Fabre has suggested (*loc. cit.* p. 366), and in various respects it is likely that the larvae respond to influences we cannot comprehend.

Burying for pupation has been described in detail by Fabre (*loc. cit.* pp. 368, 369) and also by Edwards (*loc. cit.* p. 435). I watched it frequently this year and found that as a rule all the larvae of a

circulating mass beginning to burrow together occurred much more frequently than the leader checking a procession by commencing to burrow and its followers doing so after an interval. I may here add to the descriptions already given of the action of a larva burying itself that the chief movement of the body is dorso-ventral bending as the sand is attacked with the mandibles, and that the hind end of the body and the prolegs are the chief means of bringing the excavated sand to the surface. The true legs seem to shovel the loosened sand back to the prolegs. In one instance in which the leader of a procession of seven commenced burrowing its example was not followed. The second larva partially descended the hole made by the leader but did not burrow. This continued for 20 minutes, when the remaining six larvae marched away in two processions of three each. The leader had quite buried itself in 31 minutes. In two sets of observations on the same group of larvae earlier in the day this particular larva had been remarked as erratic and repeatedly straying from its fellows.

I do not know any published statements on the time which elapses between the beginning of burrowing and complete burial, so I may mention that single larvae were seen to bury in 31 and 37 minutes, a group of 4 in 110 minutes, one of 7 in 120 minutes, one of 12 in 85 minutes, and one of 21 in 110 minutes. Seven larvae which had been buried for 48 hours were dug up; one of these completely reburied itself in 10 minutes and the other six did so in 40 minutes. These larvae were from 3 to 4 inches deep when first dug up and appeared to have added very little silk in the two days. Fabre indicates that the cocoon is completed in fifteen days. I brought a number of larvae, some of which had buried themselves by April 2nd, in a biscuit box filled with sand to Cambridge. The journey did not appear to interfere with pupation, as by April 15th several had apparently completed their cocoons. The imagos began to emerge from the sand on August 16th, and I placed them on a sapling of *Pinus pinaster* covered with muslin in the hope that they would pair and lay their eggs on the young leaves, but none of the moths lived long enough to do so.

While at Arcachon the larvae I kept in boxes refused to feed on young pine leaves, but seemed quite as active after several days starvation as when captured. Possibly feeding is given up as the time for pupation approaches.

As in 1906 and in agreement with Edwards' experience I experienced no irritation from handling the larvae. No doubt sensibility to the glandular hairs varies with the individual: Fabre's own case, so vividly described (*loc. cit.* p. 378), is probably exceptional.

Very much concerning the processional habit remains obscure.

Réaumur and all subsequent authors observed the larvae to a considerable extent under somewhat artificial conditions. Even in Fabre's glass-houses containing pine saplings and sand at Sérignan the larvae could not wander as much as in the woods. The really satisfactory method would be to follow day and night the proceedings of the occupants of a nest in the woods. This would throw light on how far they can range and yet find their nest again and to what extent the integrity of a family is maintained. The meaning of processions which do not end by burying and the advantage obtained by continuous secretion of the thread remain puzzles. So also does the frequent formation of a "circulating mass" by a procession. It seems certain that contact between larvae is of much importance and that though there is no permanent leadership the leader for the time being does really determine the behaviour of a procession because all its followers endeavour to maintain head-and-tail contact. This is seen well when the leader checks its march, for the larvae always endeavour to maintain head-and-tail contact by bending their bodies: overlapping ensues only if the leader stops for some little time. Thus mass formation is always incepted by the leader, as sometimes also is burrowing.

In my previous paper I suggested that the procession may be compared with a chain of salps or polychaetes, save that organic union is absent, in the sense that they probably all respond fairly equally to the same stimulus, hunger, sunlight, desire to pupate and so on. The observations made since this was written seem to emphasise the comparative importance of the leader for the time being.

I am indebted to Mr Edwards for much information on various matters referred to in his paper and to my wife for continuous assistance during the observations made this year.

LITERATURE.

Réaumur, 1736, *Mémoires pour l'histoire des Insectes*, II. pp. 149—161 (Paris).

Ratzeburg, 1840, *Forst-Insecten*, II. p. 128, and Taf. 8 (Berlin); also *Stettiner Entomologische Zeitung*, p. 40.

Lapaury, 1876, *Ann. Soc. Ent. de France*, XVI. p. 244 (Paris).

Fabre, circa 1898, *Souvenirs Entomologiques*, Sér. VI. pp. 298—392 (Paris).

Brindley, 1906, *Proc. Cambridge Phil. Soc.* XIV. Pt. I. p. 97.

Edwards, 1910, *Proc. Cambridge Phil. Soc.* XV. Pt. V. p. 431.

Fig. 1. Nest of *Cnethocampa pinivora* on *Pinus pinaster*, Arcachon, Gironde; April 4, 1910.

Fig. 2. Nest of *Cnethocampa pinivora* on *Pinus pinaster*, Arcachon, Gironde; April 4, 1910.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1909—1910.

ANNUAL GENERAL MEETING.

October 25th, 1909.

In the Optical Lecture Room.

Dr Hobson, Vice-President, in the Chair.

The following were elected Officers for the ensuing year:

President:

Prof. Bateson.

Vice-Presidents:

Dr Hobson.
Dr Fenton.
Prof. Seward.

Treasurer:

Prof. Newall.

Secretaries:

Mr A. E. Shipley.
Dr Barnes.
Mr A. Wood.

Other Members of the Council:

Dr Duckworth.
Mr W. G. Fearnsides.
Dr Sell.
Mr W. E. Dixon.
Prof. Wood.
Prof. Hopkinson.
Mr G. H. Hardy.
Prof. Sir J. Larmor.
Prof. Biffen.
Prof. Pope.
Mr R. H. Rastall.
Mr K. Lucas.

The following Communications were made:

1. Discussion of a difference equation relating to the tension of overhead wires supported by equidistant poles. By A. A. Robb, M.A., St John's College.

2. Note on the abnormal pair of appendages in *Lithobius*. By F. G. Sinclair, M.A., Trinity College.

3. A Class on Integral Functions. By J. E. Littlewood, B.A., Trinity College.

4. On the Scattering of the β-rays from Radium by Air. By J. A. Crowther, M.A., St John's College.

5. Note on the electrical behaviour of fluorescing iodine vapour. By R. Whiddington, B.A., St John's College. (Communicated by Professor Sir J. J. Thomson.)

6. On the reflection of sound at a paraboloid. By Rev. H. J. Sharpe.

7. On the emission of Röntgen rays from thin metallic sheets. By G. W. C. Kaye, B.A., Trinity College. (Communicated by Professor Sir J. J. Thomson.)

8. The emission of positive rays from heated phosphorus compounds. By F. Horton, M.A., St John's College.

November 8th, 1909.

In the Cavendish Laboratory.

Professor Bateson, President, in the Chair.

The following was elected a Fellow of the Society:

J. C. F. Fryer, B.A., Gonville and Caius College.

The following Communications were made:

1. Discontinuities in Light Emission. By N. R. Campbell, M.A., Trinity College.

2. The Shape of Beams of Canal Rays. By J. A. Orange, B.A., Trinity College. (Communicated by Professor Sir J. J. Thomson.)

3. The determination of solutions of the equation of wave motion which involve an arbitrary function of three variables which satisfies a partial differential equation. By H. Bateman, M.A., Trinity College.

4. On the Oscillations of Superposed Fluids. By H. J. Priestley. (Communicated by Mr W. Welsh.)

5. The stresses in a thick hollow Cylinder subjected to internal pressure. By L. B. Turner, B.A., King's College. (Communicated by Professor Hopkinson.)

6. The theory of the motion of a charged particle through a gas. By Professor Sir J. J. Thomson.

November 22nd, 1909.

In the Botany School.

The following were elected Fellows of the Society:

S. Brodetsky, B.A., Trinity College.
T. K. Shaw, B.A., Sidney Sussex College.

The following Communications were made:

1. Aldabra and neighbouring islands. (Illustrated with Lantern slides.) By J. C. F. Fryer, B.A., Gonville and Caius College.

2. Western Indian Ocean. (Illustrated with Lantern slides.) By Professor Stanley Gardiner.

3. Notes on the Larger Cetacea. By D. G. Lillie, B.A., St John's College. (Communicated by Mr A. E. Shipley.)

4. The continuations of functions defined by generalised hypergeometric series. By G. N. Watson, B.A., Trinity College.

5. On some general Properties of Mixed Solutions. By L. Vegard. (Communicated by Professor Sir J. Larmor.)

January 24th, 1910.

In the Optical Lecture Room.

Dr Hobson, Vice-President, in the Chair.

The following Communications were made:

1. On the velocities of diffusion of solutions of rubidium and caesium chlorides. By G. R. Mines, M.A., Sidney Sussex College.

2. Experimental Investigation as to Dependence of the Weight of a Body on its state of Electrification. By L. Southerns. (Communicated by Professor Sir J. J. Thomson.)

3. Note on an Attempt to Detect a Difference in the Magnetic Properties of the Two Kinds of Ions of Oxygen. By Miss D. B. Pearson. (Communicated by Professor Sir J. J. Thomson.)

February 7th, 1910.

In the Botany School.

Professor Bateson, President, in the Chair.

The following were elected Fellows of the Society:

D. Thoday, M.A., Trinity College.
R. H. Compton, B.A., Gonville and Caius College.
H. Hamshaw Thomas, B.A., Downing College.

The following Communications were made:

1. A note on some fossil plants from Newfoundland. (Illustrated by lantern slides.) By E. A. Newell Arber, M.A., Trinity College.

2. On the relation between the fossil Osmundaceae and the Zygopterideae. (Illustrated by lantern slides.) By W. T. Gordon. (Communicated by Mr E. A. Newell Arber.)

3. On a new species of *Physostoma* from the Lower Carboniferous of Pettycur (Fife). (Illustrated by lantern slides.) By W. T. Gordon. (Communicated by Mr E. A. Newell Arber.)

4. A note on *Cardiocarpon compressum* Will. By Mrs E. A. Newell Arber. (Communicated by Mr E. A. Newell Arber.)

5. On the assimilating tissues of certain Coal Measure plants. (Illustrated by lantern slides.) By H. Hamshaw Thomas, B.A., Downing College. (Communicated by Mr E. A. Newell Arber.)

6. Notes on the genus *Schizoneura* Schimper and Mougeot. By L. J. Wills, M.A., King's College. (Communicated by Mr E. A. Newell Arber.)

7. On the occurrence of *Schizoneura paradoxa* S. and M. in the Bunter of Nottingham. By R. D. Vernon. (Communicated by Mr E. A. Newell Arber.)

8. On petrified plant remains from the Upper Coal Measures of Bristol. By D. G. Lillie, B.A., St John's College. (Communicated by Mr E. A. Newell Arber.)

February 21*st*, 1910.

In the Comparative Anatomy Lecture Room.

PROFESSOR SEWARD, VICE-PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

A. B. Bruce, M.A., Peterhouse.
F. W. Dobbs, M.A., Trinity College.

The following were elected Associates:

R. T. Beatty, Emmanuel College.
W. T. David, Trinity College.
S. G. Lusby, Emmanuel College.

The following Communications were made:

1. Mimicry in Ceylon Rhopalocera, with some notes on the enemies of Butterflies. (Illustrated by lantern slides.) By Professor PUNNETT.

2. The Andaman Islands. (Illustrated by lantern slides.) By A. R. BROWN, M.A., Trinity College. (Communicated by Mr A. E. Shipley.)

3. The Procession and Pupation of the larva of *Cnethocampa pinivora*. By T. G. EDWARDS, B.A., Emmanuel College. (Communicated by Mr H. H. Brindley.)

4. On Double Sixes. By Professor W. BURNSIDE.

5. The solution of a system of Differential Equations occurring in the theory of radio-active transformations. By H. BATEMAN, M.A., Trinity College.

6. On the Change of Order of Integration in an improper repeated Integral. By Dr YOUNG.

7. The production of cathode particles by homogeneous Röntgen Radiations. By R. T. BEATTY. (Communicated by Professor Sir J. J. Thomson.)

8. On the scattering of rapidly moving electrified particles by matter and its application to determine the number of corpuscles in the atoms of the chemical elements. By Professor Sir J. J. THOMSON.

March 14th, 1910.

In the Cavendish Laboratory.

PROFESSOR BATESON, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

J. Romanes, B.A., Christ's College.
E. M. Wellisch, B.A., Emmanuel College.
L. J. Wills, M.A., King's College.

The following Communications were made:

1. On the cause of the phosphorescence of the glass in vacuum tubes when the pressure is not very low. By Professor Sir J. J. THOMSON.

2. Transmission of β-rays. By J. A. CROWTHER, M.A., St John's College.

3. Secondary X-Rays from Metallic Salts. By J. L. GLASSON. (Communicated by Professor Sir J. J. Thomson.)

4. Some experiments on Ionisation in dried Air. By S. G. LUSBY. (Communicated by Professor Sir J. J. Thomson.)

May 9th, 1910.

In the Chemical Laboratory.

PROFESSOR BATESON, PRESIDENT, IN THE CHAIR.

The President moved from the chair that in consequence of the lamented death of His Majesty King Edward VII, the Society do now adjourn, without transacting the business of the Meeting; and that the Papers put down for reading be postponed till Monday, May 23rd.

May 23rd, 1910.

In the Chemical Laboratory.

Dr Fenton, Vice-President, in the Chair.

The following was elected a Fellow of the Society:

C. E. Raven, B.A., Emmanuel College.

The following were elected Associates:

H. T. Swain, Emmanuel College.
T. Harris, Emmanuel College.

The following Communications were made:

1. The Resolution of Externally Compensated Bases into their optically active components. By Professor Pope and J. Read.

2. The Resolution of Dihydropapaverine. By Professor Pope and C. S. Gibson.

3. Further study of the Products of Chlorination of α-Picoline. By Dr Sell.

4. Formation of uric acid derivatives. By Dr Fenton and W. A. R. Wilks, B.A., Gonville and Caius College.

5. On Water of Crystallisation in Calcium Phosphate. By C. T. Heycock, M.A., King's College.

6. (1) The Diketopyrrolines and their analogues. (2) The formation of α- and γ-pyrones from acetylenic acids. By S. Ruhemann, M.A., Gonville and Caius College.

7. The reduction of nitrosyl chloride. By H. O. Jones, M.A., Clare College, and J. K. Matthews, B.A., Downing College.

8. Absorption of bromine by lime. By W. A. R. Wilks, B.A., Gonville and Caius College. (Communicated by Dr Fenton.)

9. Note on the adsorption of acids by carbohydrates. By F. Robinson, B.A. (Communicated by Dr Fenton.)

10. A hollow vortex in a polygon. By Sir George Greenhill.

11. On a dissymmetry in the emission of cathode particles excited by homogeneous Röntgen radiation. By R. T. Beatty. (Communicated by Professor Sir J. J. Thomson.)

12. Note on the transmission of β-rays. By J. A. Crowther, M.A., St John's College.

13. On the Mobilities of the Ions produced in Air by Ultra-Violet Light. By A. Ll. Hughes. (Communicated by Professor Sir J. J. Thomson.)

June 6th, 1910.

In the New Medical Schools.

PROFESSOR BATESON, PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. Further notes on the procession of *Cnethocampa pinivora*. (Illustrated by Lantern slides.) By H. H. BRINDLEY, M.A., St John's College.

2. The habits of *Musca domestica*. (Illustrated by Lantern slides.) By Dr GRAHAM-SMITH.

3. On the development of *Trypanosoma lewisi* in the Rat flea. By Dr N. H. SWELLENGREBEL and C. STRICKLAND, B.A. (Communicated by Professor Nuttall.)

4. The absorption of tetanus toxin. By Dr F. RANSOM. (Communicated by Mr W. E. Dixon.)

5. The fate of uric acid in the dog and rabbit. By HAROLD ACKROYD, M.B. (Communicated by Mr W. E. Dixon.)

6. The absence of living Tubercle Bacilli in old tuberculous lesions. By Dr COBBETT.

7. The action of potash salts taken by the mouth. By W. E. DIXON, M.A., Downing College.

8. The results of sterilization experiments on the Cambridge water. By Professor SIMS WOODHEAD.

9. On Accident in Heredity, with special reference to Right- and Left-Handedness. By F. J. M. STRATTON, M.A., Gonville and Caius College, and R. H. COMPTON, B.A., Gonville and Caius College.

10. On Right- and Left-Handedness in Barley. By R. H. COMPTON, B.A., Gonville and Caius College.

11. The development of *Gnomonia erythrostoma*, the cause of the Cherry Leaf Scorch disease. By F. T. BROOKS, M.A., Emmanuel College.

12. Jacobi's double-residue theorem in relation to the theory of point-groups. By Dr A. C. DIXON.

13. Discontinuities in Light Emission. II. By N. R. CAMPBELL, M.A., Trinity College.

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Cambridge:
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